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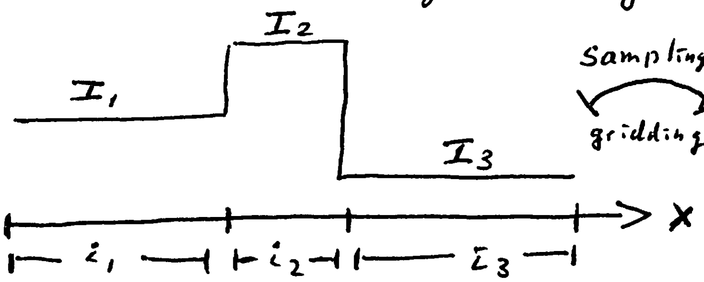
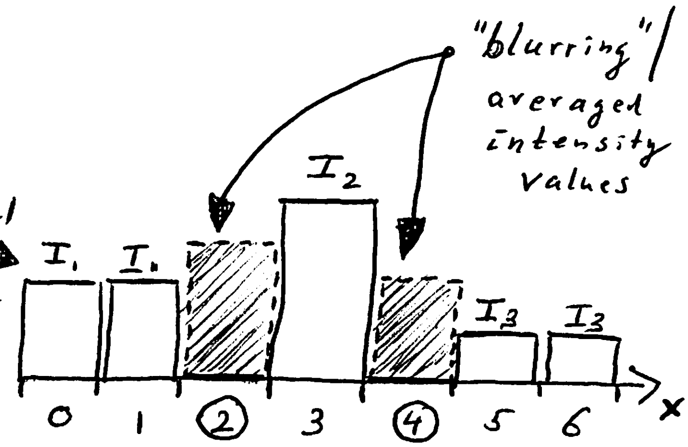
IMAGE PROCESSING AT / CLOSE TO THE RESOLUTION LIMIT: DISCONTINUITIES, SEGMENTATION, MATERIAL / OBJECT INTERFACES

• The 1D Case:

Exact discontinuous function before sampling / scanning:





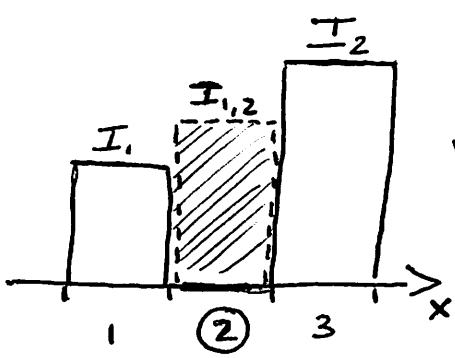
⇒ 3 distinct intervals i_1, i_2, i_3 with distinct values I_1, I_2, I_3

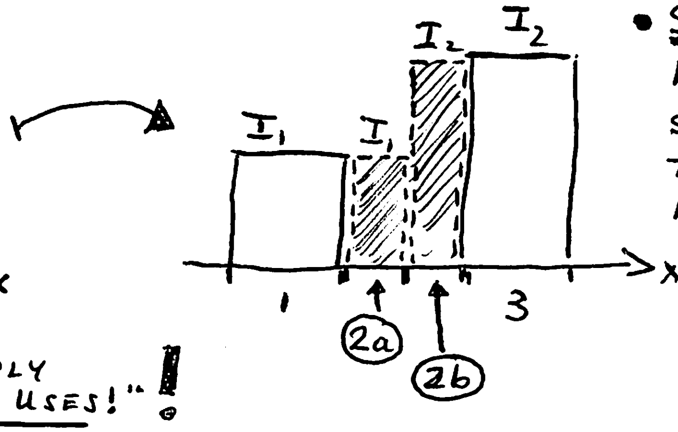
⇒ ② and ④ have associated maximal values of (absolute) slope / gradient magnitude!

