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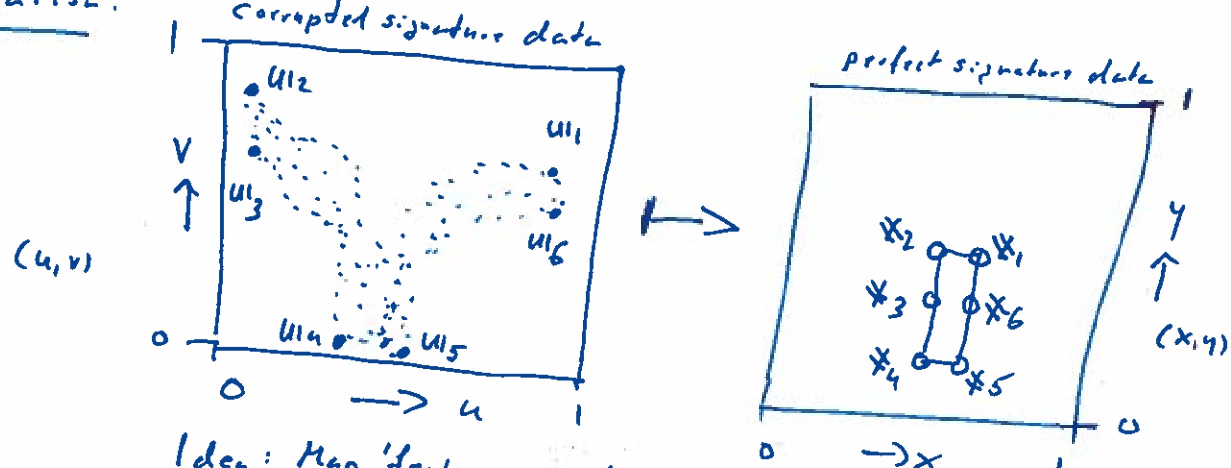
3/14/16

- Possibilities to Map Distributions (of Signature Data)

→ "Goal": Define a mapping that 'maps back' a recorded distribution of k-dimensional signature data to its 'perfect location'.
ALSO: Want to define such a mapping for just a small no. of signature dimensions - e.g., 2.

- Ex: Select 2 dimensions of recorded/imaged signature data of the same material imaged under different conditions, - and map that 2-d distribution to the 'ideal/perfect' distribution/location in 2-d space...

- Illustration:



Idea: Map 'feature points' from (u,v)-space to corresponding points in (x,y)-space

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$\Rightarrow$ Use a '<u>relatively low-degree</u>' polynomial for the mapping:

LINEAR SYSTEM

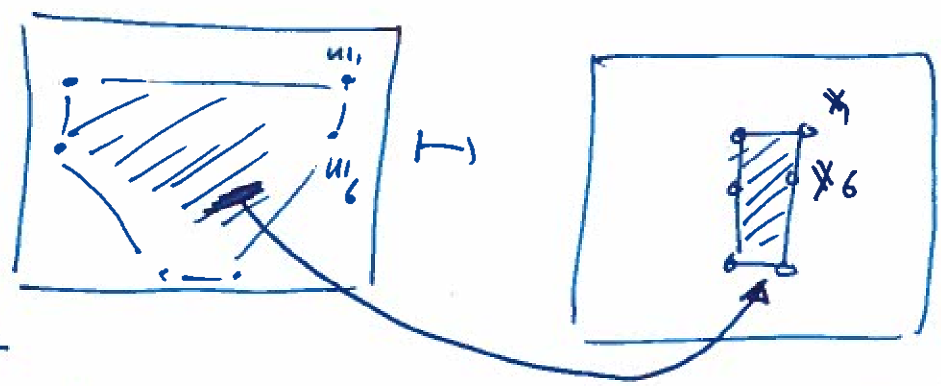
$$\begin{cases} \mathbb{X}_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = C_{00} + C_{10} u_1 + C_{01} v_1 + C_{20} u_1^2 + C_{11} u_1 v_1 + C_{02} v_1^2 \\ \qquad = \sum\limits_{\substack{i+j \le 2 \\ i,j \ge 0}} C_{i,j}\, u_1^i v_1^j \\ \quad \vdots \\ \mathbb{X}_6 = \begin{pmatrix} x_6 \\ y_6 \end{pmatrix} = \sum C_{i,j}\, u_6^i v_6^j \end{cases}$$

$$\Rightarrow \quad \boxed{\mathbb{X}_k = \sum C_{i,j}\, u_k^i v_k^j}, \quad k = 1 \ldots 6$$

