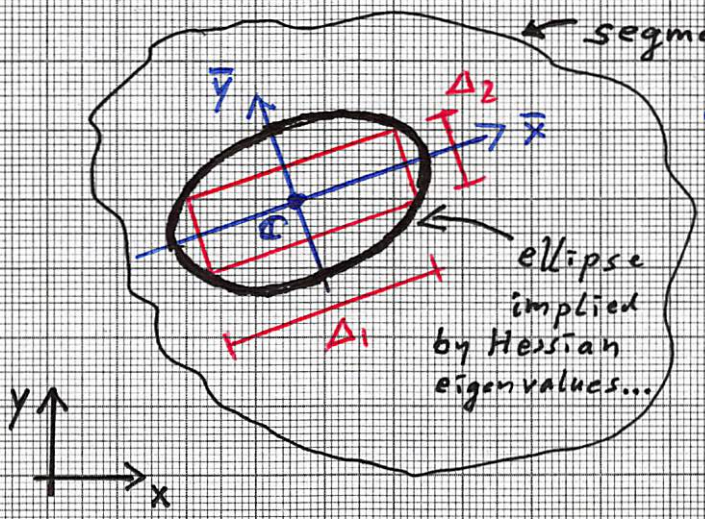
Stratovan

■ HAAR WAVELETS – CONT'D.

- Using ORIENTED and SCALED RECTANGLES as domains to perform local Haar wavelet analysis

  ⇒ SIMPLIFICATION!

IDEA: 1) In the physical (x,y)-space domain of an image segment, perform a HESSIAN-based analysis to define, locally, ORIENTATION and SCALE for a local neighborhood of intensity values:



- $\mathbb{C}$ = local center / origin for approximation / analysis
- $\bar{x}$-axis, $\bar{y}$-axis: orientation defined by Hessian eigenvectors
- Δ_1, Δ_2 = scaling parameters, based on Hessian eigenvalues, for rectangle

2) For each local rectangle domain, perform a Haar wavelet analysis directly with, for example, 16 fully mutually orthonormal basis functions:

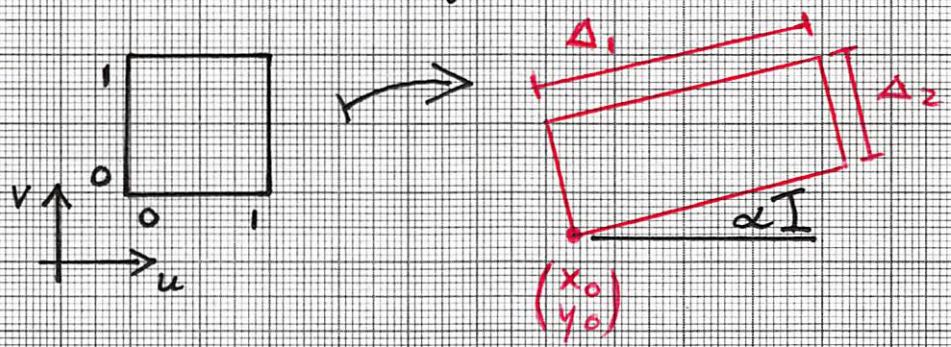
⇒ $c_i = \langle F, f_i \rangle$

Stratovan

■ HAAR WAVELETS - Cont'd.

3) Mapping of UNIT SQUARE to ARBITRARY RECTANGLE:

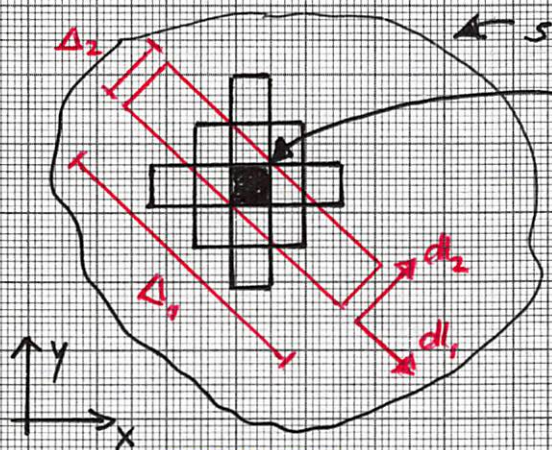
(used for change-of-variables theorem ...)



