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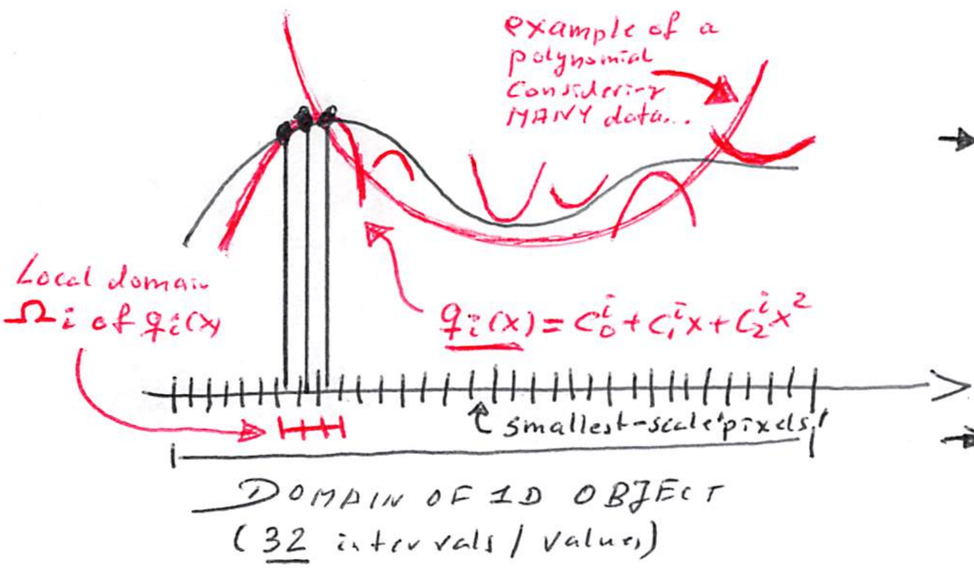
7/29/2019

■ TEMPLATES Used for Image Analysis

- IDEA: "Given an image, move a template / a filter over the image – still supporting certain 'shape parameters' for the template/filter – and determine a histogram(s) of the parameter value distributions (for each segment/object in the image)."

Example (1D case):



→ TEMPLATE = quadratic polynomial $q(x) = c_0 + c_1 x + c_2 x^2$

→ Determine LEAST SQUARES quadratic polynomials of the generic template form to a local neighborhood of data

→ Use a MULTI-RESOLUTION approach and compute these polynomials for increasingly coarser, lower-resolution versions of the image (i.e., for increasingly larger sub-regions in the image's domain)

→ Generate polynomials by considering 32, 16, 8, 4, ... values / intervals in the domain ⟺

⇒ Histograms & characterization of image at multiple scales!

→ One obtains histograms based on the c_0, c_1, c_2 coefficients of the quadratic polynomials, leading to more "meaningful" derived data values for function and first/second derivative values:

$$\overline{q_i} = \frac{1}{|\Omega_i|} \int_{\Omega_i} |q_i(x)|\, dx \;,$$

$$\overline{q_i'} = \frac{1}{|\Omega_i|} \int_{\Omega_i} |q_i'(x)|\, dx \;, \quad \overline{q_i''} = \frac{1}{|\Omega_i|} \int_{\Omega_i} |q_i''(x)|\, dx$$

TEMPLATES - Cont'd

• IDEA: [In other words]

"Define a template, e.g., a 'local TAYLOR APPROXIMATION template,' and move it across the region in an image domain that defines an object / segment; use templates that are more or less restricted (in terms of the parameter values adjustable for them), by specializing template-type and parameter-value-range to the specific object / material type, if possible."

• ISSUES: → Statistical (histogram) properties should be established for ALL GROUND TRUTH object / material types, at multiple resolutions.

→ IDEALLY, a SUFFICIENT AND NECESSARY statistical property characterization is generated (and used for later classification and recognition of types).

→ Therefore, a template / local approximation is 'ideal', when it permits the separation of all distinct types into distinct (non-overlapping) parameter-space regions of the used template parameters – and when such a separation would not be possible with less template parameters.

