Bernd Hamann

Stratovan

6/10/2019

- **GOAL**: "Generate as much differential insight for a pixel's/voxel's small neighborhood as possible – and use this insight to globally, statistically characterize an entire object."

IDEA / METHOD:

→ Characterize each pixel / voxel of an object / segment in terms of DIFFERENTIAL behavior

→ Consider a 3×3 (3×3×3) neighborhood of pixels/voxels to estimate DIRECTIONAL DERIVATIVES.

→ Employ the different-order FINITE DIFFERENCE FORMULAS to compute all possible directional derivatives.

→ Considering all pixels/voxels defining an object, generate histograms / distributions that describe the object's global, statistical variation of differential behavior.

→ Expectation: This statistical differential behavior of an object should characterize it "well" as an object of specific differential (and TEXTURE) type – well enough for classification purposes (?).

Bernd Hamann 6/3/19

Stratovan

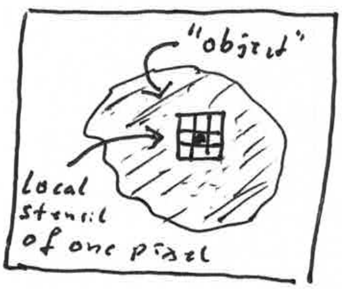
■ LDAC - Local Differential Analysis for Classification

→ IDEA: Use variety of varying-order local finite difference schemes / methods to estimate differential behavior and use histogram characterization of all such differential characteristics to describe the statistical behavior of all the pixels / voxels of a ground-truth object (= set of pixels / voxels) ...

• Given: Set of pixels/voxels defining an "object"

⇒ Characterize this set by determining histograms implied by the local differential behavior of all pixels/voxels of the "object"! (OBJECT = OBJECT WITH OR WITHOUT TEXTURE/VARYING INTENSITY...)

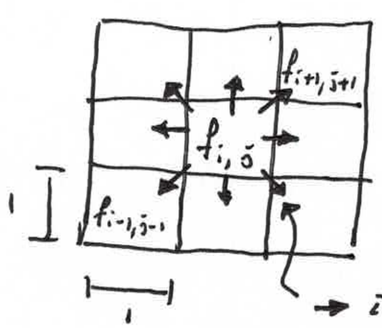
!!! DIRECTIONAL DERIVATIVE BEHAVIOR !!!



Example: 3×3 2D pixel stencil

0) Histogram 0 = histogram of all $f_{i,j}$-values of object /* usually only non-neg. $f_{i,j}$ */

1) Histogram 1A = histogram of all 8 first deriv. estimates, based on linear approximation:



→ $f_{i+1,j} - f_{i,j}$ /* derivatives can */

→ $(f_{i+1,j+1} - f_{i,j}) / \sqrt{2}$ /* be negative, zero, */

→ $f_{i,j+1} - f_{i,j}$ /* positive... */

...

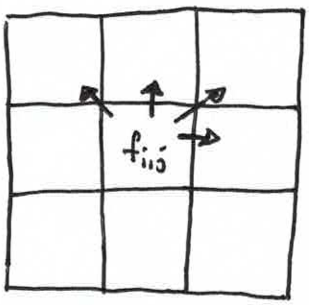
→ $(f_{i+1,j-1} - f_{i,j}) / \sqrt{2}$

→ indicates DIRECTION of derivative

Stratoran

■ LDAC - cont'd.

1) Histogram 1B = histogram of all 4 first deriv. estimates, based on quadratic approximation:



→ $(f_{i+1,j} - f_{i-1,j})/2$

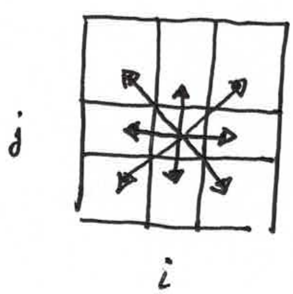
→ $(f_{i+1,j+1} - f_{i-1,j-1})/2\sqrt{2}$

→ $(f_{i,j+1} - f_{i,j-1})/2$

→ $(f_{i-1,j+1} - f_{i+1,j-1})/2\sqrt{2}$

- ISSUE: Should only the ABSOLUTE value of these estimates be considered? Is it sufficient? Is it in the spirit of invariance?