$\Rightarrow$ determine the unknown coefficients $C_{i,j} = \begin{pmatrix} c^u_{i,j} \\ c^v_{i,j} \end{pmatrix}$.

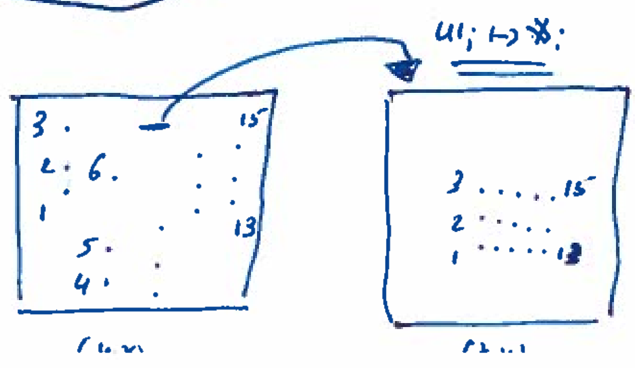
$\Rightarrow$ The mapping $(u,v) \mapsto (x,y)$ actually defines a '<u>continuous</u>' mapping for the (u,v)-plane to the (x,y)-plane:

(u,v)



(x,y)

• <u>OR</u>: Could <u>AUTOMATICALLY(!)</u> Generate <u>MANY</u> corresponding point pairs $(u,v)_i$, $(x,y)_i$ and determine a <u>LEAST-SQUARES</u> low-degree polynomial for a mapping:



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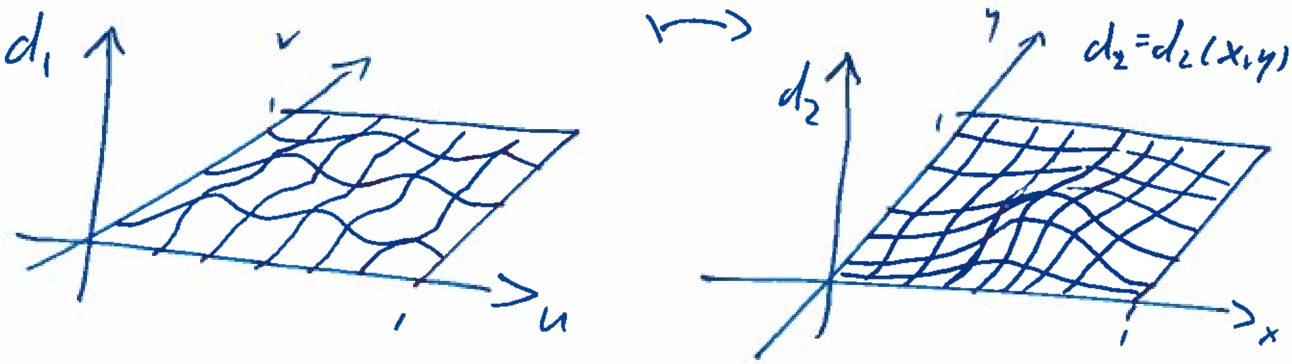
• Different Viewpoint

