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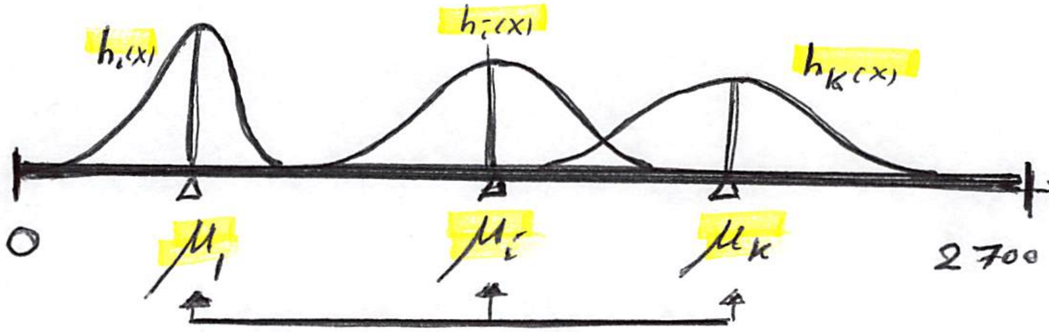
8/14/17

BLENDING HISTOGRAMS

- Given: → Set of "prototype" histograms $h_1, \ldots, h_K$ of K materials:



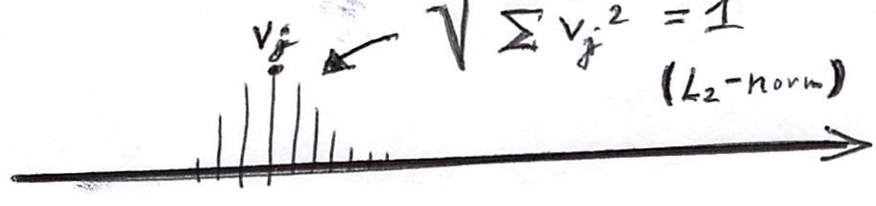
K mean values

→ Histogram $h(x)$ to be analyzed/classified:

→ Histograms are normalized: $\| h_i(x) \| = \| h(x) \| = 1$:

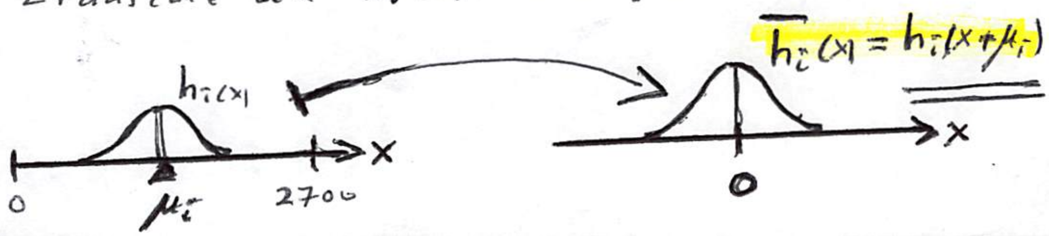
$$\left\{ \sqrt{\sum v_j^{\,2}} = 1 \right\}$$

(L2-norm)

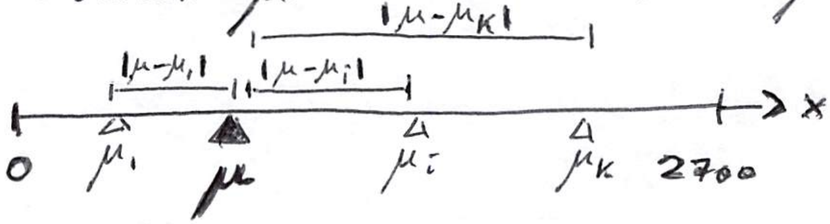


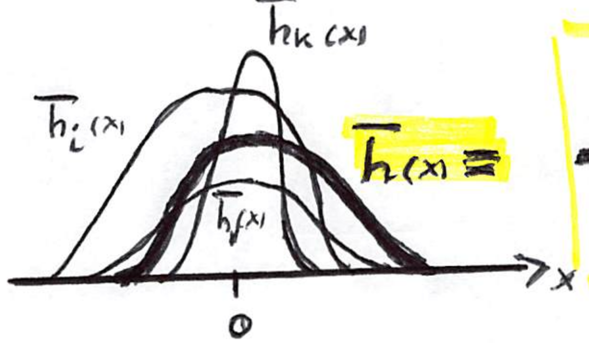
- **Operations:**
  - Translation: translate all $h_i(x)$ to origin:

$$\bar{h}_i(x) = h_i(x + \mu_i)$$



• **Blending:** linearly blend the $\overline{h}_i(x)$, using "distance weights" considering the distance between μ and the respective μ_i-values, to define $\overline{h}(x)$, a new function obtained by blending the $h_i(x)$ functions for value μ.