• **ASSUMPTION**: The discrete intensity value samples belong to DISTINCT OBJECTS with DISTINCT (HOMOGENEOUS) INTENSITY values;

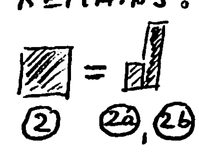
⇒ OBJECT / MATERIAL INTERFACES ARE IN REGIONS WITH LARGE GRADIENTS!

⇒ Possible "reconstruction" of proper discontinuity:





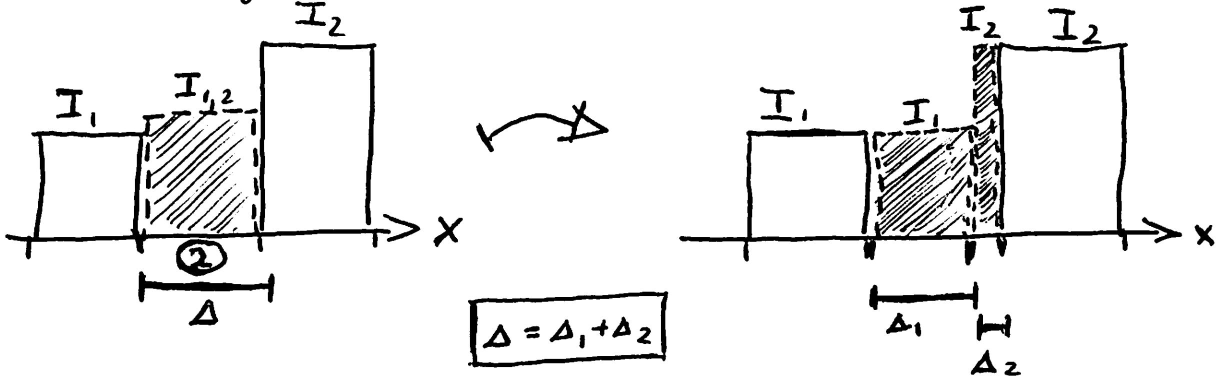
• SPLIT ② INTO ②a, ②b SUCH THAT THE AREA / INTEGRAL REMAINS:



"DO NOT SPLIT ② – SIMPLY BE AWARE OF ITS TWO USES!"

DISCONTINUITIES ... cont'd.

⇒ Determing the exact <u>split point</u> / interface location:



area preservation: $\Delta\, I_{1,2} = \Delta_1 I_1 + \Delta_2 I_2$

$$= \Delta_1 I_1 + (\Delta - \Delta_1) I_2$$

$$\Longleftrightarrow \quad \Delta I_2 - \Delta I_{1,2} = \Delta_1 I_2 - \Delta_1 I_1$$

$$\Delta\,(I_2 - I_{1,2}) = \Delta_1 (I_2 - I_1)$$

$$\Rightarrow \quad \underline{\underline{\Delta_1 = \Delta \cdot \frac{I_2 - I_{1,2}}{I_2 - I_1}}} \, ,$$

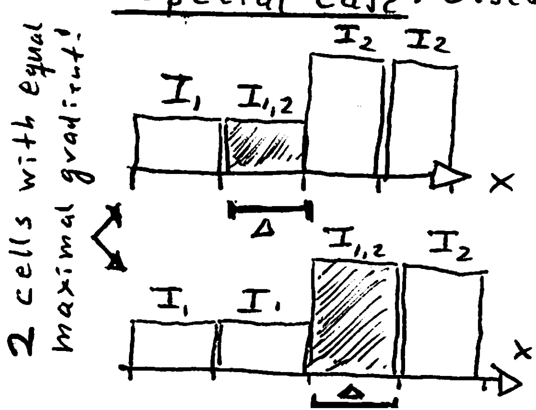
$$\underline{\underline{\Delta_2 = \Delta - \Delta_1}} \qquad \square$$

⇒ Meaning – "Uses of sample 2":

"Cell 2 contains $\frac{\Delta_1}{\Delta}$ of material type I_1
and $\frac{\Delta_2}{\Delta}$ of " " I_2."

