BERND HAMANN — BH

STRATOVAN

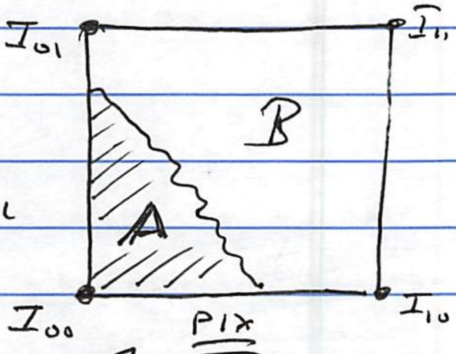
BOUNDARIES

① 12/7/2015

- 2D Case: → ONLY NEEDED
  - ① THERE EXISTS A BOUNDRY.
  - ② FRACTIONS OF MATERIALS

[NO NEED FOR INTERFACE GEOMETRY]

[Bonnell et al., Dillard et al. – POTTS model, ...]



◦ "Two-material case"

← Pixel with intensity I being the "mixture" / weighted average of intensity for "pure A" and "pure B":

$$I_{PIX} = w_1 \cdot I_A + w_2 \cdot I_B,$$

(where $w_2 = 1 - w_1$)

$$= w_1 \cdot I_A + (1 - w_1) I_B,$$

$$(0 \le 1 \le w_1)$$

$$= w_1 (I_A - I_B) + I_B$$

$$\Rightarrow \boxed{w_1 = \frac{I_{PIX} - I_B}{I_A - I_B}}$$

- **NEED DEF:**

  A PIX IS A BOUNDARY PIX :⟺ THE LENGTH OF ∇ Bilin $(I_{00}, I_{10}, I_{01}, I_{11}) = \|\nabla BL(x,y)\| > \varepsilon$

- Extreme Cases: (i) most COMPACT object/material geometry
  (ii) resolution-scale THIN object/geometry

(i)

⇐ boundary pixels = minimal in number

(ii)

⇐ boundary = entire pixel set = maximal in number

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BOUNDARIES ...

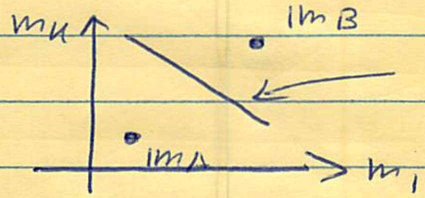
- Two materials $\Rightarrow$ 2 feature vectors $\mathbb{m}^A, \mathbb{m}^B$:

$$\mathbb{m}^A = (m_1^A, \dots, m_k^A), \quad \mathbb{m}^B = (m_1^B, \dots, m_k^B)$$

PERFECT 'PROTO' DEFS. OF A & B!

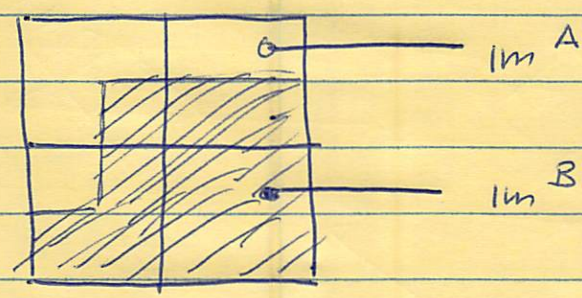
– m_i-values: intensity, variance, ...

$\Rightarrow$ Can separate A & B in K-dim feature space:



bisector separating A & B in K-dim space

- Ex:



$\Longleftrightarrow$

$(\frac{3}{4}, \frac{1}{4})$	$(\frac{1}{2}, \frac{1}{2})$
$(\frac{1}{4}, \frac{3}{4})$	$(0, 1)$

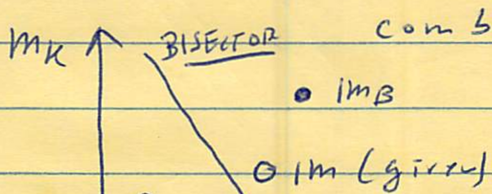
material fractions w^A, w^B; $\underline{w^A + w^B = 1}$