$$\overline{h}(x) = \frac{\sum_{i=1}^{K} \frac{1}{|\mu-\mu_i|} \cdot h_i(x)}{\sum_{i=1}^{K} \frac{1}{|\mu-\mu_i|}} \quad (*) \quad (+)$$

(*) or use: $|\mu-\mu_i|^2$

(+) OR: "TPS"

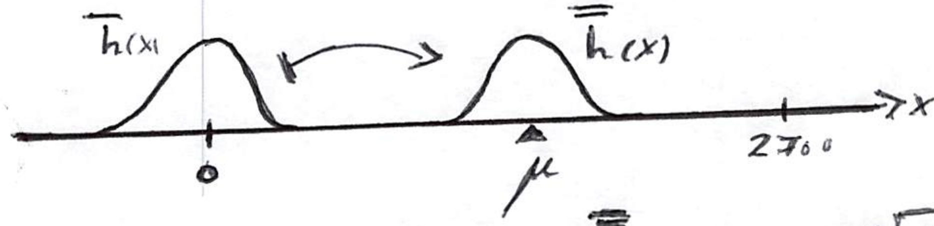
+ PLUS: → Is mean of $\overline{h}(x) = 0$?

- Yes → ✓
- No → determine $\overline{\mu}$ = mean of $\overline{h}(x)$

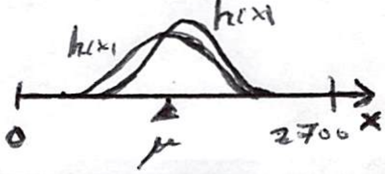
+ PLUS: → Normalize $\overline{h}(x)$ → $\|\overline{h}\| = 1$

• **Translation:** translate $\overline{h}(x)$ s.t. (new) mean is μ:

$$\overline{\overline{h}}(x) = \overline{h}(x-\mu)$$



• **Difference:** compute difference between $h(x)$ and $\overline{\overline{h}}(x)$:

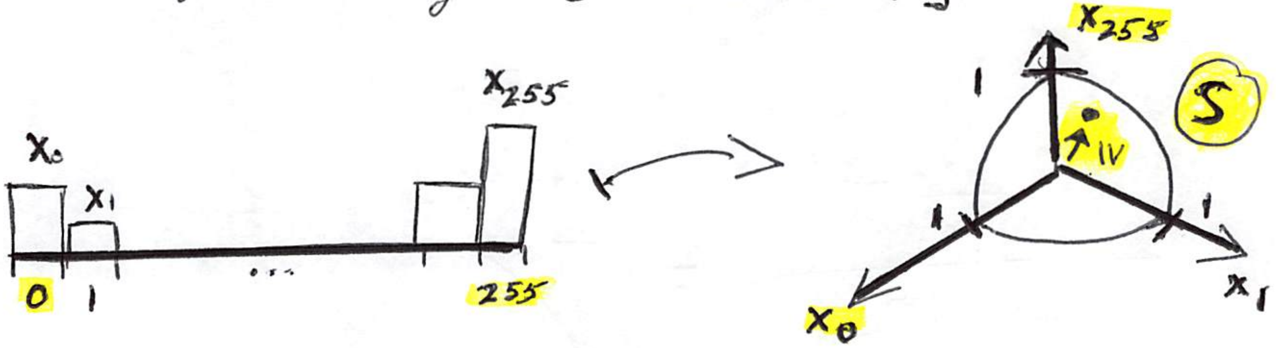


$$\delta = \sqrt{\int (h-\overline{\overline{h}})^2}$$

"IF $\delta < \varepsilon$ THEN threat"?

