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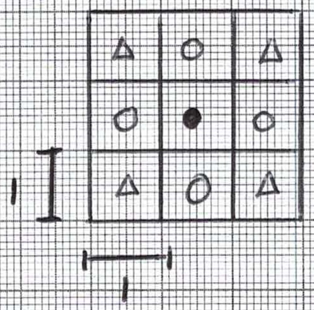
VARIATION, Interfaces & Classification

1) Definition [uniform, equidistant Cartesian grid]

→ 2D Case:

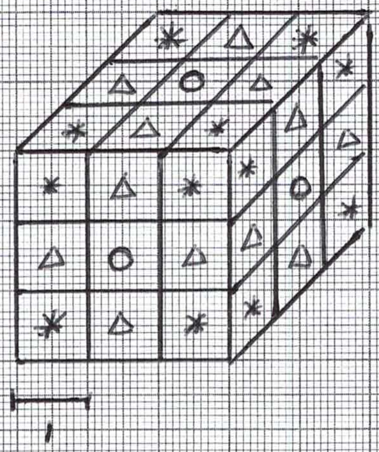


VARIATION V =

$$= \sum_{\text{all 'o'}} |f(\bullet) - f(o)|$$

$$+ \frac{\sqrt{2}}{2} \sum_{\text{all '}\Delta\text{'}} |f(\bullet) - f(\Delta)|$$

→ 3D Case: [voxel at center: '•']



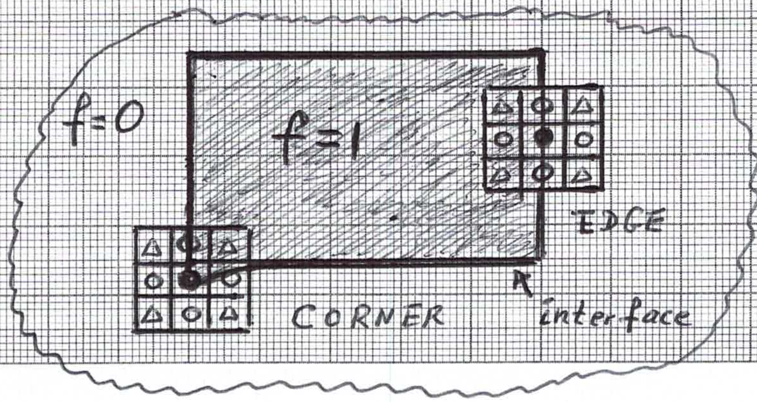
$$V = \sum_{\text{six 'o'}} |f(\bullet) - f(o)|$$

$$+ \frac{\sqrt{2}}{2} \sum_{\text{twelve '}\Delta\text{'}} |f(\bullet) - f(\Delta)|$$

$$+ \frac{\sqrt{3}}{3} \sum_{\text{eight '}*\text{'}} |f(\bullet) - f(*)|$$

2) Use of V: Interface Detection & Classification

→ 2D Case:



EDGE: • on edge here:

1	1/2	0
1	1/2	0
1	1/2	0

$\Rightarrow V = 1 + \sqrt{2}$

→ 2D case cont'd:

CORNER: • is a "corner"

here:

0	1/2	1
0	1/4	1/2
0	0	0

$\Rightarrow$ $V = 1 + 3\sqrt{2}/4$

(material density values (averaged) of pixels of a 3-by-3 mask)

NOTE: The EDGE and CORNER cases are "idealized" cases, where

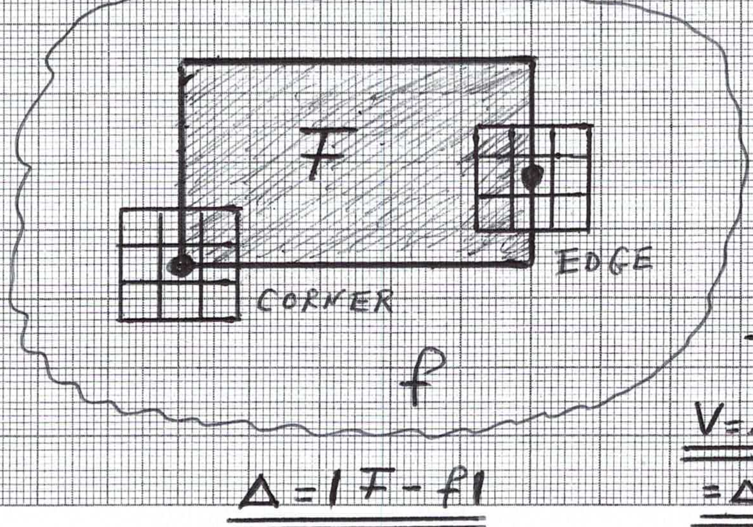
(i) '•' lies exactly on an edge of the interface,

or

(ii) '•' is a corner of the interface, and

(iii) the "transition" from one matrial type to another material type is "abrupt" and captured by a 3-by-3 pixel neighborhood.

