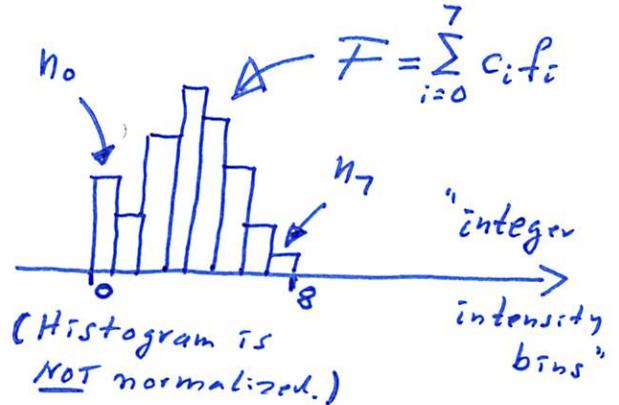■ **WAVELETS** (cont'd)
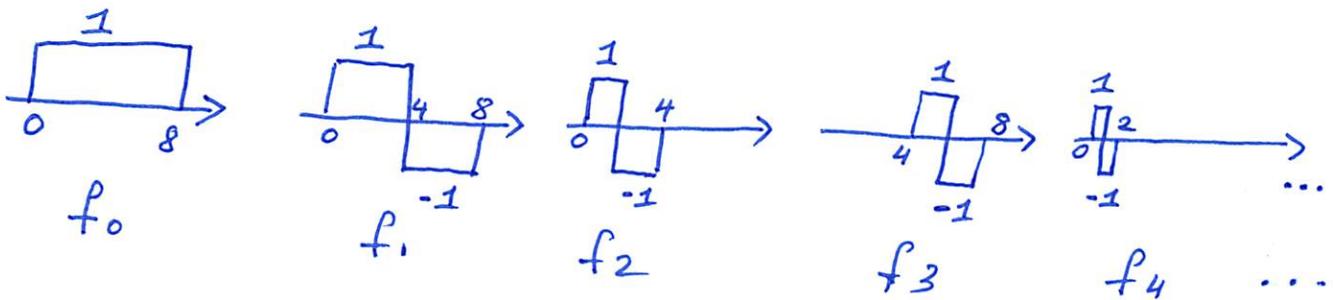
→ **INTEGER**-based wavelet representation

→ **WAVELET SPECTRA** for signature definition and detection

● **INTEGER**-based wavelets

Ex.: → Integer histogram of 8 value $n_0, \dots, n_7$:

n_0 $\quad$ $F = \sum_{i=0}^{7} c_i f_i$

n_7

"integer intensity bins"

(Histogram is **NOT** normalized.)

→ Haar wavelet basis functions (**not** normalized):

$f_0 \qquad f_1 \qquad f_2 \qquad f_3 \qquad f_4 \quad \dots$

$$\Rightarrow \left(\langle f_i, f_j \rangle \right) \begin{pmatrix} c_0 \\ \vdots \\ c_7 \end{pmatrix} = \left(\langle F, f_i \rangle \right): \begin{pmatrix} 8 & & & & \sigma \\ & 8 & & & \\ & & 4 & & \\ & & & 4 & \\ & & & & 2 \; 2 \; 2 \\ \sigma & & & & 2 \end{pmatrix} \begin{pmatrix} c_0 \\ \vdots \\ c_7 \end{pmatrix} = \begin{pmatrix} n_0 + n_1 + \dots + n_7 \\ n_0 + \dots + n_3 - (n_4 + \dots n_7) \\ n_0 + n_1 - (n_2 + n_3) \\ n_4 + n_5 - (n_6 + n_7) \\ n_0 - n_1 \\ \dots \\ n_6 - n_7 \end{pmatrix}$$

$$\Leftrightarrow \quad 8 \cdot \mathbb{C} = \begin{pmatrix} \tilde{n}_0 \\ \vdots \\ \tilde{n}_7 \end{pmatrix}$$