- GIVEN IN PRACTISE:

ONE CELL WITH ONE $\mathbb{m}$-tuple (for its center): $\boxed{\mathbb{m}}$

$\Rightarrow$ SITUATION: $\mathbb{m}$ is NOT a linear combination of $\mathbb{m}^A, \mathbb{m}^B$ in K-space!

$$\mathbb{m} \neq \alpha \mathbb{m}_A + \beta \mathbb{m}_B$$

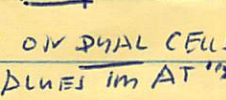


THIS BISECTOR IN K-SPACE DEFINES THE BOUNDARY OF A & B IN CELL GIVEN!

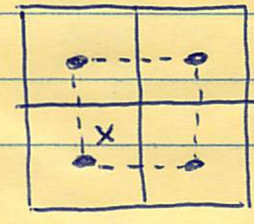
! "Where is this bisector in the cell with value $\mathbb{m}$?"

- PROCEDURE:

"CELL SUBDIVISION":



$\Rightarrow$ BILINEAR INTERPOL ON DUAL CELLS TO OBTAIN VALUES $\mathbb{m}$ AT "x":



---- DUAL CELL

x-value = BiLin (4 •-values)

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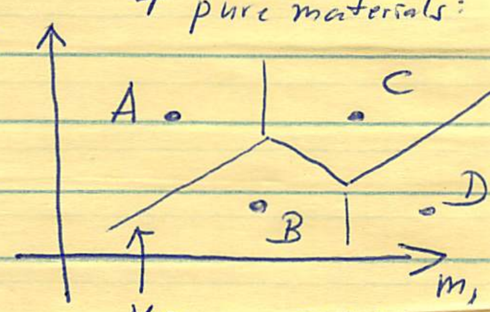
■ BOUNDARIES...

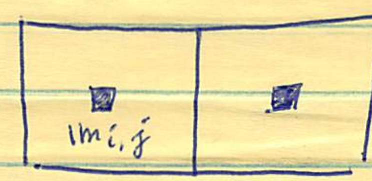
1) "Proto-type"/pure materials are defined by K-tuples determining their feature vectors:

4 pure materials:



VORONOI DIAGRAM

2) The original voxel data define/have voxel-centered K-tuples associated with them (NOT defining a pure material):



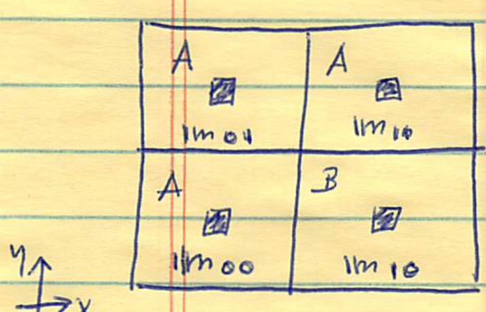
3) BOUNDARY DEF: For each cell/$\mathrm{im}_{i,j}$ determine to which of the pure materials (A, B, C, D, ...) it is closest; 2 cells sharing either an edge or a single vertex contain a boundary (surface) if the associated tiles into which their im-values lie are different.

⟹ SPLIT such cells and determine material type/association of child cells:

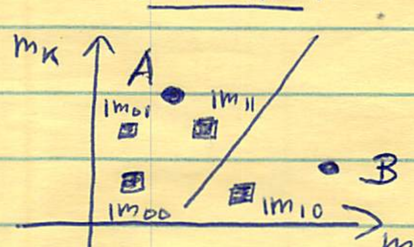
4) SPLITTING – EXAMPLE OF 2-MATERIAL CASE:

PURE MATERIAL DIAGRAM:

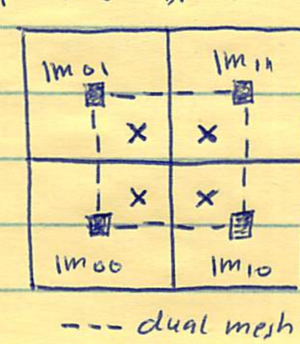
USE im_{ij} values on dual mesh for interpolation onto subdivision points "x":



given cells with associated K-tuples @ centers



im_{10} is "type B"
⟹ where is the boundary between A & B in x-y-space?