•Example: Idealized materials:

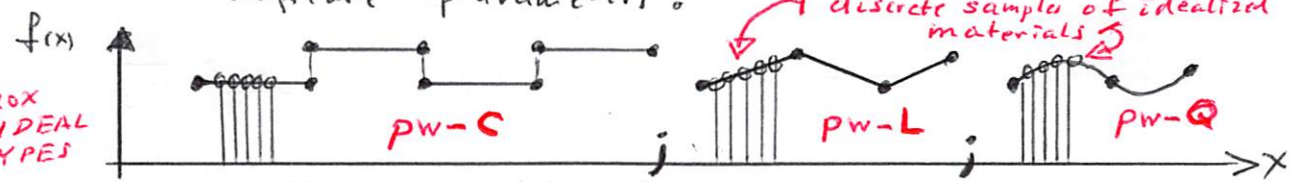
pw-C = piecewise CONSTANT

pw-L = piecewise LINEAR

pw-Q = piecewise QUADRATIC

⇕

QUADRATIC TAYLOR APPROX TEMPLATES IDEAL FOR THESE TYPES



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8/5/2019

■ Haar Wavelets (as Tensor Product Building Blocks) for TEMPLATE-based Image Analysis

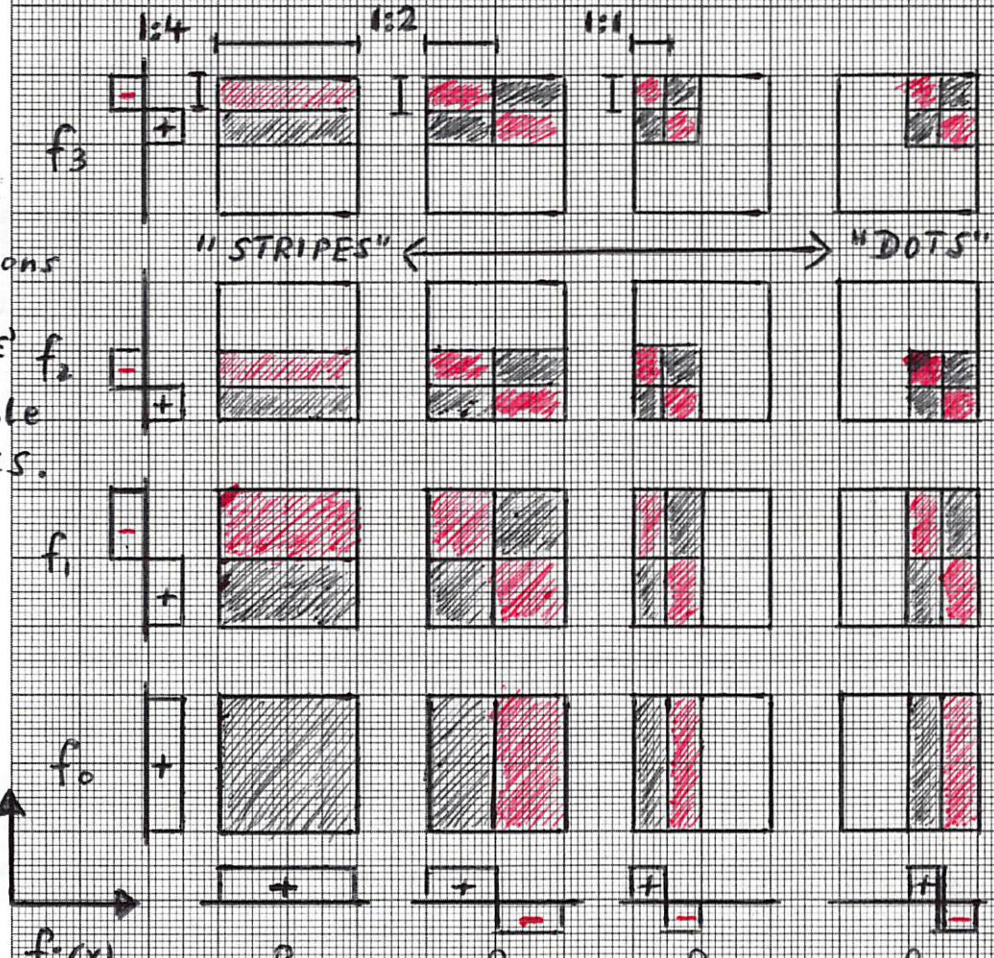
("HaWaLDAC" = Haar Wavelet LDAC)

[CUBIC]?