$$\Rightarrow \quad 8 F = \sum_0^7 8 c_i f_i \quad \text{etc.}$$

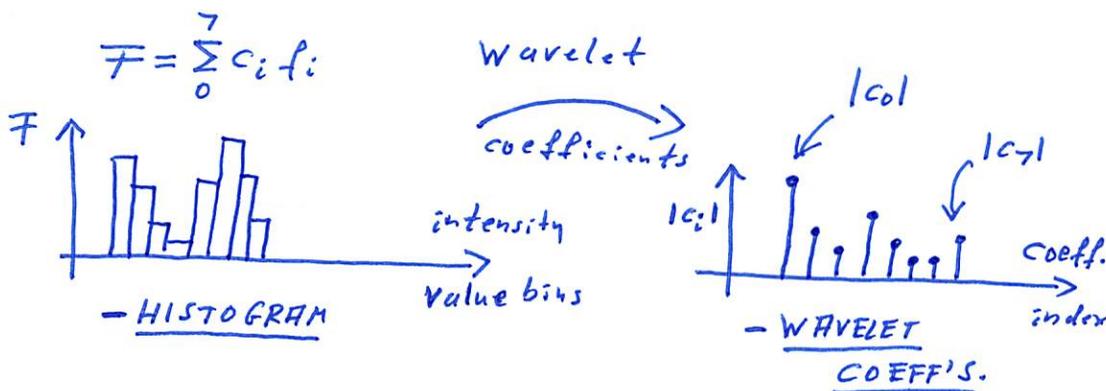● QUESTION: IS SUCH A PURELY INTEGER-BASED APPROACH DESIRABLE IN THE INTEREST OF COMPUTATIONAL EFFICIENCY OR **NOT**?

■ WAVELETS (cont'd.)

● WAVELET SPECTRA

→ Motivation / Thought:

○ Using spectral analysis, each material type/element is identified with its unique signature in frequency/spectral space. WE ARE ALSO CONCERNED WITH DISTINCT MATERIAL TYPES!

○ A wavelet decomposition/representation of a material's imaged intensity histogram defines the material's intensity spectrum. THE MATERIAL'S SIGNATURE CAN BE DEFINED AS ITS WAVELET COEFFICIENT SPECTRUM.

○ QUESTION: CAN SUCH A WAVELET SPECTRUM BE USED FOR A BETTER AND MORE EFFICIENT DETECTION OF THE MATERIALS OF SEGMENTED OBJECTS?

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■ ILLUSTRATION:

$$\mathcal{F} = \sum_0^7 c_i f_i$$

Wavelet coefficients

$\mathcal{F}$ ↑

— HISTOGRAM

intensity value bins

$|c_0|$

$|c_i|$

$|c_7|$

coeff. index

— WAVELET COEFF'S.

→ A SMALL NUMBER OF COEFF'S MIGHT SUFFICE TO DEFINE A MATERIAL UNIQUELY...

~ BH

■ <u>WAVELETS</u> / <u>SPECTRA</u> / <u>SPECTRAL ANALYSIS</u> (cont'd.)

➡ <u>Wavelet-based</u> frequency <u>analysis</u> of histograms produces "<u>spectra</u>", defining the main frequencies occurring in a measured histogram.

➡ Can principles from <u>spectral analysis</u> be used to identify the specific types of <u>materials</u> captured <u>in an intensity</u> (or density) <u>histogram</u> produced by imaging? (Think of the element-specific <u>spectral lines</u> seen in absorption and emission spectra...)

➡ <u>OR</u>: <u>Mass spectrometry</u> produces <u>mass spectra</u> used to determine the elements present in a chemical sample.

in-ten-sity → mass (mass-to-charge)

• <u>QUESTION</u>: Given the mass spectrogram of an imaged sample, <u>how are the individual elements identified in the spectrum</u>?

• To be tested!
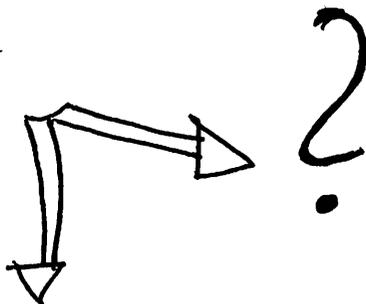
➡ <u>CAN THIS BE APPLIED TO THE ANALYSIS OF WAVELET-BASED SPECTRA OF IMAGED OBJECTS OF SPECIFIC MATERIALS?</u>

?! ➡ <u>ARE HAAR-WAVELED-BASED FREQUENCY SPETRA OF INTENSITY (OR DENSITY) HISTOGRAMS UNIQUE AND DEFINE 'HIGHLY RELIABLE' SIGNATURES FOR THE GIVEN K TYPES OF MATERIALS THAT MUST BE DETECTED</u>?