--- dual mesh

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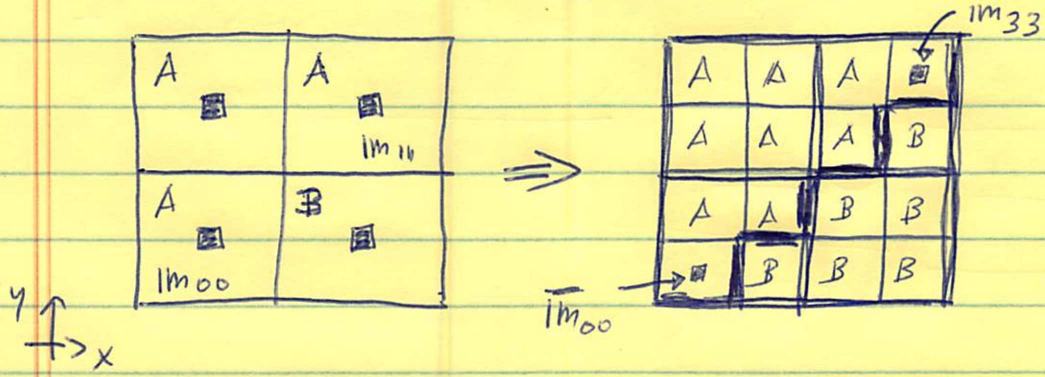
▪ BOUNDARIES...

5) RESULTING SUBDIVISION MESH AND INTERPOLATED im-VALUES:



⇒ Look up material type A or B for all $\overline{im}_{ij}$

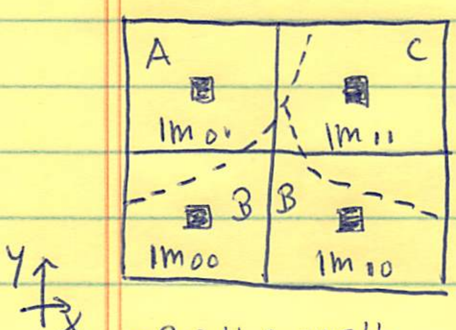
⇒ Labels for new subdivision cells

6) TERMINATION: Stop subdividing after M subdivision steps; boundary of 2 material types implied when edge shared by 2 cells has 2 different types on its 2 sides.

⇒ OBTAIN VOLUME ESTIMATE of regions occupied by type A (B) by summing up volumes of all respective type-A (type-B) cells.

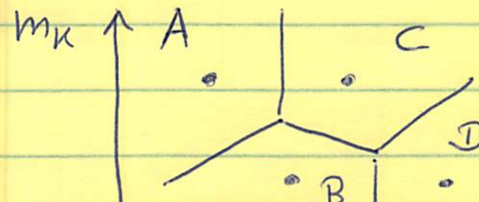
▪ GENERAL BOUNDARY-MAPPING PROBLEM:

- Physical xy (xyz)-space:



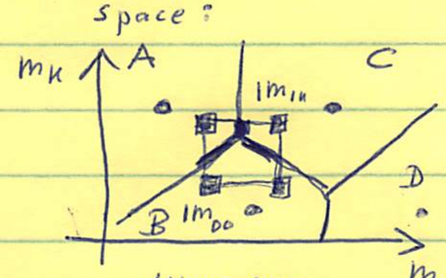
– cells with cell-centered material K-tuples

- K-dim. feature space of 4 pure materials:



– Voronoi dia. of 4 pure types

- im_{ij} tuples in feature space:



– MUST MAP BOUNDARY DEFINED IN K-DIM. FEATURE SPACE TO PHYSICAL SPACE! USING:

$im = w_A im_A + \ldots + w_D im_D$;

$\to w_A + \ldots + w_D = 1$; $w_A, \ldots, w_D \geq 0$