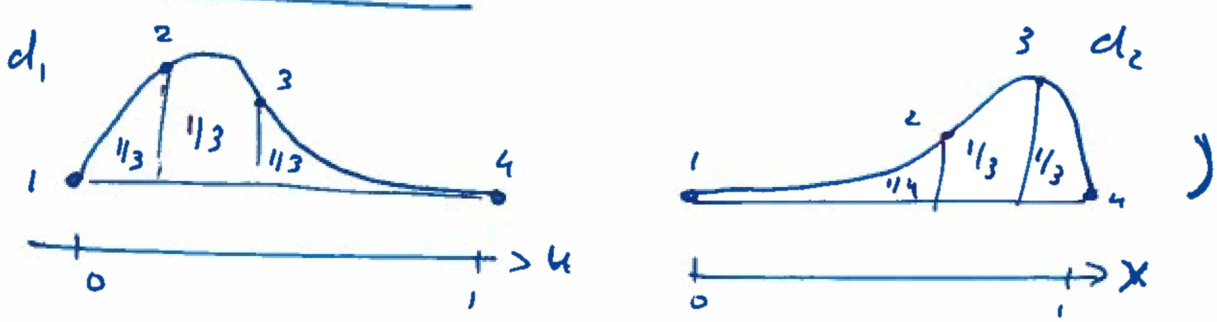
→ View the problem as a CONTINUOUS problem where one continuous distribution/density must be mapped to another



Corrupted/imaged

Perfect/Ideal

⇒ Use an INTEGRAL-based method to map d_1 to d_2 (similarly to mapping 2 1-d histograms to each other via integral-based correspondences:



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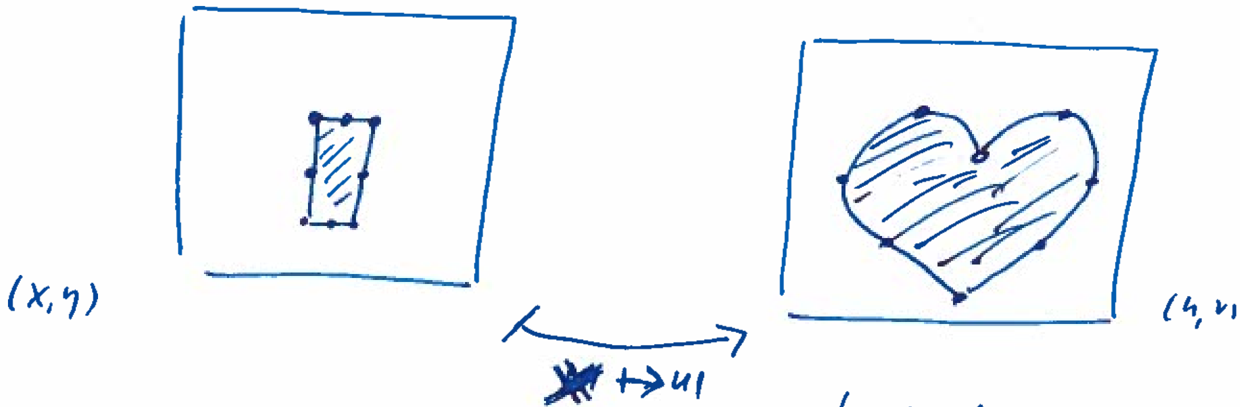
- Can "insight" be gained by defining/understanding the mapping from ✱ (predicted) to ui (imaged)?

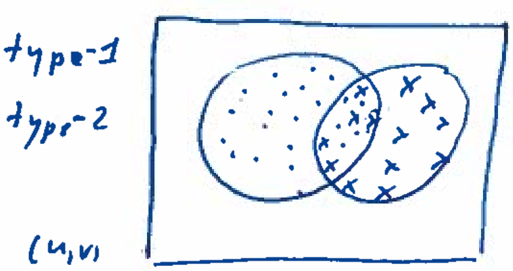


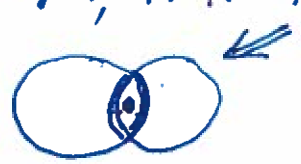
helpful mapping ?

- CONTEXT: The ideal/correct signature data of 2 materials "do not overlap" in (x,y)-space – but they 'do overlap in (u,v)-space'?

QUESTION: Can a mapping from (u,v)- to (x,y)-space be used to RESOLVE THE AMBIGUITY in (u,v)-space??? WHEN A (u,v)-tuple lies in overlap region, is it type-1 or type-2???