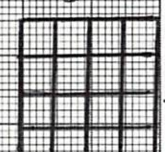
- **PRINCIPLE:** The IMAGES representing a wavelet basis define TEMPLATES that do – or do not – occur in a given image; thus images of basis functions serve classification.

- **2D EXAMPLE:** 16 basis functions of Haar wavelet:

Basis functions define 'TEXTURE' at multiple SCALES.



→ 4x4 pixel region:



→ orthonormal basis functions:

$b_{ij}(x,y) = f_i(x) \cdot f_j(y)$

→ $I(x,y) = \sum_j \sum_i c_{ij}\, b_{ij}(x,y)$

→ $\{b_{ij}\}$ = TEXTURE TEMPLATES! at multiple scales!!

⇒ **USE 4x4 WINDOW, COMPUTE HISTOGRAMS FOR c_{ij}'s**

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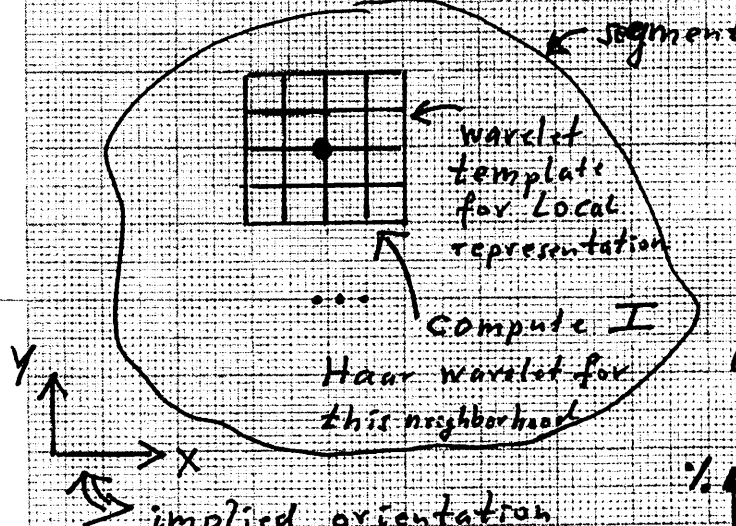
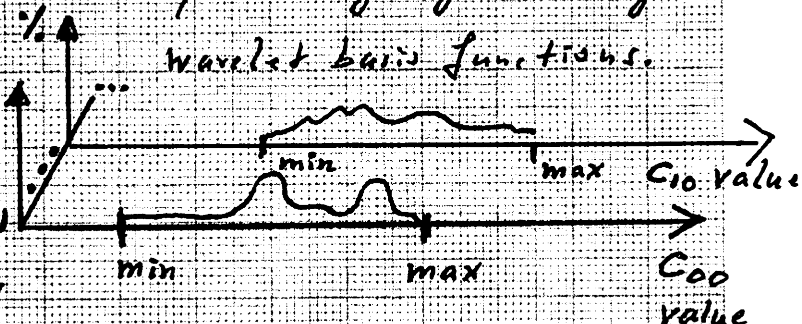
■ Haar Wavelets ... for Image Analysis – Cont'd.

• **IDEA:** Wavelet coefficient histograms to define and classify texture



(i) Compute wavelet coeff's. for all 4x4 templates with centers inside segment

(ii) Define coeff. histograms by meaningfully ordering the wavelet basis functions.



For each pixel: $\underline{I = \sum_j \sum_i c_{ij}\, b_{ij}(x,y)}$

⇒ c_{ij}-values are signed and have different value ranges.

⇒ 16 coeff value histograms

• **ADVANTAGE:** The multi-scale Haar wavelet basis implies coefficient value histograms capturing the multi-scale image texture.

→ "The Haar basis functions can be viewed as 'templates' that capture large- and small-scale behavior (thick/thin structure)."