→ 2D case cont'd: GENERAL CASE of densities F and f



$\Delta = |F - f|$

EDGE:

F	$\frac{F+f}{2}$	f
F	$\frac{F+f}{2}$	f
F	$\frac{F+f}{2}$	f

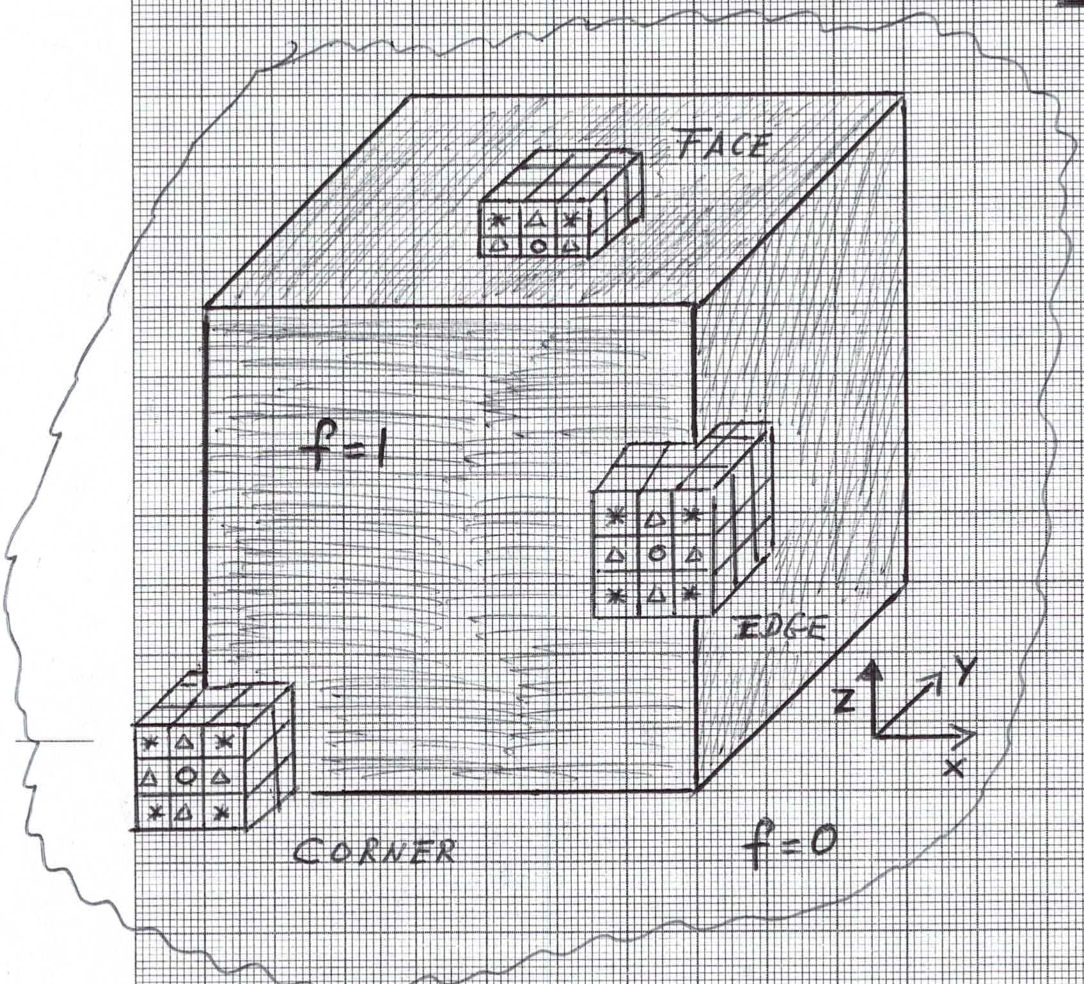
$V = \Delta + \Delta\sqrt{2}$
$= \Delta(1+\sqrt{2})$