- • type-1
- × type-2





"Map (u,v)-tuple • to (x,y)-space using ALL mappings for all material types; minimal "energy"-mapping

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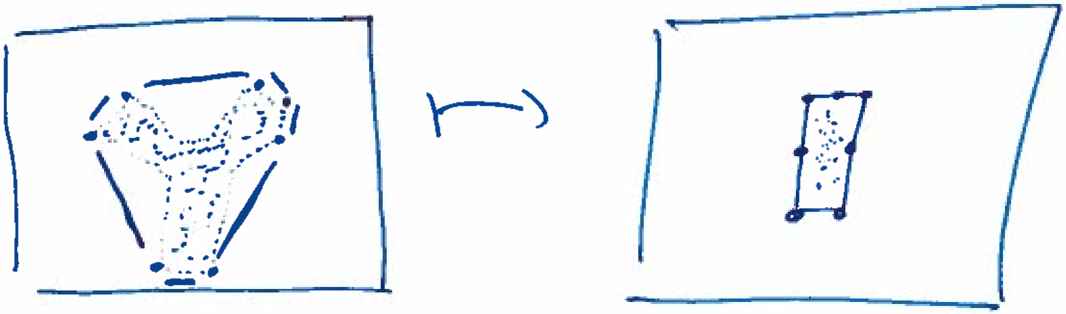
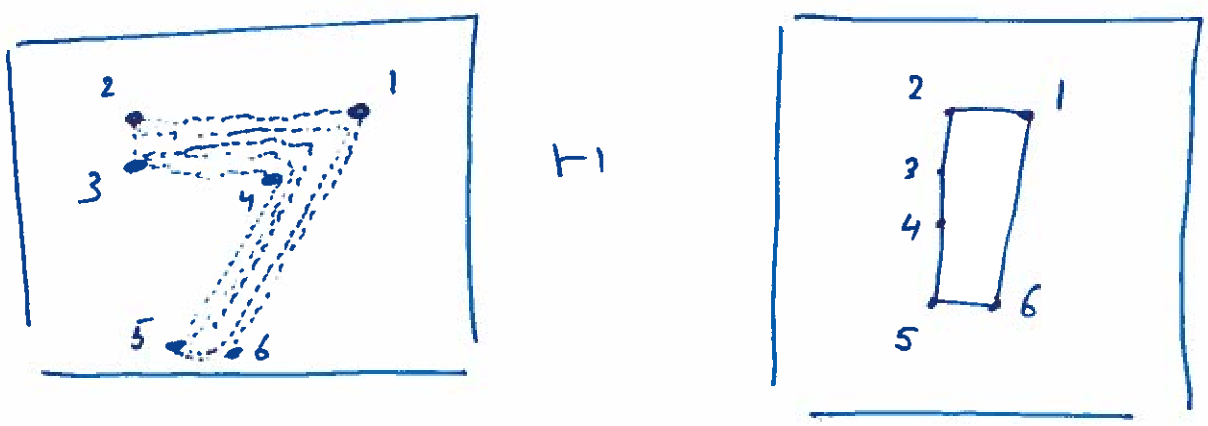
• CONVEX HULL HELPFUL?



(u,v) (x,y)

• CONFORMAL MAP?

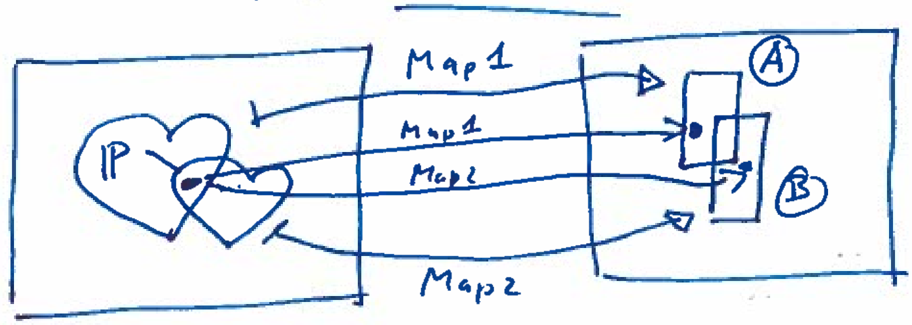
"Map points on non-convex outlines of boundaries to each other".



