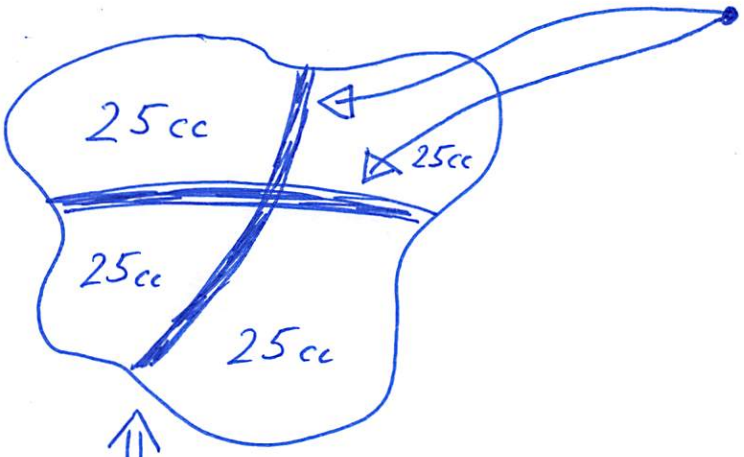
"Object = Multi-Fragment Object"

→ Issue: Volume of object estimated well!

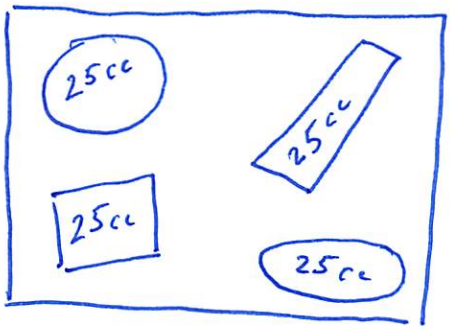
- Illustration:



These regions could be:
- background
- noise
- streaks
- gaps
- "texture" ...

What interpretation is right?
(i) 4 objects, each of 25cc ???
(ii) 1 object, of 100cc ???

- Illustration:



"CLEAR Situation":
→ 4 objects, each of 25cc

Why?

"The DISTANCE between [the centers of] the 4 regions/objects is > ε, considering all mutual pairwise distances."

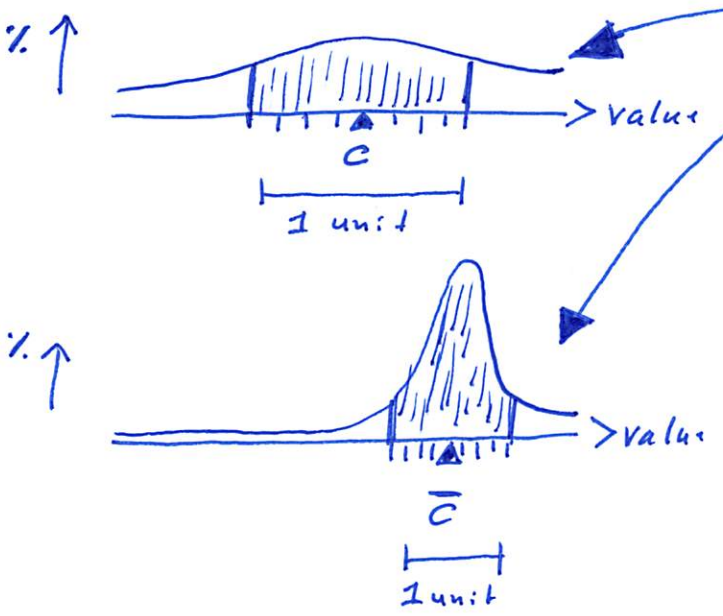
- IDEA:
→ CONSIDER COMPLETE GRAPH OF ALL REGION/OBJECT CENTERS TO DETERMINE WHICH REGIONS BELONG TO SAME OBJECT!