CORNER:

f	$\frac{F+f}{2}$	F
f	$\frac{F+3f}{4}$	$\frac{F+f}{2}$
f	f	f

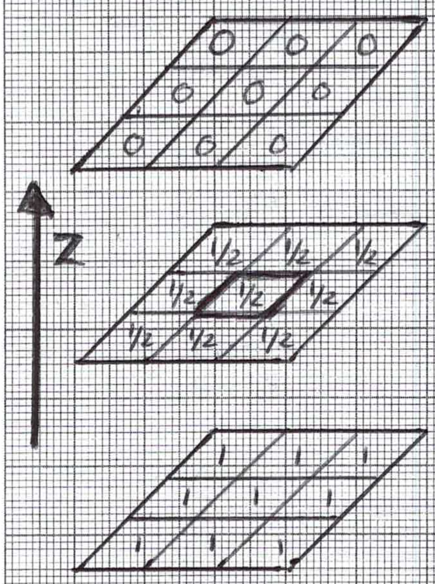
$V = \Delta + 3\Delta\sqrt{2}/4$
$= \Delta(1 + 3\sqrt{2}/4)$

→ 3D case:

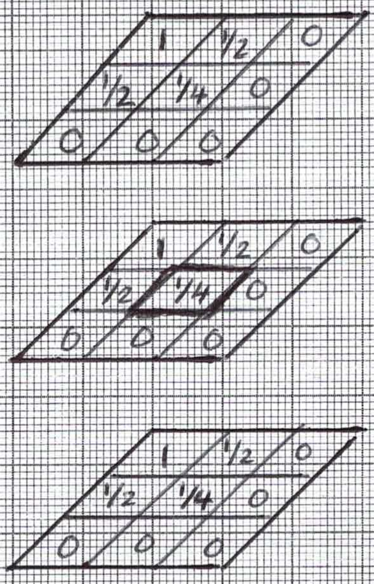


• Below:
The 3 "z-layers" of voxels of the 3-by-3-by-3 mask for the "idealized" cases of interface types; the center voxel '•' lies exactly on a face or an edge or is a corner of the object.

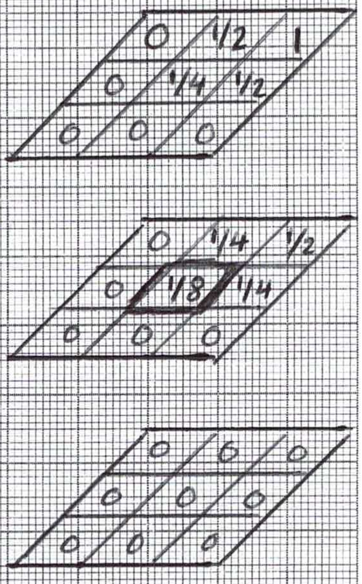
FACE:



EDGE:



CORNER:



here: ⇒ $V = 1 + 2\sqrt{2} + 4\sqrt{3}/3$ | $V = 1 + 7\sqrt{2}/4 + \sqrt{3}$ | $V = 3/4 + 9\sqrt{2}/8 + 7\sqrt{3}/12$

→ 3D case – cont'd.: GENERAL case...

FACE:

f	f	f
f	f	f
f	f	f

$\frac{f+F}{2}$	$\frac{f+F}{2}$	$\frac{f+F}{2}$
$\frac{f+F}{2}$	$\boxed{\frac{f+F}{2}}$	$\frac{f+F}{2}$
$\frac{f+F}{2}$	$\frac{f+F}{2}$	$\frac{f+F}{2}$

F	F	F
F	F	F
F	F	F

EDGE:

F	$\frac{f+F}{2}$	f
$\frac{f+F}{2}$	$\frac{3f+F}{4}$	f
f	f	f

F	$\frac{f+F}{2}$	f
$\frac{f+F}{2}$	$\boxed{\frac{3f+F}{4}}$	f
f	f	f

F	$\frac{f+F}{2}$	f
$\frac{f+F}{2}$	$\frac{3f+F}{4}$	f
f	f	f

CORNER:

f	$\frac{f+F}{2}$	F
f	$\frac{3f+F}{4}$	$\frac{f+F}{2}$
f	f	f

f	$\frac{3f+F}{4}$	$\frac{f+F}{2}$
f	$\boxed{\frac{7f+F}{8}}$	$\frac{3f+F}{4}$
f	f	f

f	f	f
f	f	f
f	f	f

z

$$\Rightarrow \underline{\underline{V=\Delta\cdot\left(1+2\sqrt{2}+\tfrac{4}{3}\sqrt{3}\right)}} \qquad \underline{\underline{V=\Delta\cdot\left(1+\tfrac{7}{4}\sqrt{2}+\sqrt{3}\right)}} \qquad \underline{\underline{V=\Delta\cdot\left(\tfrac{3}{4}+\tfrac{9}{8}\sqrt{2}+\tfrac{7}{12}\sqrt{3}\right)}}$$