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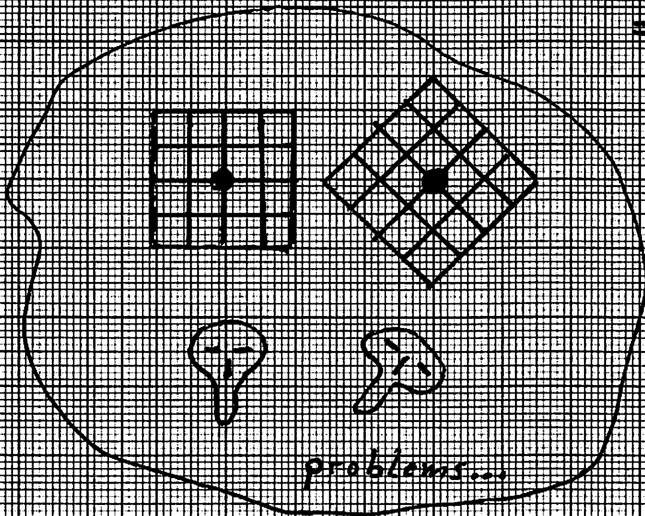
Haar Wavelets – Cont'd.

- PROBLEM: DEPENDENCE ON ORIENTATION



⟹ Wavelet coefficient value histograms DEPEND on ORIENTATION!

GOAL:
Wavelet coeff. value histograms that are computed in a (nearly) orientation-invariant way!

{NOTE: It is NOT desirable to use, for example, 180 different orientations of the 4×4 'wavelet basis grid', re-sample the given data onto all their rotated 4x4 grids, and then compute coeff. value histograms for all 180 orientations.
— TOO EXPENSIVE, TOO MUCH ERROR!}

- SOLUTION:
  1) Establish a local neighborhood-inherent orientation for a DISK
  2) Establish a POLAR (LOG-POLAR) MAP disk to rectangle domain and vice versa.
  3) Compute Haar wavelet coeffs. for RECTANGLE domain.

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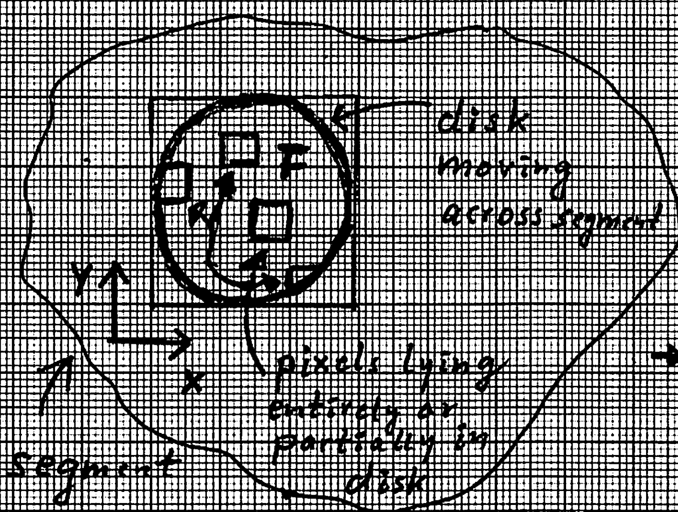
■ Haar Wavelets: ROTATION-INVARIANT SIGNATURES

1) Compute local least-squares QUADRATIC POLYNOMIAL, considering the given pixel intensity values inside a DISK. Thus, the given values define a piecewise-constant function defined for the disk, to be optimally approximated by the quadratic polynomial $q(x,y) = \sum_{\substack{i+j\le 2\\ i,j\ge 0}} c_{ij}\, x^i y^j$.



→ Piecewise-constant function defined by intensity values: F

→ The basis functions of q are $b_{ij} = b_{ij}(x,y) = x^i y^j$.