(u,v) (x,y)

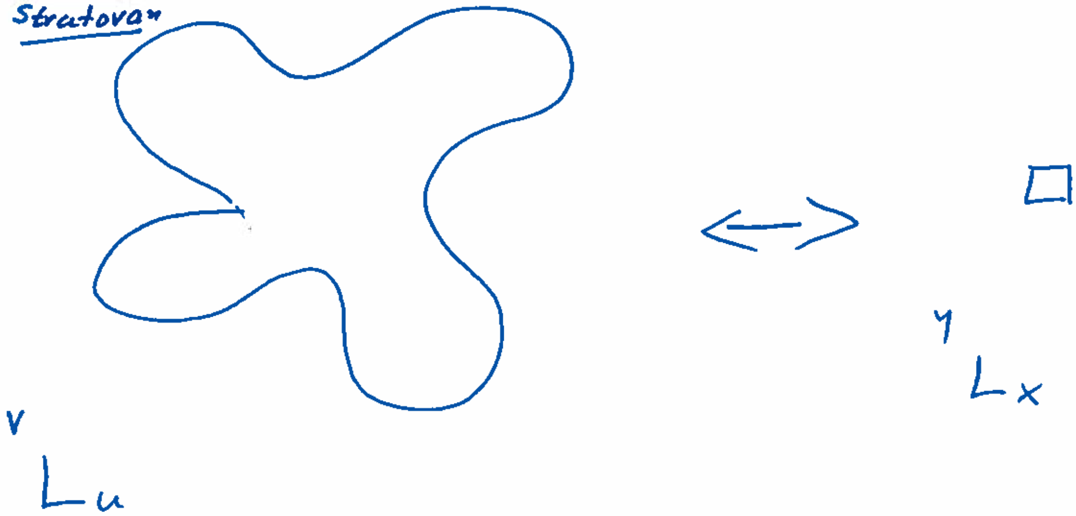
• Must resolve the AMBIGUITY:

(u,v)

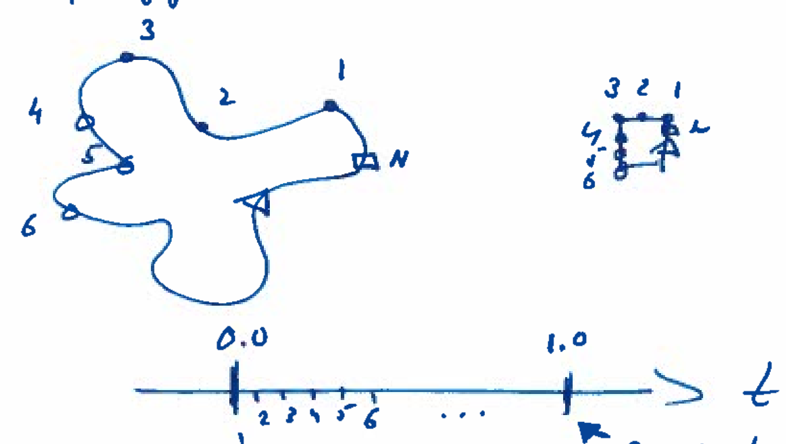


(x,y)

P maps to A and B
=> WHAT TYPE IS IT?



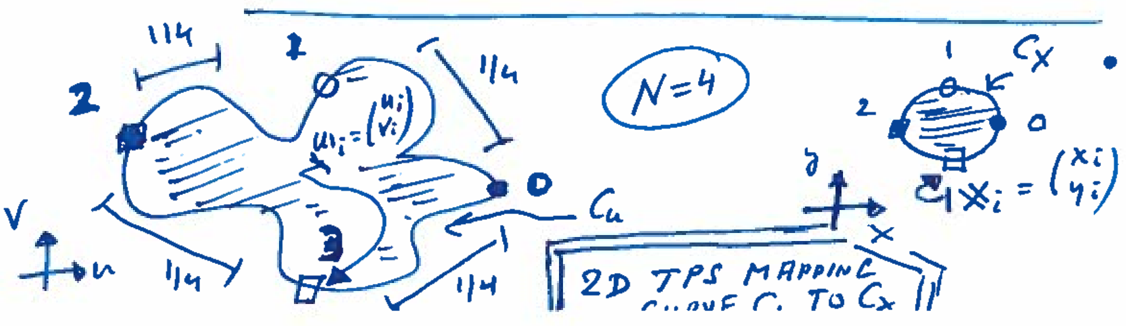
- chord-length parametrize the boundary polygons
- establish AUTOMATIC correspondence between points on outline polygons.



normalized arc/chord length

• **Map Closed Curve to Circle**

→ Correspondence via arc length.