$$\begin{pmatrix} u \\ v \end{pmatrix} \mapsto \text{Translate}\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \cdot \text{Rotate}(\alpha) \cdot \text{Scale}(\Delta_1, \Delta_2) \cdot \begin{pmatrix} u \\ v \end{pmatrix}$$

$$= \begin{pmatrix} x_0 + u\Delta_1 \cos\alpha - v\Delta_2 \sin\alpha \\ y_0 + u\Delta_1 \sin\alpha + v\Delta_2 \cos\alpha \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

4) Algorithmic design decisions:



← segment

■ = pixel and "dish-like neighborhood" of pixels

- values of ■ and its neighbors used to define LOCAL LEAST SQUARES QUADRATIC APPROXIMATION:

$$app(x,y) = \sum_{\substack{i,j \geq 0 \\ i+j \leq 2}} c_{i,j}\, x^i y^j$$

- ISSUES:

(i) How many '□' to use?

(ii) How to scale Δ_1, Δ_2?

$\Rightarrow$ Hessian of $app(x,y) = \begin{pmatrix} 2c_{2,0} & c_{1,1} \\ c_{1,1} & 2c_{0,2} \end{pmatrix}$

$\Rightarrow$ eigenvalues and eigenvectors of Hessian

Stratovan

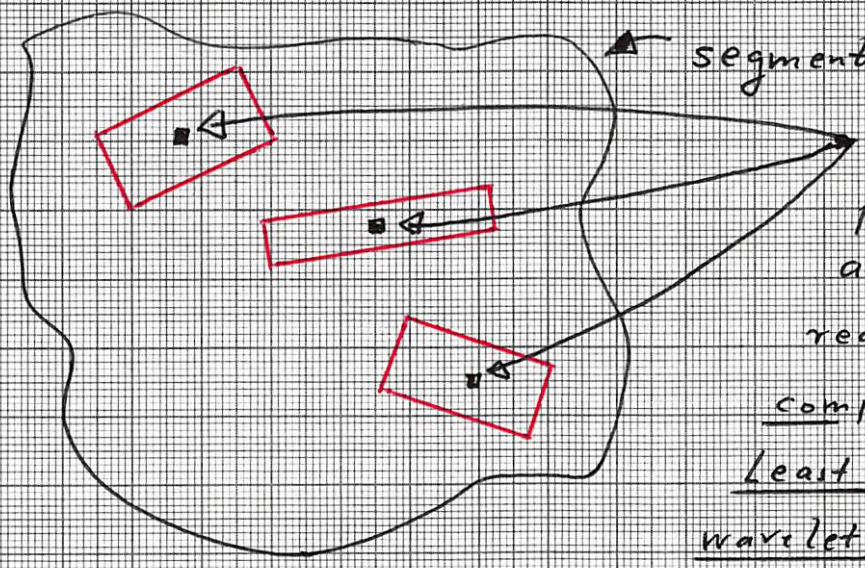
■ HAAR WAVELETS - Cont'd.

• Related paper: Feng, Hotz, Hamann, Joy - "Anisotropic Noise Samples"

goal: construction of set of anisotropic elliptical regions - non-overlapping and dense - for texturing.

OUR goal: construction of set of anisotropic elliptical regions - dense and possibly overlapping - for defining Local least squares Haar wavelet image approximations and wavelet-coefficient-based "signatures"

⇒ Simplification: USE RECTANGLES



for each (?) pixel '■' define an associated rectangle and compute the best, Least squares Haar wavelet approximation.

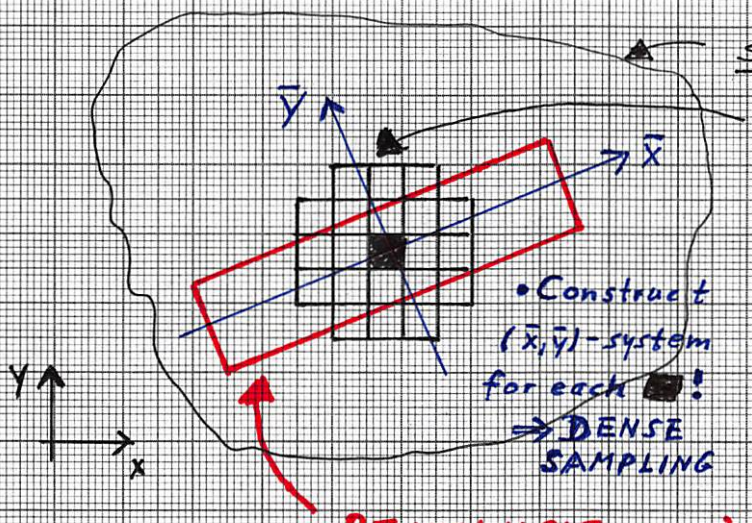
⇒ Consider the given intensity values inside the rectangles for each Haar wavelet approximation.

