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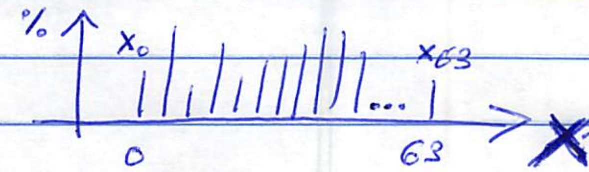
# THOUGHTS: HISTOGRAMS, POINTS, & MULTI-DIM. ANALYSIS

## 1) Histograms ⟺ Points

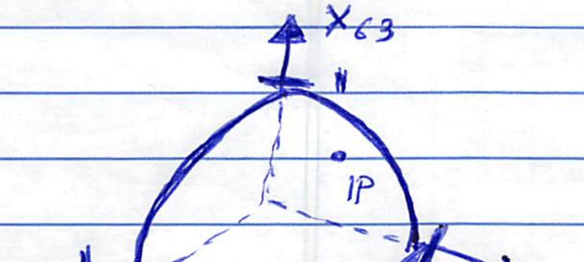



$\Leftrightarrow$ Point $p = \begin{pmatrix} x_0 \\ \vdots \\ x_{63} \end{pmatrix}$

64 bins

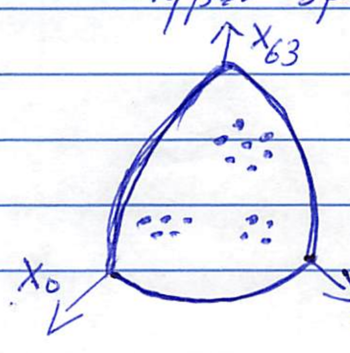
- Possibility for Normalization:
  normalize $p$ s.t. $\|p\| = \sqrt{p^T \cdot p} = 1$

- Can view / understand all histograms / points as locations on unit hyper-sphere in 64 dimensions:




All $x_i$: $0 \le x_i \le 1$

- Very similar histograms should define point clusters on this hyper-sphere:




Here: 3 clusters (= material classes)

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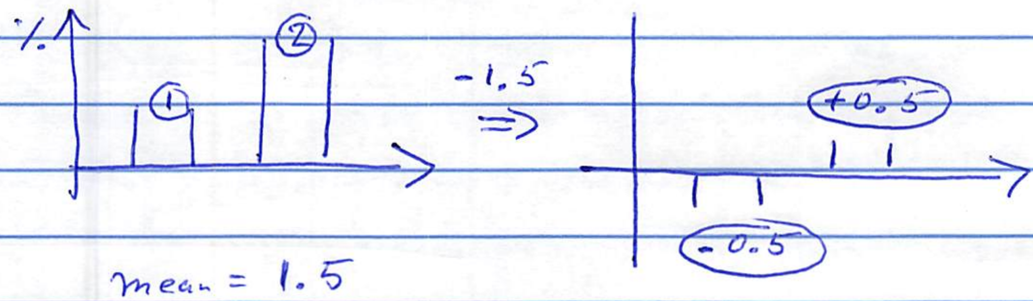
2) <u>Issues:</u> Should perform MEAN-SUBTRACTION?

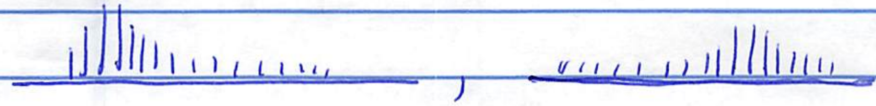



mean = 1.5

⇒ Obtain points on the entire 64-dim. hypersphere - not just the "positive hyper-octant"...

⇒ Distance of points on the hyper-sphere = geodesic distances ...

⇒ "Quite different histograms (and points) might still represent same material. - How to "recognize" this?