→ To obtain the best quadratic approximation of F one must solve

$\langle\cdot,\cdot\rangle$ inner product

$$\begin{pmatrix} \langle b_{00}, b_{00}\rangle & \cdots & \langle b_{00}, b_{02}\rangle \\ \vdots & & \vdots \\ \langle b_{02}, b_{00}\rangle & \cdots & \langle b_{02}, b_{02}\rangle \end{pmatrix} \begin{pmatrix} c_{00} \\ \vdots \\ c_{02} \end{pmatrix} = \begin{pmatrix} \langle F, b_{00}\rangle \\ \vdots \\ \langle F, b_{02}\rangle \end{pmatrix}$$

→ Compute the CONSTANT HESSIAN of $q(x,y)$. Determine the 2 eigenvalues of the Hessian. Establish a 'standard' to be used to define the eigendirection used as 'data-inherent x-direction'.

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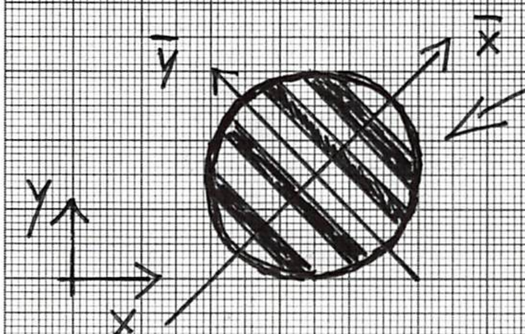
Haar Wavelets: ...

2) • The intermediate result of step 1) is a data-inherent local coordinate system for the disk.

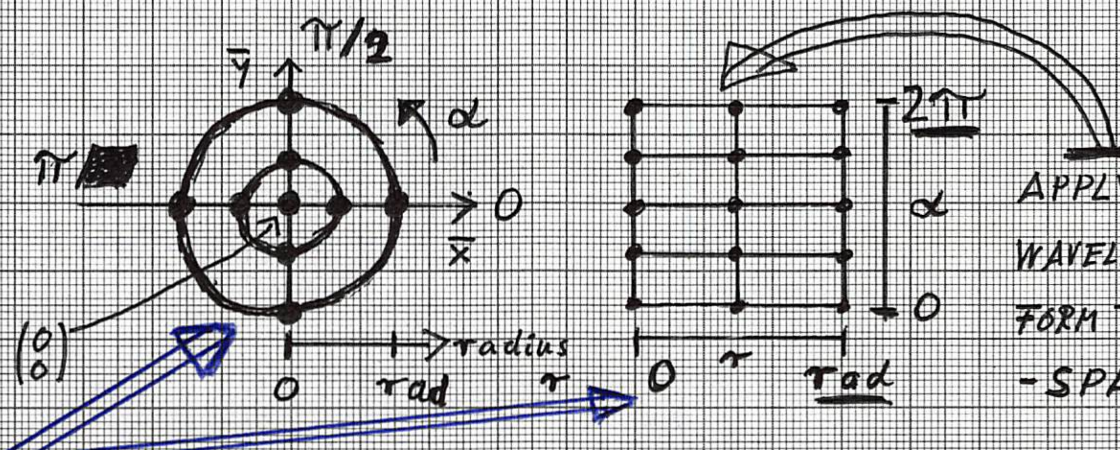


"General example / idea": A function/image with this parallel-stripes pattern has highest variation in its own inherent $\bar{x}$-direction and no variation (here) in $\bar{y}$-direction.

$\Rightarrow$ Similarly, the largest eigenvalue of the Hessian of $g(x,y)$ defines the inherent $\bar{x}$-direction.

• The Cartesian-grid-based nature of the tensor product Haar wavelet transformation requires us to map the DISK TO A RECTANGLE:

MUST RE-SAMPLE PIXEL VALUES INSIDE DISK ONTO POWER-OF-TWO GRID OF (LOG-) POLAR RECTANGLE !!



APPLY HAAR WAVELET TRANSFORM TO (r,α)-SPACE !!!

(i) Polar coordinate map: $\begin{pmatrix} r \\ \alpha \end{pmatrix} \mapsto \begin{pmatrix} r\cos\alpha \\ r\sin\alpha \end{pmatrix} = \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$

(ii) Log-polar coord. map(*): $\begin{pmatrix} r \\ \alpha \end{pmatrix} \mapsto \begin{pmatrix} e^r\cos\alpha \\ e^r\sin\alpha \end{pmatrix} = \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$

(*) Compensates for the fact that concentric circles increase in length with increasing radius...

~BH