⇒ <u>Material fraction information!</u>

- <u>Special case</u>: Discontinuity perfectly preserved in discrete data:



$\underline{I_{1,2} = I_1} \Rightarrow \Delta_1 = \Delta \frac{I_2 - I_1}{I_2 - I_1} = \Delta, \; \Delta_2 = 0$ <u>correct!</u>

$\boxed{\textbf{NO NEED TO SPLIT!}}$

$\underline{I_{1,2} = I_2} \Rightarrow \Delta_1 = \Delta \frac{I_2 - I_2}{I_2 - I_1} = 0, \; \Delta_2 = \Delta$ <u>correct!</u>

MATERIAL INTERFACE DETECTION... cont'd.

General Algorithm ... 1D Case

- **Step 1:** Determine intervals indicative of a "jump" from one type (I_1) to another type (I_2); e.g., consider gradient magnitude to define intervals that contain a material interface/discontinuity.
- **Step 2:** For all these intervals (resulting from Step 1) compute the optimal split points. (See previous pages.)

  ALTERNATIVELY:

  Compute material fractions $\frac{\Delta_1}{\Delta}$ and $\frac{\Delta_2}{\Delta}$ for all intervals containing material interfaces; store these fractions for 2-material intervals.

- **OUTPUT:** Intervals and sub-intervals (resulting from splitting) with one specific material type with associated distinct intensity value I_k

  OR

  Material fraction values for all original intervals, with only 2-material intervals having 2 associated material fraction values. □

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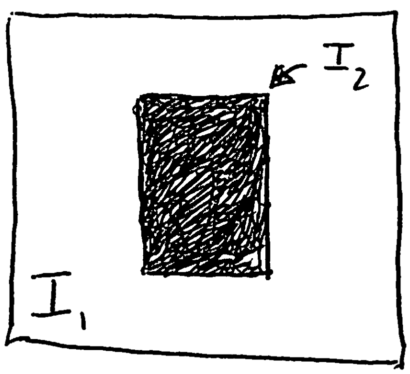
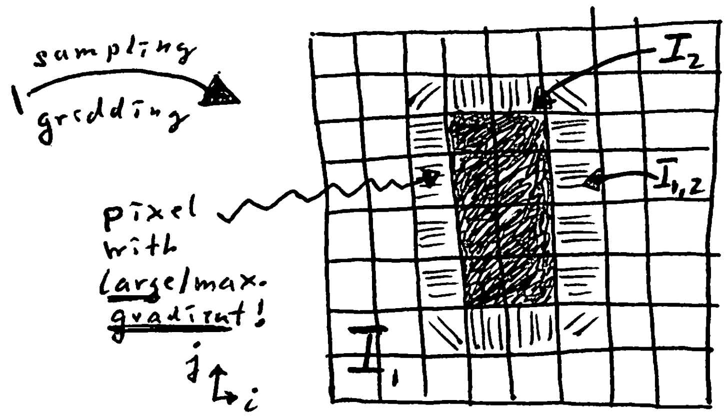
The 2D Case – Material Fraction Computation

- **Exact**, original discontinuous bivariate function:

- Discrete / sampled / reconstructed version of function:

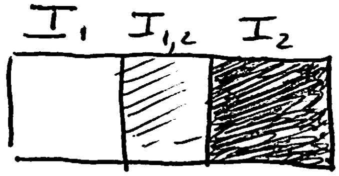




2 materials: I_1, I_2

$\Rightarrow$ "Blurred" boundary region / pixels exist around the I_2-material.

$\Rightarrow$ Determine material fractions in blurred inter-face pixels:



- Pixels have area a.
- Pixel $I_{1,2}$ must be split into (sub-pixels with) areas a_1 and a_2:

$$a\, I_{1,2} = a_1 I_1 + a_2 I_2$$
$$= a_1 I_1 + (a - a_1) I_2$$

$$\Rightarrow \quad \underline{\underline{a_1 = a \cdot \frac{I_2 - I_{1,2}}{I_2 - I_1}}} \, ,$$

$$\underline{\underline{a_2 = a - a_1}} \qquad \square$$

$\Rightarrow$ Compute and store these material fractions for all pixels. ISSUE (theoretical!): Pixels can exist where more than two materials come together... [For practical purposes this is irrelevant.???]