2 different histogram
- but SAME material
(obtainable via shifting + scaling)

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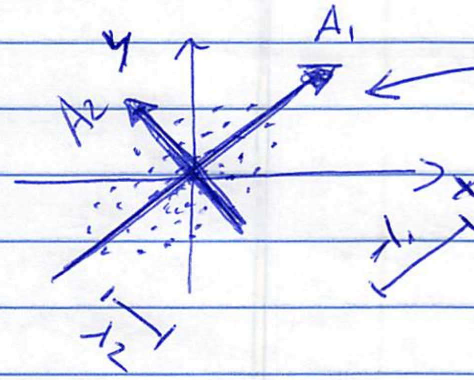
## 3) CONSIDERATIONS CONCERNING GEOMETRY & MATRIX ALGEBRA:

E.g., point set in 2D plane with mean = 0:




Points $p_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}$, $i = 1 \ldots N$

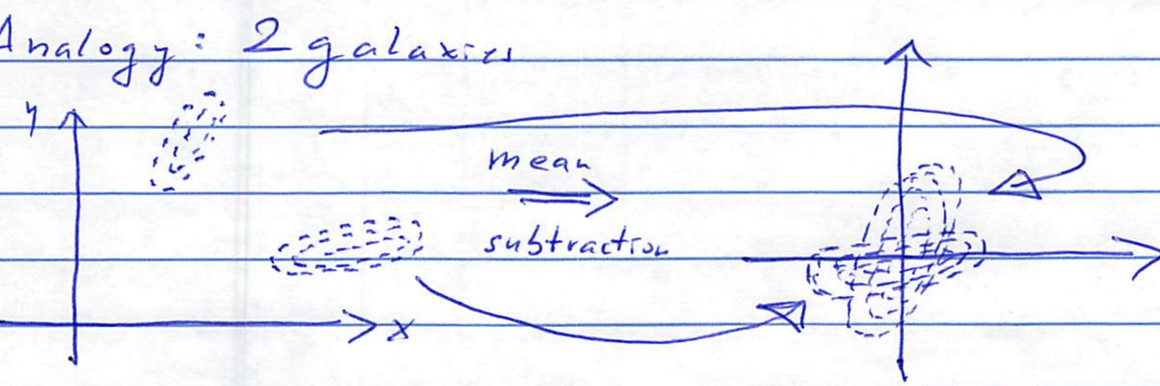
$$\Rightarrow COV = \begin{bmatrix} x_1 & \cdots & x_N \\ y_1 & \cdots & y_N \end{bmatrix} \begin{bmatrix} x_1 & y_1 \\ & \vdots \\ x_N & y_N \end{bmatrix}$$

$$= \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

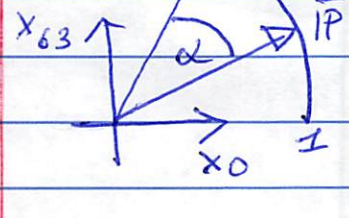
$$\Rightarrow \lambda_1, \lambda_2, A_1, A_2$$

"principal components"

- Analogy: 2 galaxies




## 4) DISTANCE ON HYPER-SPHERE (NORMALIZED POINTS) VIA "GEODESIC / DOT PRODUCT DISTANCE":




$$\boxed{\cos(\alpha) = p \circ \bar{p}}$$

$\alpha \in [0°, 180°]$

$\cos \in [-1, +1]$

$\Rightarrow dist(p, \bar{p}) = \cos(\alpha)$

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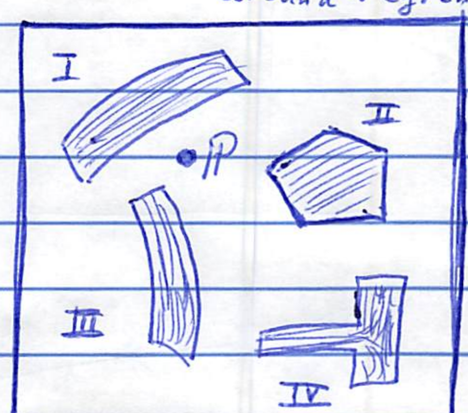
■ **THOUGHT**: "Representing Signatures in High-dim. Space as **B-SPLINE Solids**"
[discussion with Ben Gregorski]

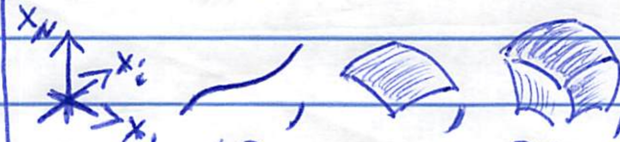
① Determine dimensionality of manifold (defining a valid signature) that is embedded in the high-dim. signature space defined by all signature attributes/dimensions.

