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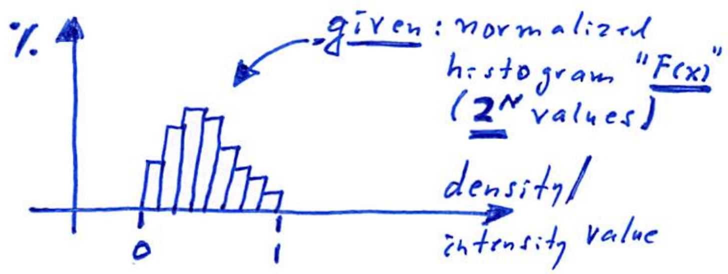
22 AUG 2016

Wavelets: A Tool for Improving Computational Efficiency – And Possibly Classification & Detection?

→ **ISSUE:** Histograms are represented as power-of-2 "bar diagrams" = piecewise-constant functions.

⇒ Haar wavelets are ideal to represent these histograms/functions – and approximate them at a desired/necessary degree of precision.

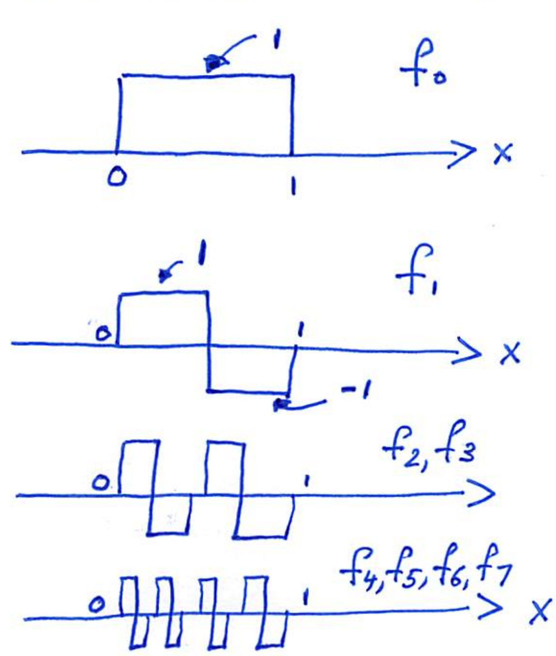
→ Illustration: [1D case; normalized histogram and basis functions defined over domain [0,1]]



(i) Basis functions f_i are normalized and mutually orthogonal to each other:

$$\langle f_i, f_j \rangle = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{otherwise} \end{cases}$$

- Haar wavelet basis functions:



(ii) Calling the original histogram "F" and the independent variable "x", one can write F(x) as:

$$F(x) = \sum_{i=0}^{2^N-1} c_i \, f_i(x), \text{ where}$$

$$c_i = \langle F, f_i \rangle = \int_0^1 F \cdot f_i \, dx, \text{ see}$$

Stollnitz, DeRose, Salesin...

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■ Haar Wavelets (cont'd.)

Thoughts: 1) Haar wavelets decompose the histogram of an object/material type into a set of increasingly higher-frequency basis functions.

2) The projection of an original histogram onto the set of Haar wavelet functions produces a spectrum of coefficients c_i for the expansion $\sum_{i=0}^{2^N-1} c_i f_i(x)$;

question: DOES A SMALL SUBSET OF THESE COEFFICIENTS SUFFICE TO 'CAPTURE' THE IMPORTANT INFORMATION DEFINING AN OBJECT/MATERIAL TYPE?

3) CHALLENGE: One must determine the minimal set of wavelet coefficients c_i that uniquely defines an object/material type (well enough) to allow one to perform correct classification and detection.

• WHY? → Computational complexity will depend on the resolution N of a 'binned' 1D, 2D, multi-D histogram. Complexity could be $O(N)$, $O(N^2)$, ...

→ IF K WAVELET COEFFICIENTS ($K \ll N$) SUFFICE TO REPRESENT AN OBJECT/TYPE, COMPUTATIONAL COST WILL DECREASE.

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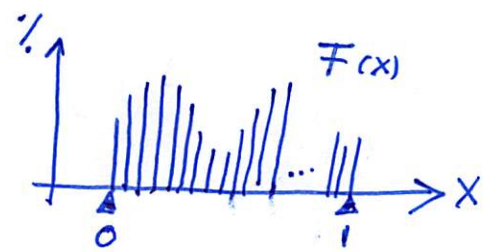
■ Wavelets... (cont'd.)

[Reference: Eric Stollnitz, Tony DeRose, David Salesin – Wavelets for Computer Graphics: A Primer, Part 1]

• Given histogram



• Wavelet representation

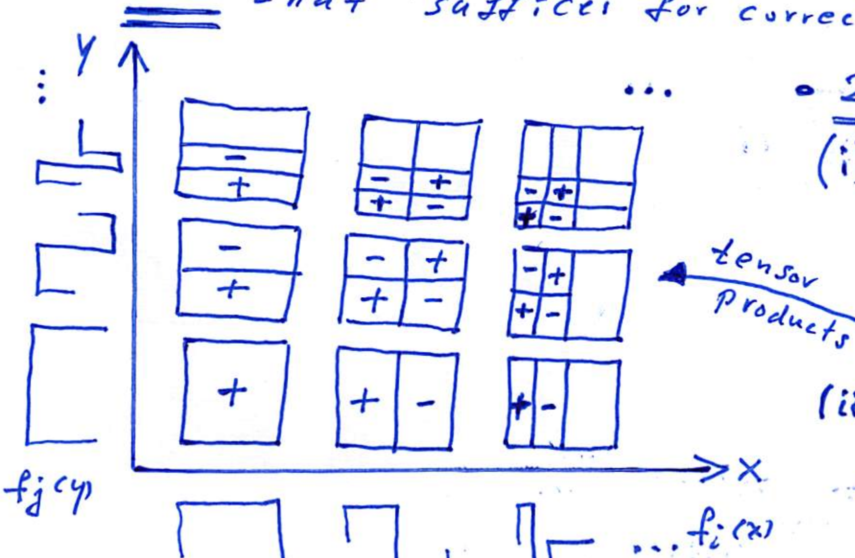
$$F(x) = \sum_{i=0}^{2^N-1} c_i \cdot f_i(x)$$

⇒ How to use the coefficients c_i?

1) Order coefficients c_i by absolute magnitude and use only that subset of that ordered set of coefficients that "suffices for correct classification and detection."

2) Use only the subset of coefficients that belongs to a (or several) specific frequency(ies)/band(s) that "suffices for correct..."

3) Determine the minimal subset of coefficients that "suffices for correct..."



• 2D case:

(i) Use basis functions $f_i(x)$ and $f_j(y)$ to define bi-variate basis functions:

$$f_{i,j}(x,y) = f_i(x) \cdot f_j(y)$$

(ii) 2D wavelet representation:

$$F(x,y) = \sum_i \sum_j c_{i,j} \cdot f_{i,j}(x,y)$$

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