Stratovan

- ISSUES:
  - → Must consider all N pixels?
  - → Good value of K = ?
  - → Good no. of pixels in rectangle = ?

HAAR WAVELETS - Cont'd.

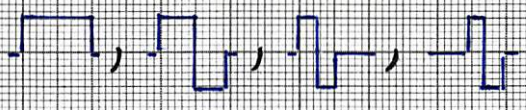
5) Summary of Haar wavelet analysis with RECTANGLES for SCALE- and ORIENTATION-INDEPENDENT MULTI-SCALE characterization of a segment



K pixels in a "disk-like neighborhood of ■, used to construct Local quadratic least squares approximation

RECTANGLE scaled and oriented based on Eigenvalues and eigenvectors of Hessian of quadratic approximation

- Compute a local least squares (best) Haar wavelet approximation for all given intensity values inside the rectangle.

  NOTE: Use as basis functions the 16 tensor product functions based on these 4 univariate functions: 

- One obtains 16 Haar wavelet coefficients per pixel. (All pixels should be considered as 'centers' for a Haar wavelet approximation...) Determine 16 HISTOGRAMS of coefficint values, one histogram per coefficient.
  → MULTI-SCALE "TEXTURE SIGNATURE" ∴ BH

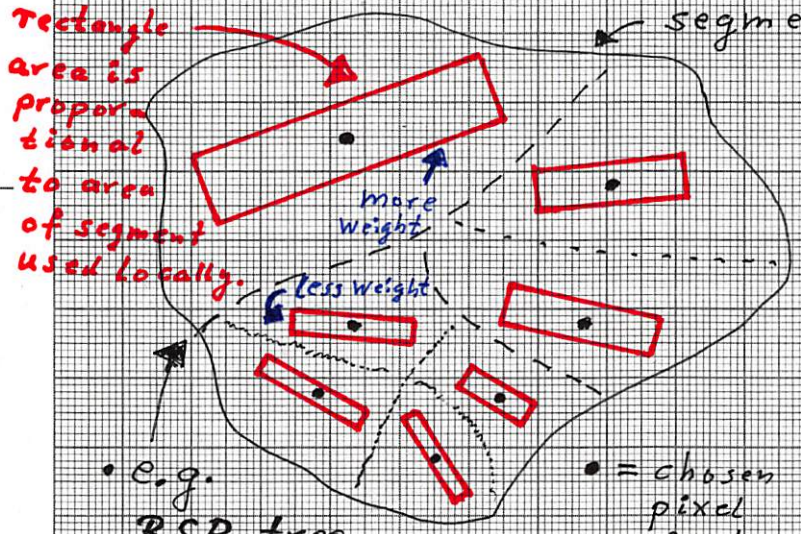
Stratovan

■ HAAR WAVELETS – Cont'd.

- **COMPUTATIONAL COST / EFFICIENCY** of wavelet coefficient computation!

Goal: Establishment of histograms for values of wavelet coefficients c_i that are "good enough" for classification!

Efficiency – Approaches:



- e.g. BSP tree subdivision (rectangles based on more local pixels affect histograms more)
- = chosen pixel for local wavelet analysis

→ Perform wavelet approximation adaptively.

→ Do NOT perform local wavelet approx. for every pixel's neighborhood; select/distribute pixels (•) in segment based on "variation"!

→ SIMPLE: Select randomly a sufficiently large set of (•).

→ ITERATIVE REFINEMENT: Continue selecting additional (•) until histograms (normalized) no longer change.

→ SPATIAL TREE-BASED REFINEMENT:

Use binary space partition (BSP) approach to refine set of (•) until histograms no longer change.