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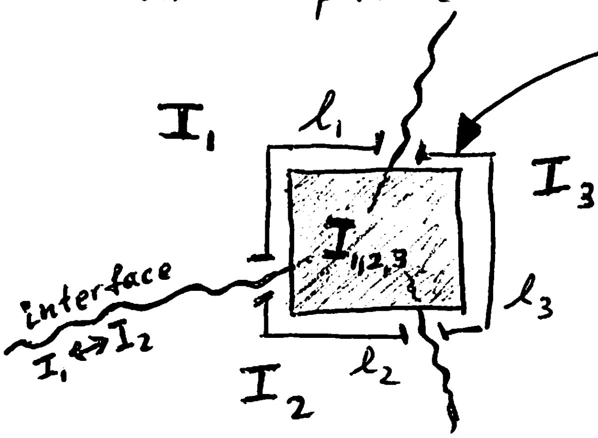
The 2D Case – Cont'd. ...

→ The case of more than 2 material types in a pixel:

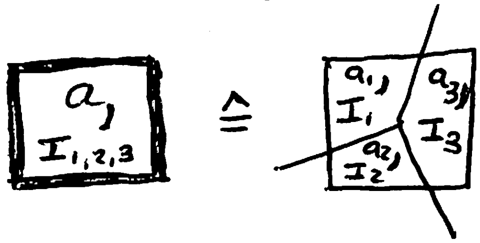


pixel with area a where 3 material types with individual, "pure" intensities I_1, I_2 and I_3 come together; pixel itself having a "blended" intensity value $I_{1,2,3}$

→ "Crude approximation" of needed material fractions: Consider the lengths l_1, l_2, l_3 on pixel's boundary and define approximations for area ratios/area fractions

$$\hat{a}_i = \frac{l_i}{L},$$

where $L = l_1 + l_2 + l_3$. The condition that should be satisfied is given by:



$$a\, I_{1,2,3} = a_1 I_1 + a_2 I_2 + a_3 I_3 .$$

The value $a \cdot \frac{l_i}{L}$ approximates the needed area a_i !!!

(Generally, this equation will **NOT** be true:

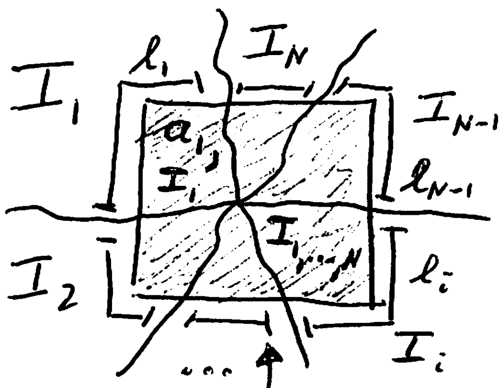
$$a_1 I_1 + a_2 I_2 + a_3 I_3 = a \frac{l_1}{L} I_1 + a \frac{l_2}{L} I_2 + a \frac{l_3}{L} I_3$$

$$\Leftrightarrow \quad \sum_i \frac{a_i}{a} I_i = \sum_i \frac{l_i}{L} I_i$$

For practical purposes, the difference can be neglected.)

The 2D & 3D Cases – Cont'd. ...