- **Alternative View**

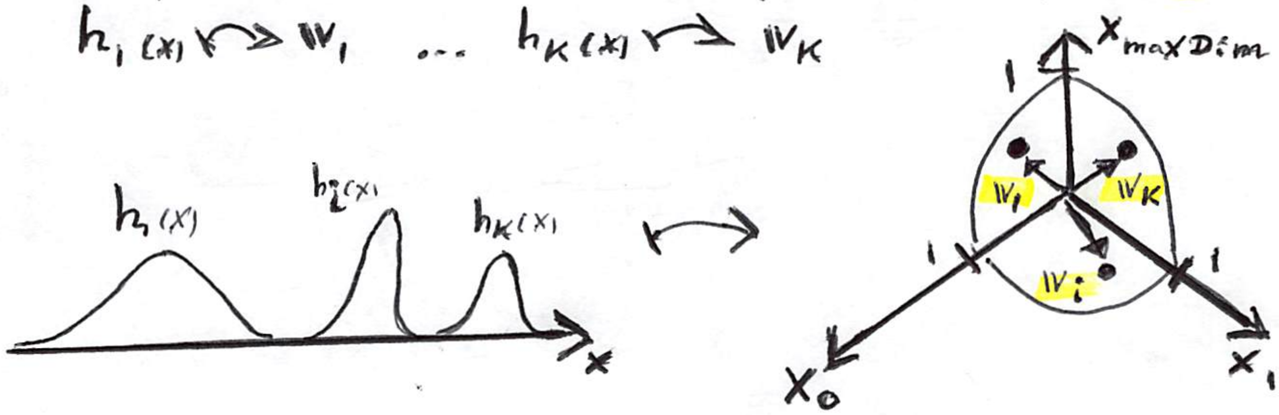
" Histograms viewed as (positional) unit vectors emanating from the origin and ending on the part of a (unit) hyper-sphere [consisting of points with only non-negative coordinates]:



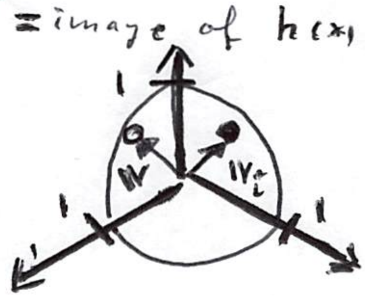
$$\mathbb{V} = \begin{pmatrix} x_0 \\ \vdots \\ x_{255} \end{pmatrix} \quad \text{and} \quad \|\mathbb{V}\| = 1$$

⇒ Prototype histograms viewed as points on sphere S:

$h_1(x) \mapsto \mathbb{V}_1 \quad \ldots \quad h_K(x) \mapsto \mathbb{V}_K$



⇒ Consider GEODESIC DISTANCE or ANGLE (COS) between $\mathbb{V}$ (= image of $h(x)$) and $\mathbb{V}_i$: $\mathbb{V}$ and $\mathbb{V}_i$ s.t. $\|\mathbb{V}\| = \|\mathbb{V}_i\| = 1$



⇒ $\cos = \mathbb{V} \cdot \mathbb{V}_i \in [0, 1]$

cos close to 1 ⇒ $\mathbb{V}$ & $\mathbb{V}_i$ are close!

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■ HISTOGRAM APPROXIMATION – cont'd.

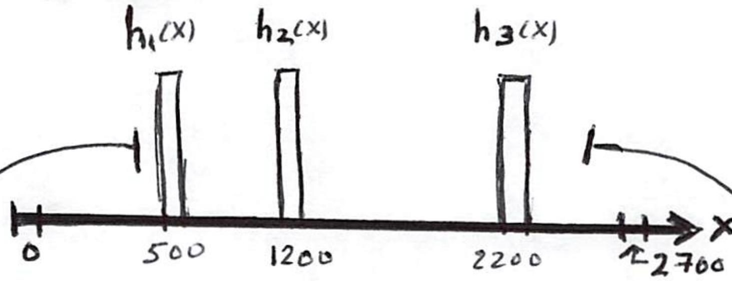
→ given: 1) IDEAL histograms (= perfect density / intensity)

2) SCANNER- and PHANTOM-SPECIFIC histograms (of the same materials)

3) HISTOGRAM of unknown material (obtained via the specific scanner)

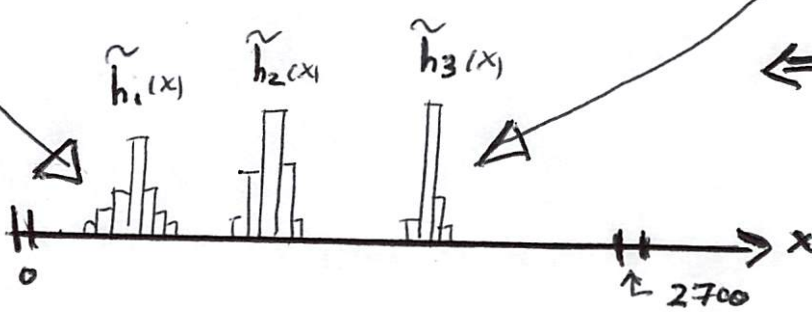
→ question: "Can the histogram from 3) be viewed as one of the histograms from 2), or possibly as a combination of histograms from 2)?"