■ DISTANCE METRIC INHERENT TO A DATA SET

[Paper: Feng, Hotz, Hamann, Joy "Anisotropic Noise Samples"]

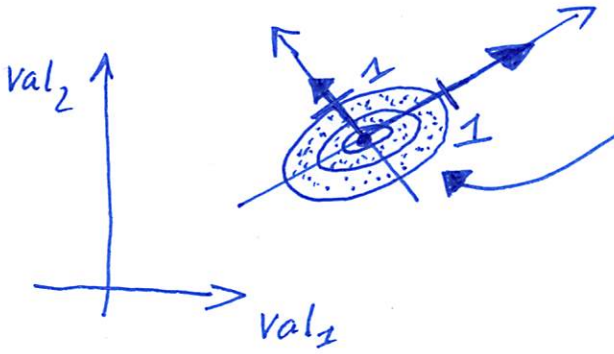
→ A point set, distribution [function] implies a "metric tensor" that is data/function-specific !

• Illustration:



- Two different distributions / histograms: Defining inherent "UNIT"
- RELEVANCE: MUST DETERMINE DISTANCE of a specific value v to some center value c (c̄) of a specific distribution

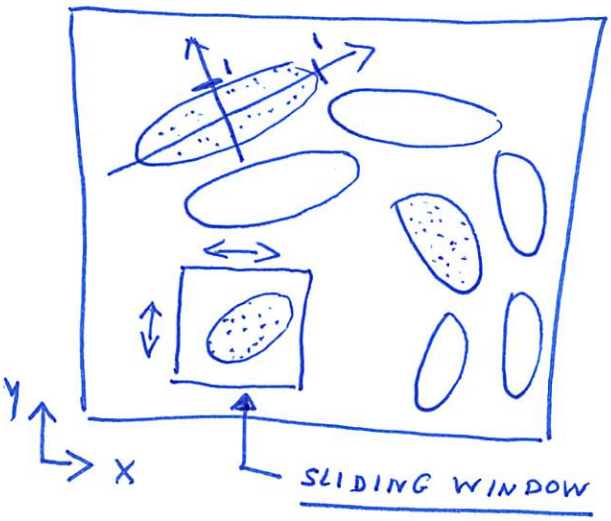
• 2D case: POINT SET'S INHERENT METRIC IMPLIED BY PCA



PCA → 2 eigenvalues, 2 eigendirections / -vectors
→ point set-implied meaning of UNIT
→ definition of DISTANCE based on PCA-induced metric [tensor]

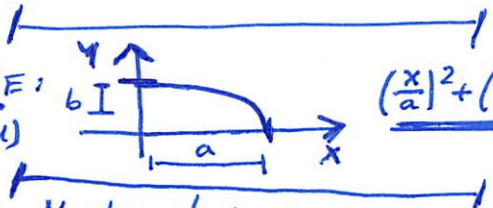
Metric, Metric Tensor, Distance etc.

- Example: Point set/distribution in Plane
  - → Consider point sub-sets in local neighborhoods to establish local PCA-based coord. systems/metrics



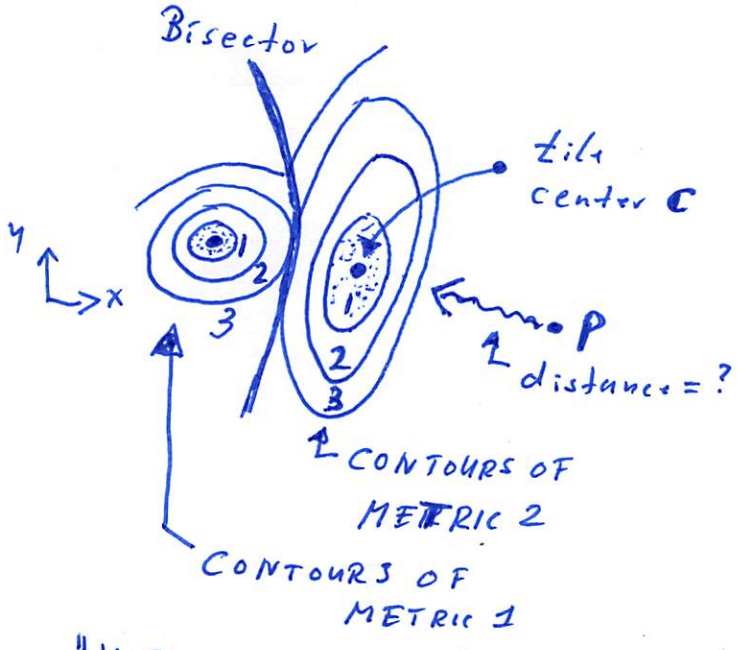
→ Use 'sliding window' to move across xy-plane; determine local point set/distribution in each window; determine local PCA-based coordinate system and metric.