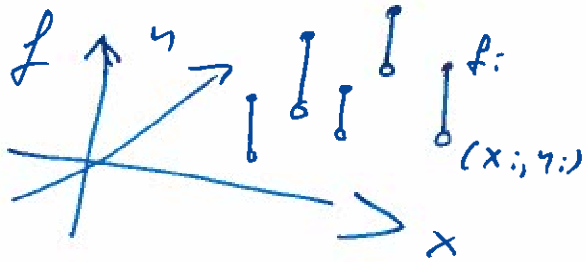


• More typical: $N = 100$

$u_i \leftrightarrow x_i,\quad i = 0 \ldots N-1$

Using Standard TPS for Mapping

- **Review:** TPS (Scattered Data Setting)



$$\left\{ \sum w_i = 0, \quad \sum w_i x_i = 0, \quad \sum w_i y_i = 0 \right\}$$

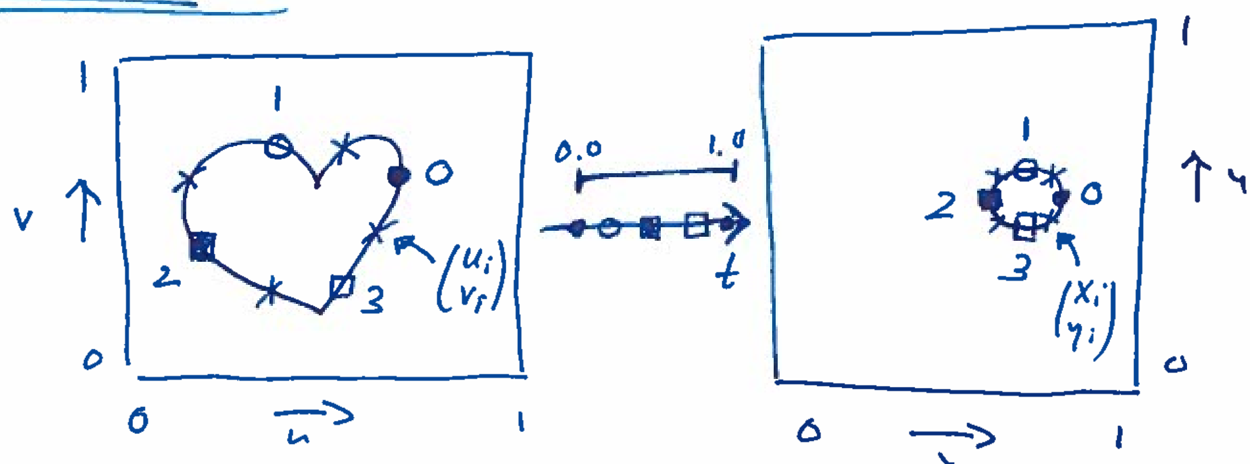
- TPS = $f(x,y) = a + bx + cy + \sum_{i=1}^{N} w_i \cdot f_i(x,y)$

$$f_i(x,y) = d_i^2 \cdot \log(d_i)$$

$$d_i^2 = d_i^2(x,y) = (x - x_i)^2 + (y - y_i)^2$$

- Compute coefficients a, b, c, w_i
  s.t. function $f(x,y)$ interpolates the given function values.

- **Our Setting:**



– Points are UNIFORMLY distributed on the 2 curves.
– 2 points 'correspond' when they have the same t-value
– Compute the 2 TPS FUNCTIONS:

$x = x(u,v)$, $y = y(u,v)$

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