1) IDEAL histograms:



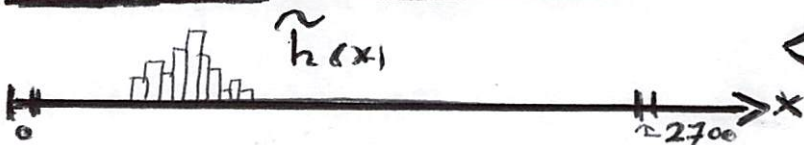
⇐ 2701 bins, each one for ONE integer-value density

2) SCANNER-specific histograms:



⇐ 3 materials producing "translated and spread-out versions" $\tilde{h}_j(x)$ corresponding to the ideal $h_j(x)$ histograms

3) Histogram of UNKNOWN MATERIAL:

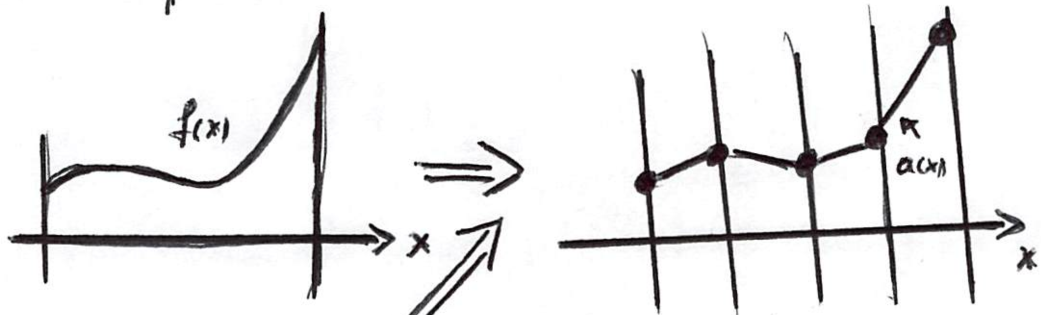


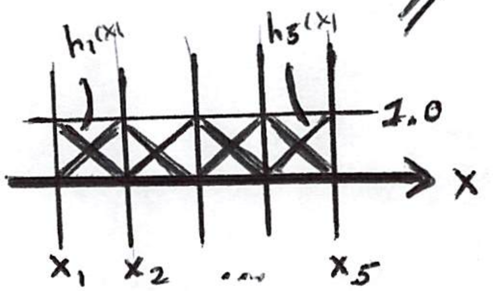
⇐ Does $\tilde{h}(x)$ "resemble" one of the $\tilde{h}_j(x)$ histograms, or a "combination"?

■ HISTOGRAM APPROXIMATION - Cont'd.

➡ Compute **BEST APPOXIMATION** of $\widetilde{h}(x)$ using method of **LEAST SQUARES** applied to the histograms, understood as piecewise constant functions.

{ **Reminder:**





Best approximation $a(x)$ of $f(x)$ as a combination of **hat basis functions** $h_i(x)$:

$$a(x) = \sum_{i=1}^{5} c_i\, h_i(x),$$

where

$$\begin{pmatrix} \langle h_1,h_1\rangle & \dots & \langle h_1,h_5\rangle \\ \vdots & & \vdots \\ \langle h_5,h_1\rangle & \dots & \langle h_5,h_5\rangle \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_5 \end{pmatrix} = \begin{pmatrix} \langle f,h_1\rangle \\ \vdots \\ \langle f,h_5\rangle \end{pmatrix}$$

⇒ least squares approximation }

➡ **Here**: • All functions $h_j(x)$, $\widetilde{h}_j(x)$, $\widetilde{h}(x)$ are:

- ≥ 0 and
- normalized, i.e., $\|\cdot\|_2 = \sqrt{\int f \cdot g} = 1$

⇒ example above:

$$\boxed{\begin{pmatrix} \langle \widetilde{h}_1,\widetilde{h}_1\rangle & \dots & \langle \widetilde{h}_1,\widetilde{h}_3\rangle \\ \vdots & & \vdots \\ \langle \widetilde{h}_3,\widetilde{h}_1\rangle & \dots & \langle \widetilde{h}_3,\widetilde{h}_3\rangle \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} \langle \widetilde{h},\widetilde{h}_1\rangle \\ \vdots \\ \langle \widetilde{h},\widetilde{h}_3\rangle \end{pmatrix}}$$

("$\langle \cdot,\cdot \rangle$" denotes the inner product of two functions.)

⇒ Compute coefficient vector $(c_1,c_2,c_3)^T$;

Determine those coefficients $c_j \geq 0$ for which $|1-c_j| < \varepsilon$

⇒ $\widetilde{h}(x)$ "represents" those "material types j" ■ BH