→ General case: N material types coming together:



pixel of area a, intensity value $I_{1,\dots,N}$

- **Condition:**

$$a\, I_{1,\dots,N} = \sum_{i=1}^{N} a_i I_i \,,$$

where $a = \sum_{i=1}^{N} a_i$

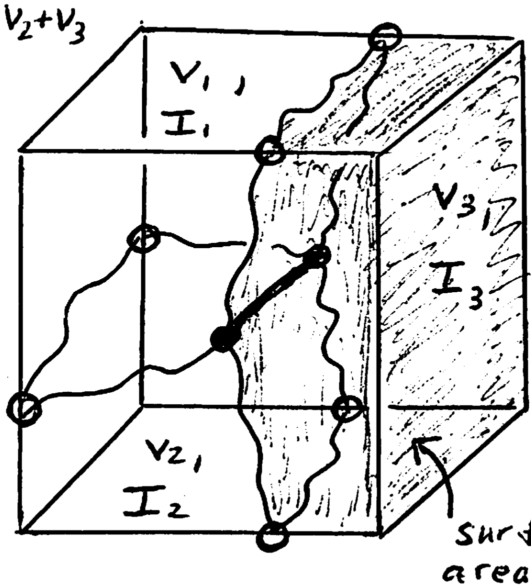
- Approximation of a_i:

$$a_i \approx \frac{l_i}{L}\, a \,, \quad L = \sum_{i=1}^{N} l_i$$

→ **3D Case**: General case: N material types coming together in ONE VOXEL:

$V = v_1 + v_2 + v_3$

Voxel's intensity: $I_{1,2,3}$



surface area s_3

voxel of volume v, with three materials merging: I_1, I_2, I_3

⇒ Condition:

$$v\, I_{1,2,3} = v_1 I_1 + v_2 I_2 + v_3 I_3$$

- Approximation of v_i:

$$v_i \approx \frac{s_i}{S}\, v \,, \quad S = \sum_i s_i$$

- s_i = "voxel's surface area for I_i"

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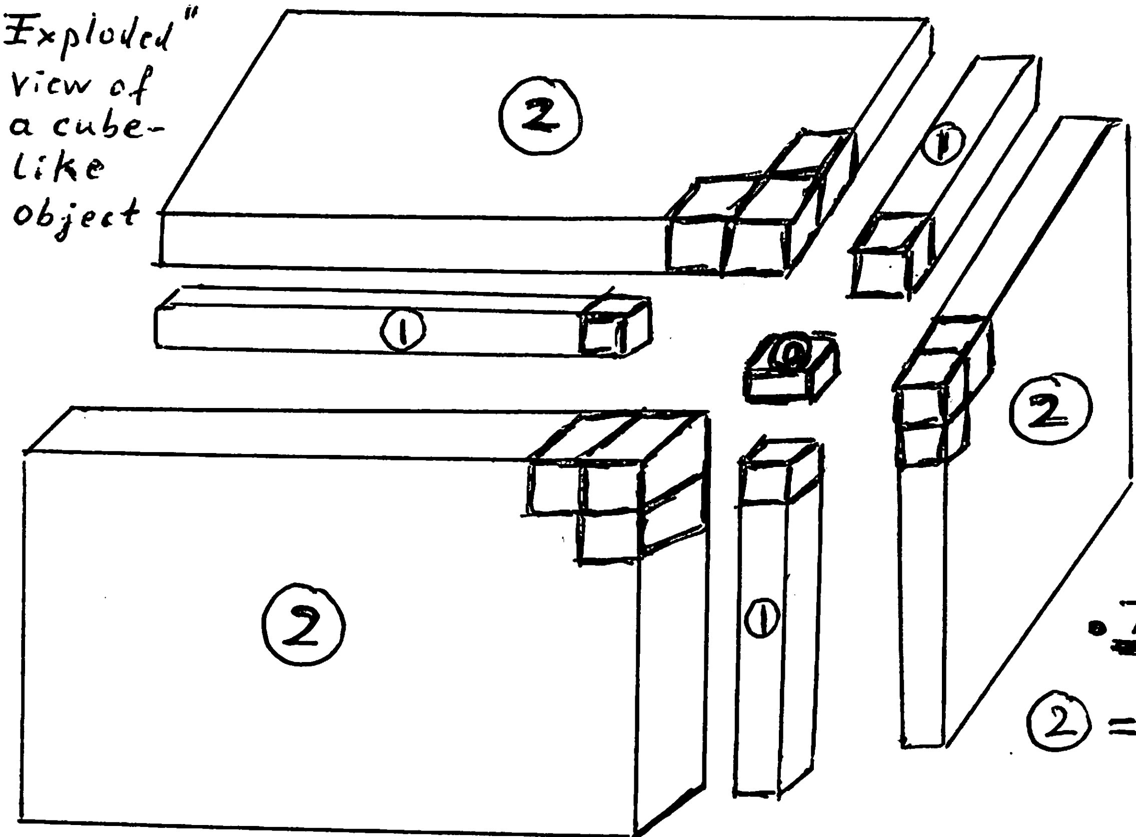
3D Case – Cont'd. ...

