Bernd Hamann, Stratovan 12 OCT. 2015

■ ISSUES RELATED TO SHAPE CHARACTERIZATION & CLASSIFICATION

1) MOMENTS

→ ZERNIKE MOMENTS can be used to characterize extracted/segmented regions in an image. Advantages: INVARIANT under TRANSLATION, SCALE, ROTATION.

→ IDEA: Moments are derived from "projections" of an extracted object's image function $f(x,y)$ onto specific classes of basis functions – e.g., polynomials in x and y.

Ex:

$$\text{moment}_{i,j}(x,y) = \int_{\substack{\text{region} \\ \text{of object}}} (x^i y^j) \cdot f(x,y)\, dx\, dy$$

(= inner products of $x^i y^j$ and $f(x,y)$)

• NOTE: Consider relationships between

- Zernike moments,
- spherical harmonics, (= basis functions)
- wavelets etc. / vanishing moments of wavelets.

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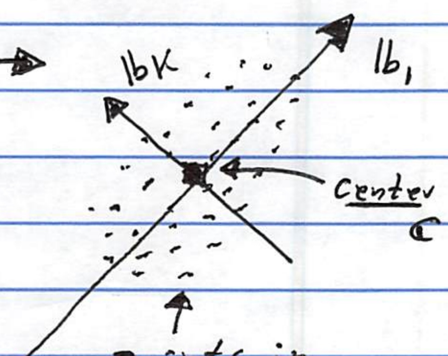
2) SPECTRAL ANALYSIS

→ Consider the application of FOURIER or WAVELET analysis to an extracted 3D object/segment – using the resulting coefficients (at varying scales) as object "signature".

3) BEST BASIS / PRINCIPAL COMPONENTS ANALYSIS / Karhunen-Loève Transform

→ 

points in high-dimensional "signature space" – representing tuples given by the K attributes stored for a voxel belonging to a specific material class

■ Context:

→ $\{\mathbb{b}_1, \dots, \mathbb{b}_K\}$ = best orthogonal basis of a high-dim. point cloud defing attribute tuples of voxels of specific material type.

→ Center $\mathbb{C}$ and associated best ortho basis for each point cloud/material type should be distinct – and allow for the classification of voxels.

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4) **BEST APPROXIMATION** of a function $f(x,y)$ or $f(x,y,z)$ by a set of basis functions $f_1,\dots,f_K$ spanning the linear space of approximating functions $f_{app}=\sum_{i=1}^{K} c_i f_i$:

$$\begin{pmatrix} \langle f_1, f_1 \rangle & \dots & \langle f_1, f_K \rangle \\ \vdots & & \vdots \\ \langle f_K, f_1 \rangle & \dots & \langle f_K, f_K \rangle \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_K \end{pmatrix} = \begin{pmatrix} \langle f, f_1 \rangle \\ \vdots \\ \langle f, f_K \rangle \end{pmatrix}$$

→ Solving this linear system for the unknowns $c_1,\dots,c_K$ will define the BEST-POSSIBLE approximation f_{app} of f. The error (L_2-norm) E would be

$$E = \| f_{app} - f \| = \int_{volume} (f_{app}-f)^2 \, dx\,dy\,dz$$

→ CONTEXT: Again, an extracted object in 3D space defines an image intensity function $f(x,y,z)$ over the object's region/volume; expanding this function in terms of "well-chosen" basis functions $f_1,\dots,f_K$ could lead to material-specific coefficients $c_1,\dots,c_K$.

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5) Relationship to CLASSIFICATION

→ IDEA:

(i) For a specific object/material/image "class" determine a class-specific, class-dependent best basis;

(ii) determine such a basis for a "relatively lower-dimensional function space" in which certain members of a class could be represented/approximated well (with small error);

(iii) when the best (-possible) approximation of a newly extracted object/material must be classified, compute the approximation error relative to all known/stored material classes to decide whether it is "close" to any of the known/stored classes.

→ **NOTE**: "Best approximation" / "best basis" should be considered both in the "physical image space" (2D, 3D, 4D) and the "high-dim. signature space."

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