→ ELLIPSE: (standard)



$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=1$$

- Consider local metrics to establish families of metric-induced DISTANCE-CONTOURS



"VORONOI DIAGRAM-LIKE STRUCTURE"

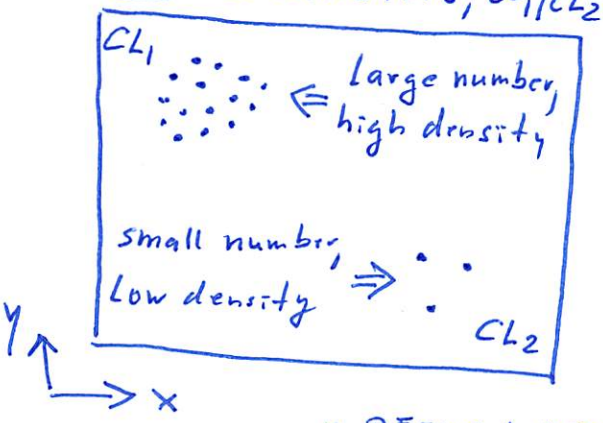
→ • Metric/distance measure varies (smoothly or discontinuously) across xy-plane!

- DISTANCE COMPUTATION:
  Given a point P, determine to which tile center C is closest – using the center-/tile-specific metric induced by local point distributions.
- TWO RELEVANT QUESTIONS ANSWERED:
  (i) To which center C/point set is point P closest to?
  (ii) To which point inside C's tile is point P closest to?

Distance, Clustering, Errors etc. (cont'd.)

→ Issue: Considering CLUSTERS and the number of points associated with them and density of points, should number and density influence the distance measure used to determine to what cluster a (new) point is closest to?

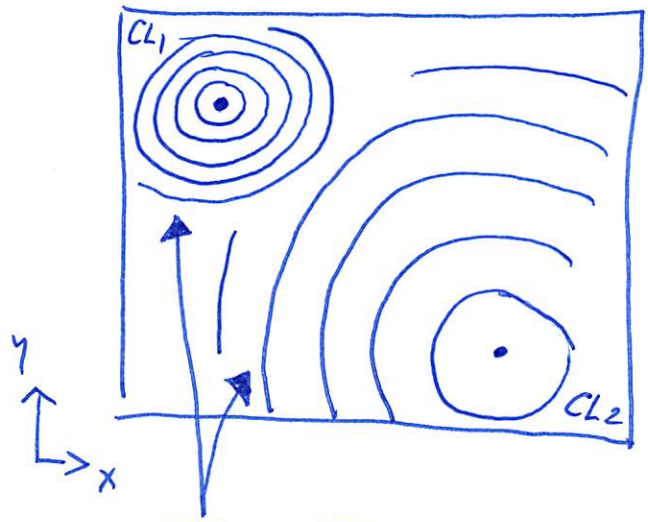
Ex: 2 clusters, CL_1/CL_2



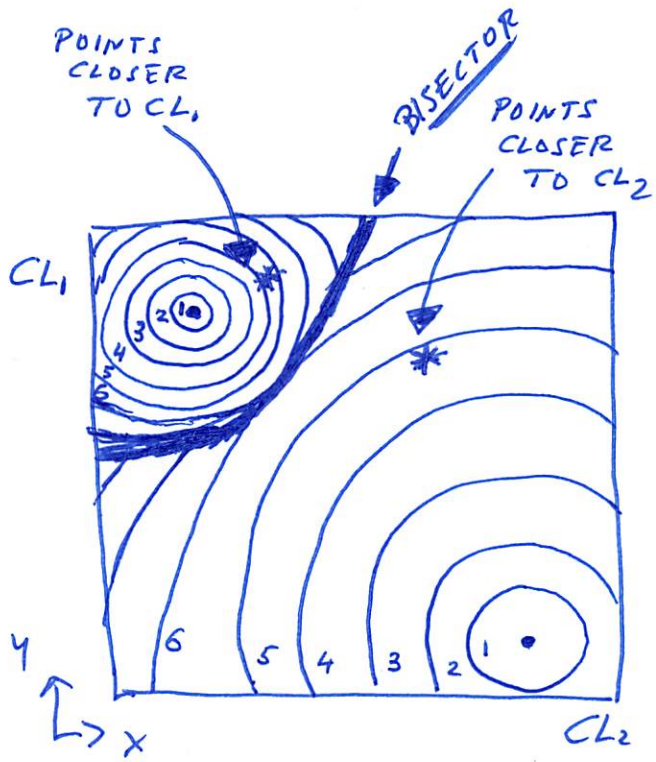
⇒ GOAL:

Define a distance metric that "properly considers" point number and density of all clusters! How?

DEFINE A METRIC (TENSOR) THAT COULD HAVE THESE DISTANCE-ISO-LINES:



ISO-LINES OF CONSTANT DISTANCE VALUES RELATIVE TO CL_1 AND CL_2.

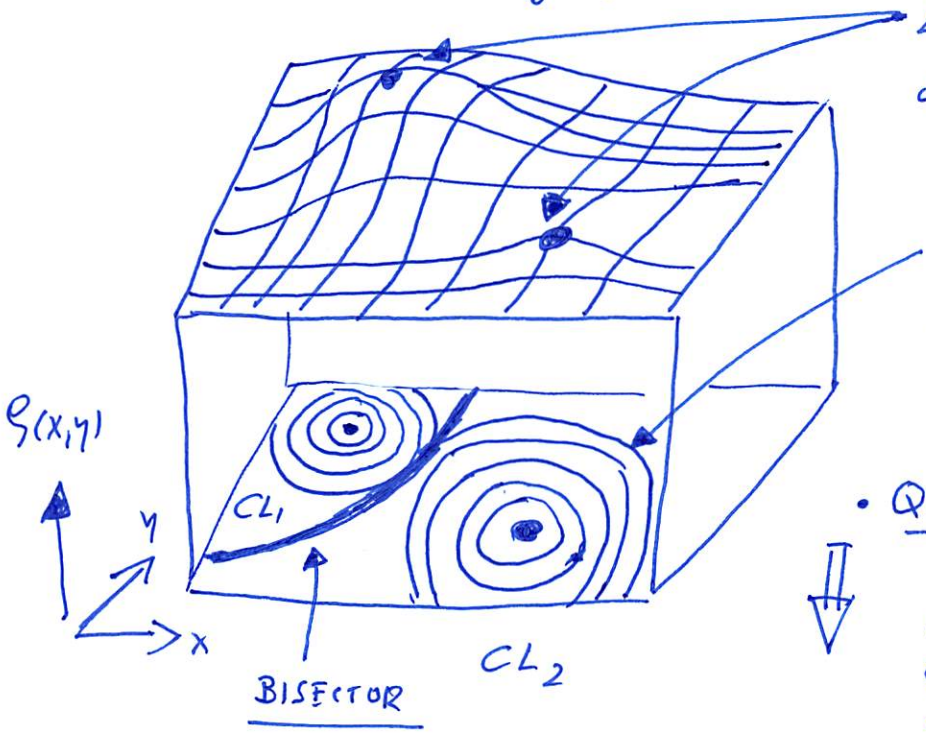


⇒ POINTS ON BISECTOR HAVE EQUAL DISTANCE TO CL_1 & CL_2.

■ Distances... (cont'd.)

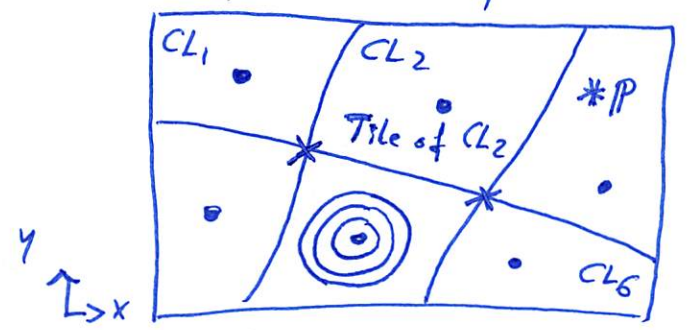
⇒ Is it possible to use the probability density function of the set of all points to establish a "separatrix structure" based on the critical points of the density function? Is the BISECTOR related to separatrix structure?