→ "Types of Interface Voxels"

"Exploded" view of a cube-like object



- Three types:

  ② = 2-manifold "Layer"

  ① = 1-manifold "stack"

  ⓪ = 0-manifold "voxel"

- Process for handling material-interface voxels:
  - i) Identify all material-interface voxels;
  - ii) Determine proper material-type fractions for all material interface voxels;
  - iii) Consider three cases / types:
    - ② "Layer": two materials come together.
    - ① "Stack": any number of materials come together.
    - ⓪ "Voxel": " " " " " "
  - iv) Voxel with volume v and intensity value I has volume / material fractions v_i for the K materials with intensities I_i:

$$vI = \sum_{i=1}^{K} v_i I_i \; ; \quad \text{APPROXIMATE:} \; \boxed{v_i = \frac{s_i}{\sum s_i} v} \; ;$$

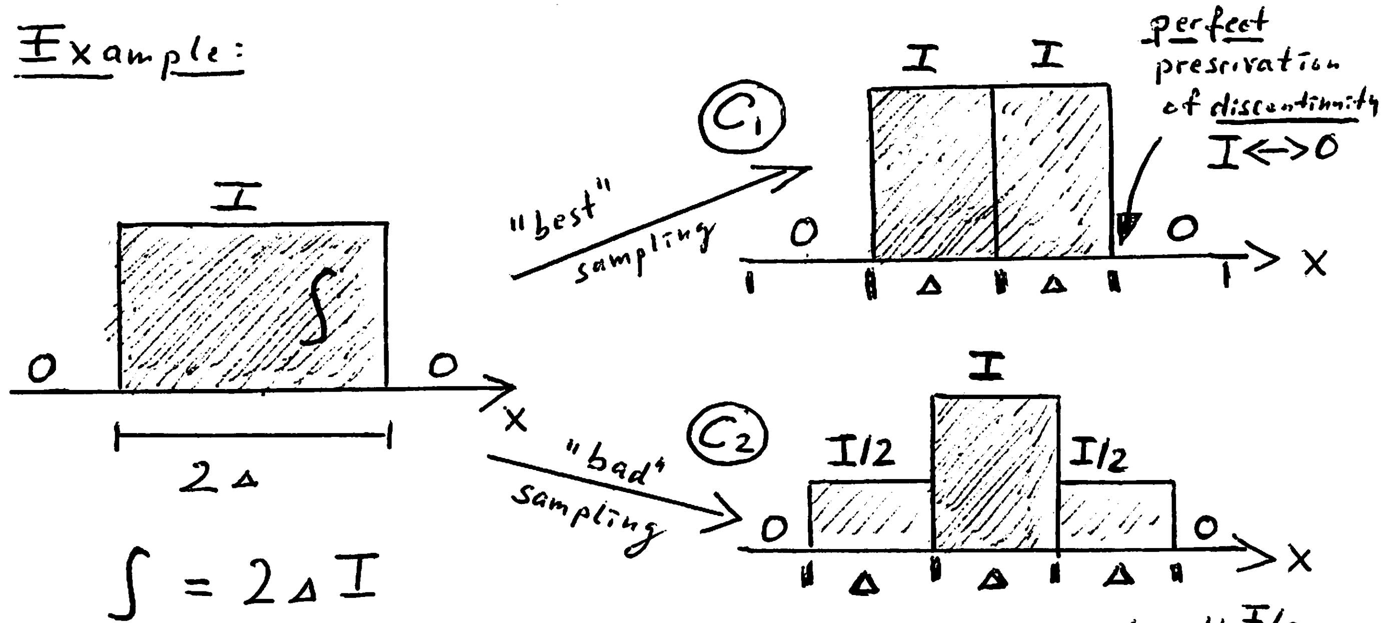
s_i = voxel's surface area for type I_i

$S = \sum s_i$ = voxel's surface

Resolution Issues – Limit

- (1D case) "Object must be at least TWICE the width of the grid spacing to allow proper processing."

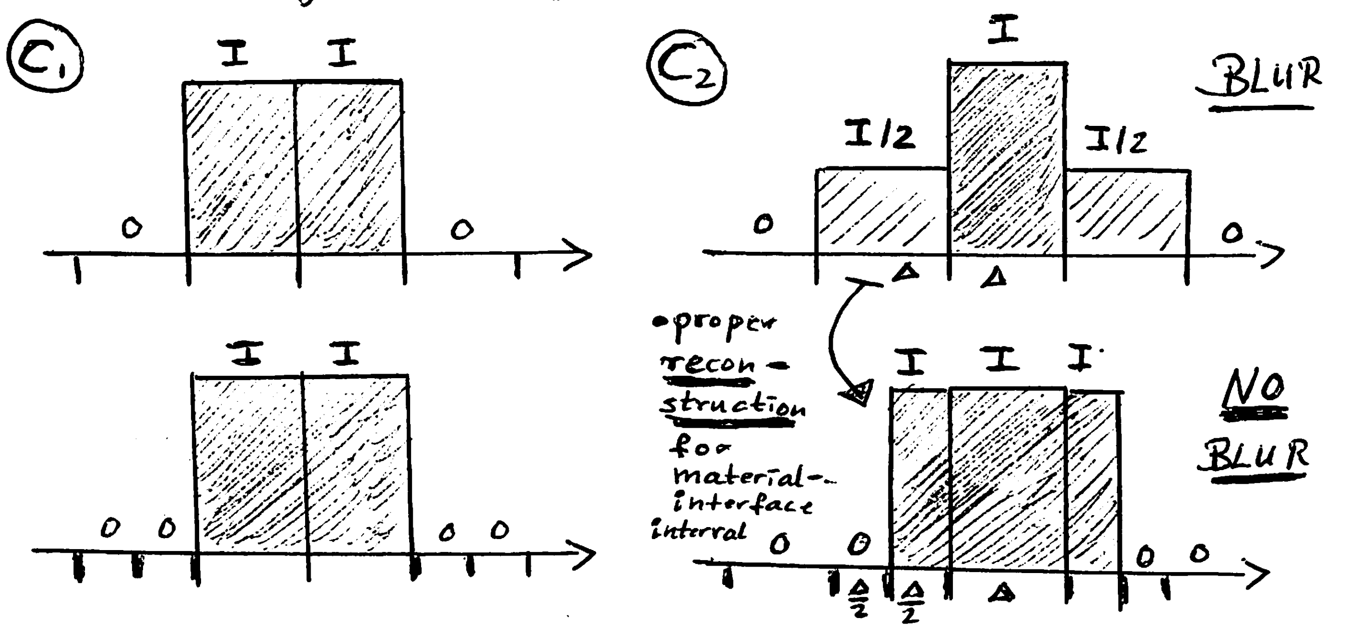
Example:



$$\int = 2\Delta I$$

⇒ boundary "blurred" with I/2 samples ...

⇒ Processing cases C_1 and C_2 in "interface intervals":



⇒ In both cases the INTEGRAL condition $\int = 2\Delta I$ can be satisfied – and the BLURRED behavior can be fixed!

BH