2) Histogram 2 = histogram of all 4 second deriv. estimates, based on quadratic approximation:



→ $(f_{i-1,j} - 2f_{i,j} + f_{i+1,j})$

→ $(f_{i-1,j-1} - 2f_{i,j} + f_{i+1,j+1})/2$

→ $(f_{i,j-1} - 2f_{i,j} + f_{i,j+1})$

→ $(f_{i-1,j+1} - 2f_{i,j} + f_{i+1,j-1})/2$

- NOTE: FOR SMOOTH, DIFFERENTIABLE FUNCTIONS $f(\mathbb{P})$:

$$\boxed{D_{\vec{dl}}(\mathbb{P}) = \vec{dl} \cdot \nabla f(\mathbb{P})}$$

$\mathbb{P}$ point
$\vec{dl}$ direction
D value of direct. deriv.

BUT: WE DEAL WITH DISCRETE DATA, WITH DISCONTINUOUS FUNCTIONS, WITH ABRUPT CHANGES IN f-VALUES!

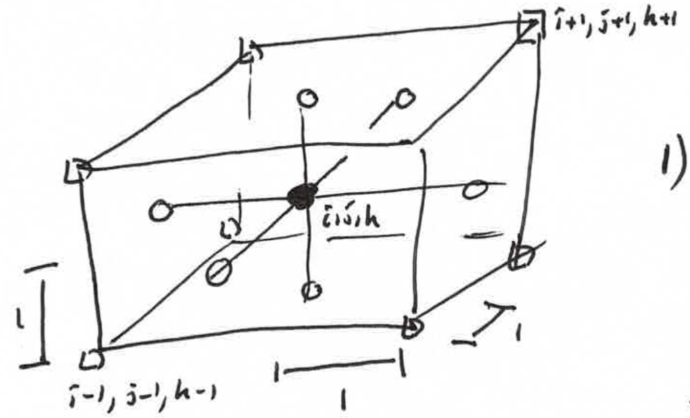
Bernd Hamann

Stratovan

6/3/19

LDAC - Cont'd

3D case, using a 3×3×3 voxel neighborhood



0) $\underline{Hist0}$ = hist. of all $f_{i,j,k}$-values

1) $\underline{Hist1A}$ = hist. of all $\underline{\underline{26}}$ first deriv. estimates (linear approximation):

→ $f_{i+1,j,k} - f_{i,j,k}$, ..., $(f_{i+1,j+1,k+1} - f_{i,j,k})/\sqrt{3}$, ...

$\underline{Hist1B}$ = hist. of all $\underline{\underline{13}}$ first deriv. estimates (quadratic approximation):

→ $(f_{i+1,j,k} - f_{i-1,j,k})/2$, ..., $(f_{i+1,j+1,k+1} - f_{i-1,j-1,k-1})/2\sqrt{3}$, ...

2) $\underline{Hist2}$ = hist. of all $\underline{\underline{13}}$ second deriv. estimates (quadratic approx.):

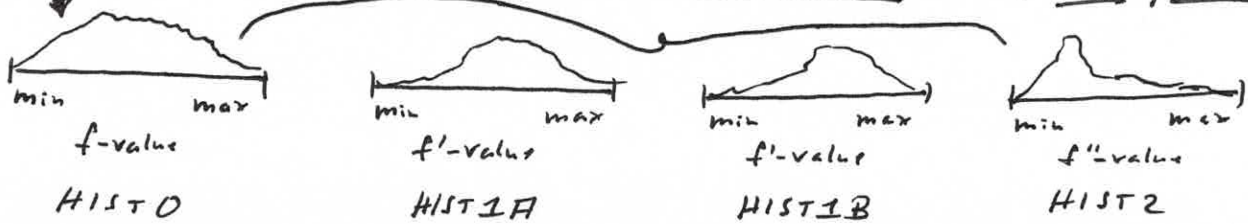
→ $(f_{i-1,j,k} - 2f_{i,j,k} + f_{i+1,j,k})/1$, ...

→ $(f_{i-1,j-1,k-1} - 2f_{i,j,k} + f_{i+1,j+1,k+1})/3$, ...

⇒ **RESULT**: $\underline{4}$ histograms "summarizing" the differential / variational behavior of the pixels / voxels constituting an "object":

!!! THESE $\underline{4}$ HISTOGRAMS TOGETHER DEFINE OBJECT / MATERIAL

2D; 3D — N $f_{i,j}$-values



2D ⇒	N $f_{i,j}$-values	$8N$ $f'_{i,j}$-values	$4N$ $f'_{i,j}$-values	$4N$ $f''_{i,j}$-values
3D ⇒	N "	$26N$ "	$13N$ "	$13N$ "

!!! ALL DERIV. ESTIMATES ARE DIRECTIONAL, THUS UNIVARIATE, ESTIMATES !!