Ex: Viewing a 2D density function as a height field:



- Question: Can one use the tile and separatrix structure implied by the critical points of the density function to define a 'good' distance metric?

⇒ Consider SEPARATRIX structure of a bivariate density function in xy-plane:



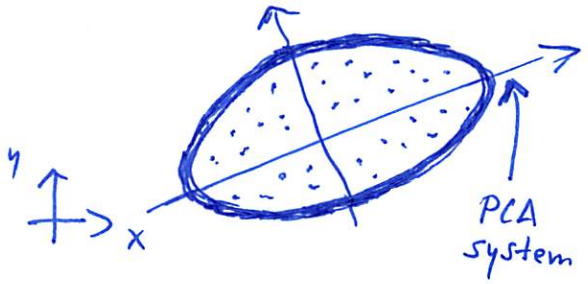
- local max. of density fct.
× saddle point of density fct.
✳ separatrix structure

Here:
- 6 clusters
- "P in tile of CL_i ⇔ P is closest to cluster CL_i

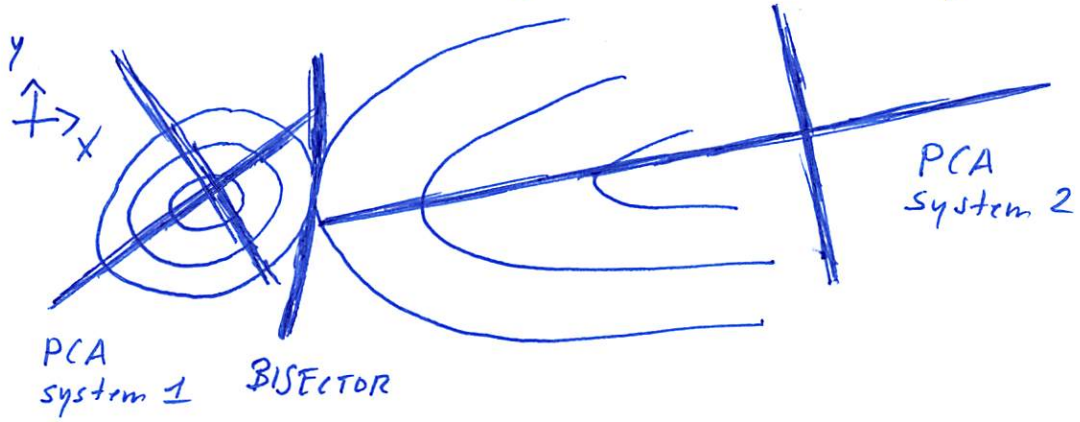
The SHAPE of Point Distributions

→ Implications on Local Distance Metrics

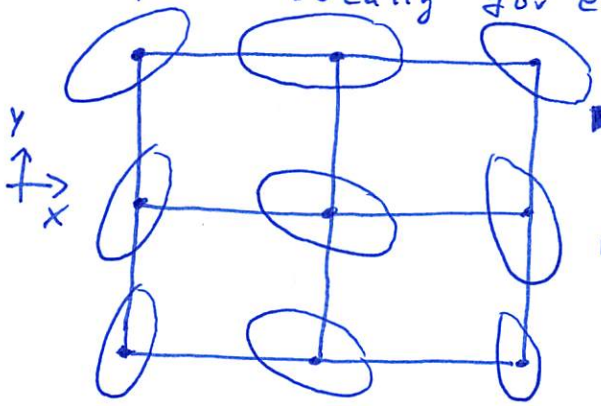
I) The local metric tensor: PCA analysis defines the metric tensor desired to define distance values relative to the PCA-induced coord. system and units



II) Two local metric tensors define tessellation and bisector - defining to what cluster a point is closest to



III) Can determine many locally defined PCA systems - by using a fixed overlaid grid and compute a PCA system locally for each grid point (considering the points in the grid point's neighborhood)



Locally computed PCA system and implied distance metric

→ NO NEED TO COMPUTE AND STORE THE ACTUAL VORONOI DIAGRAM DEFINED BY THE VARYING METRIC. — CAN COMPUTE DISTANCES DIRECTLY BASED ON ALL LOCAL METRIC TENSORS! BH