- **REMARKS:**

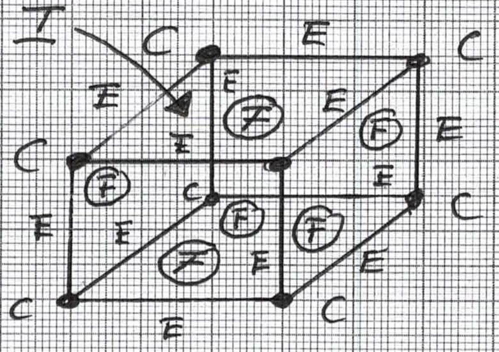
1) These formulas apply to the "idealized case": faces are planar; along edges the difference (angle) between two face normals is 90°; at corners three planar faces come together whose normals differ (in direction) by 90°.
GENERALLY, faces are curved surfaces; the face normal difference along an edge varies; an arbitrary number of faces can share a corner...

- **REMARKS – cont'd.:**

BUT: The V-values resulting from the 3D, 3-by-3-by-3 mask should still capture face, edge or corner "behavior."

2) The V-values (= V-value ranges/intervals) should generally indicate whether
   - a voxel is an INTERIOR voxel (of an object),
   - a voxel is an EDGE voxel,
   - a voxel is a FACE voxel, or
   - a voxel is a CORNER voxel.

3) Based on 2) the topology of an object is defined and can be extracted/represented:



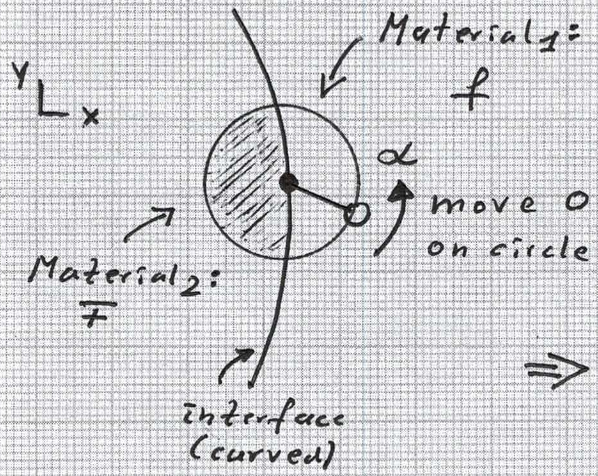
I – Interior
F – Face
E – Edge
C – Corner
} voxels

4) Based on the V-values, one can devise a GENERALIZED SEGMENTATION STRATEGY: Since certain V-value intervals define interior, face (= material interface), edge, or corner voxels of an object, one can compute the "3D, 2D, 1D and 0D SEGMENTS" defining the object.

• REMARKS – cont'd.:

5) The "ideal" way to compute the variational measure V should be an integral measure:

2D case:



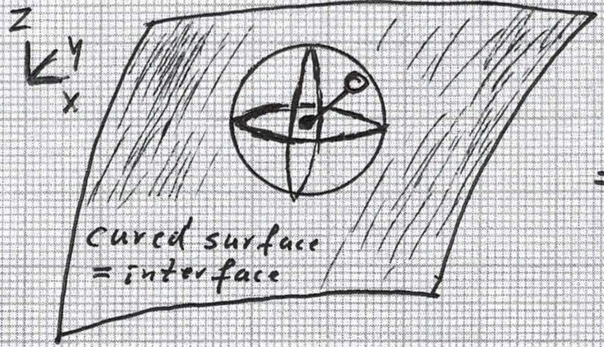
• point on interface

o point that is "close to •", lying on a circle with center •

$$\Rightarrow V = \int_{\alpha=0}^{2\pi} \left| \text{value}(\bullet) - \text{values}(\circ) \right| d\alpha$$

Value of V should determine whether point • is an interior, interface, or corner point of an object (2D case).

3D case:



• point on interface

o point "close to •", lying on sphere with center •

$$\Rightarrow V = \int_{\text{sphere}} \left| \text{value}(\bullet) - \text{value}(\circ) \right| d\,\text{area}$$

6) OUR DATA CONCERNS DISCONTINUOUS MATERIAL PROPERTIES!

⇒ "Least-squares smoothing" must not be done when estimating variational measure V!

– BH