Bernd Hamann

Stratovan

6/3/19

■ LDAC - cont'd.

Various Issues and Thoughts

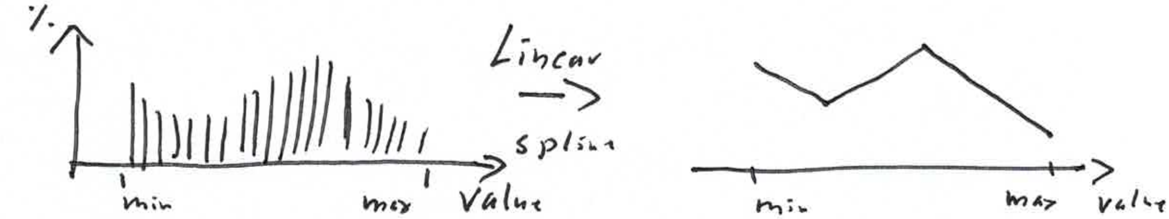
1) Directional deriv. estimates should be (or not?) ORIENTATION-INVARIANT !!??

⇒ BUT: We DO NOT want to construct smooth local functions from the given data - correct? Our OBJECTS ARE DISCONTINUOUS!

2) Result: An object is "described" (statistically) by multiple distributions: DISTRIBUTIONS of DIRECTIONAL DERIV. / DIFFERENTIAL BEHAVIOR, i.e., hist(f), hist(f'), hist(f''), ...

3) Is it of interest / desirable / good to consider "replacing" the histograms by, e.g., (non-negative) piecewise linear best spline approximations?

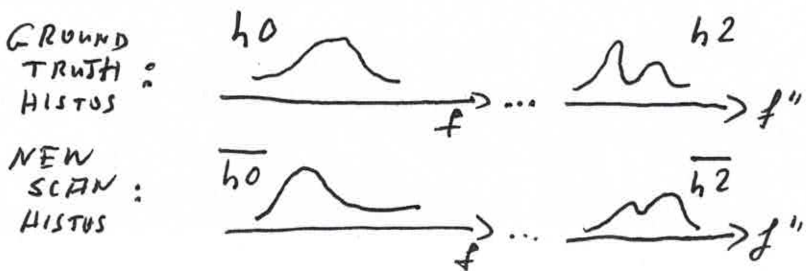
Example:



• Using linear spline ⇒ speed? better comparison of histograms?

4) Store this DIFFERENTIAL BEHAVIOR HISTOGRAM DATA for all "GROUND TRUTH" OBJECTS / MATERIALS! ⇒ DATA TO BE ANALYZED MUST BE COMPARED AGAINST "GROUND TRUTH HISTO'S"

5) COMPARISON VIA INNER PRODUCTS:



• CONDITIONS TO USE FOR "RECOGNITION":

$\underbrace{\langle h0, \overline{h0} \rangle < \varepsilon_0, \ \ldots, \ \langle h2, \overline{h2} \rangle < \varepsilon_2}$

⇒ Using logical AND or OR to recognize a ground truth material?

■ LDAC - cont'd.

6) For example, one could "concatenate" or "combine" all these differential histograms - yielding a "statistical definition" of a material.

Thus: • K ground truth materials ⇒ K ground truth histograms, called $H_1, .. H_K$.

- New segment extracted from a new scan to be analyzed - whether it matches one of the ground truth materials.
- $H_1, ..., H_K$ define a set of **BASIS HISTOS** that can be used to represent a new histogram to be analyzed, denoted as $\mathbf{H}$. One can solve:

$$\begin{pmatrix} \langle H_1, H_1 \rangle & & \langle H_1, H_K \rangle \\ \vdots & \cdots & \vdots \\ \langle H_K, H_1 \rangle & & \langle H_K, H_K \rangle \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_K \end{pmatrix} = \begin{pmatrix} \langle H, H_1 \rangle \\ \vdots \\ \langle H, H_K \rangle \end{pmatrix}$$

⇒ "Condition for recognition":

$\mathbf{H}$ represents a ground truth H_i iff nearly all its inner products with the H_i's are zero, except for one H_j! ~

⇒ OR: Assuming that all histos H_i and H are normalized, i.e., $\|H_i\| = \|H\| = 1$, $\langle H, H_i \rangle = 1$ means "perfect match" between "type H" and "type H_i" ~ BH