• **ILLUSTRATION:**
"2D signature space with embedded 2D manifold regions (= signature regions)" + EPSILON tolerance around regions

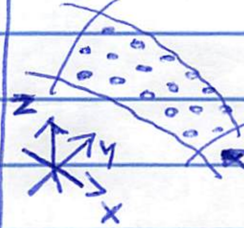



⇒ To what signature region does P belong to?
⇒ Need QUICK inside-/outside-test!
→ USE LOCAL CONVEX HULL PROPERTY!

② Define the manifolds/signature regions analytically as continuous B-spline solids. The solids can be 1D/2D/3D curve/surface/volume solids (or higher-dim. solids):




**Ex:** 3D signature points of 1 material defines 2D surface/manifold in 3D space




Signatures/signature points on 2D sheet.

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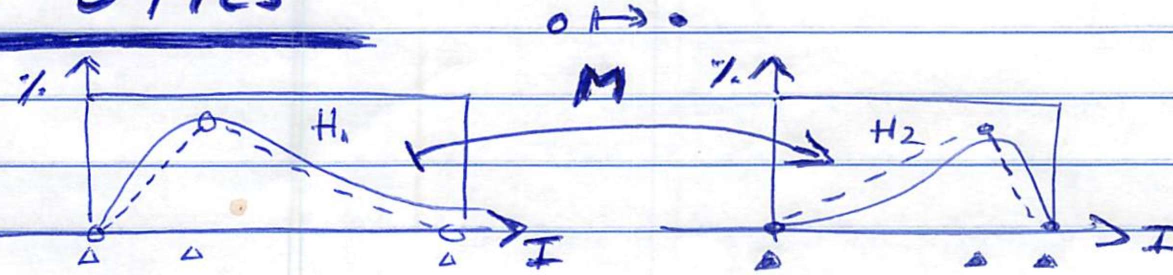
- **TOPICS**

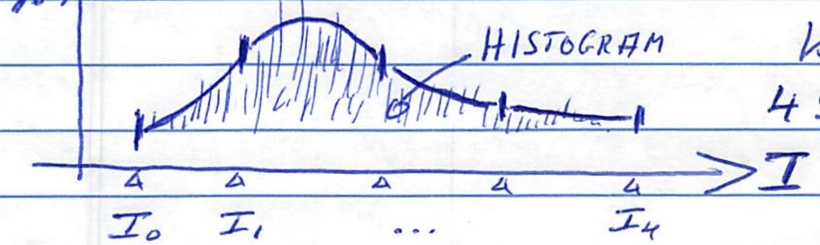



⇨ FIND AN OPTIMAL PIECEWISE LINEAR MAPPING M !
[NEED "LANDMARKS" ?!]

⇨ DEFINE AN "ENERGY TERM" MEASURING THE "ENERGY" NEEDED TO MAP HISTOGRAM $H_1$ TO $H_2$ ; HOW LARGE CAN THE ENERGY VALUE BE?

! ⇨ USE MULTI-RES. [WAVELETS] THROUGHOUT! → SPECTRAL ANALYSIS!

⇨ MORE GENERAL: COMPUTE OPTIMAL (=LEAST SQUARES) B-SPLINE APPROX. FOR $H_1$ & $H_2$ AND COMPARE — E.G., CUBIC B-SPLINE WITH K UNIFORMLY SPACED KNOTS...




K = 5
4 SEGMENTS

⇨ USING THIN-PLATE SPLINE FOR M ?
[=ENERGY-MINIMIZING]

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