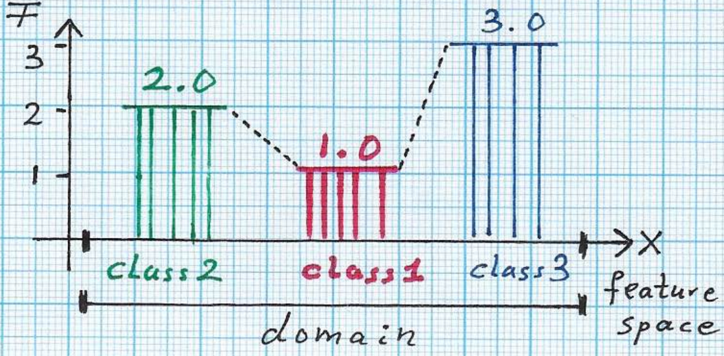
RBFs AND CLASSIFICATION – Cont'd.

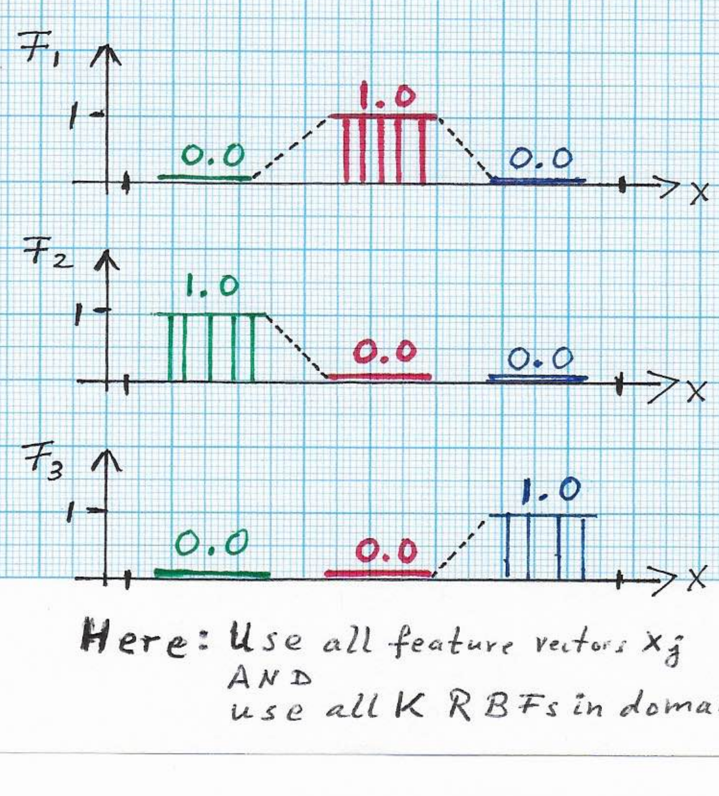
→ More specific issues...

i) **LOCATION / POSITION:** Depending on the classification problem, it could be desirable to use a pixel's/voxel's positional information as part of the feature vector used to define the pixel's/voxel's characteristics.

ii) **SCALAR- AND VECTOR-VALUED RBF EXPANSION:**



- Construct a single scalarvalued classifier function $\mathcal{F}(x)$:
  $$\mathcal{F}(x) = \sum_{k=1}^{K} c_k \, \tau_k(x),$$
  where $\mathcal{F}(x_j) \in \{1,2,3\}$, i.e., $\mathcal{F}(x_j)$ = class index of x_j.



- Construct three class-specific functions $\mathcal{F}_i(x)$ defining a vectorvalued classifier function $\mathbb{F}(x)$:
  $$\mathbb{F}(x) = \left(\mathcal{F}_1(x), \mathcal{F}_2(x), \mathcal{F}_3(x)\right)^T,$$
  where $\mathcal{F}_i(x_j) = 1$, if x_j's class index is i (= 0, otherwise).

Here: Use all feature vectors x_j
AND
use all K RBFs in domain.

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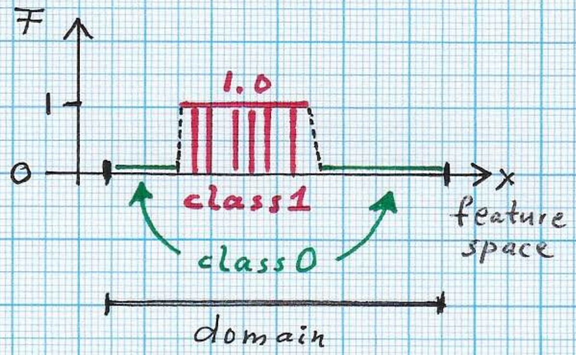
■ RBFs AND CLASSIFICATION – Cont'd.

➔ More issues...

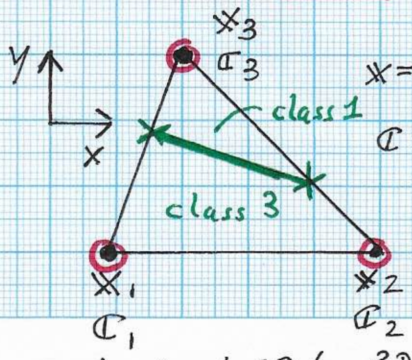
iii) "**NOT-A-CLASS CLASS**": • Use also training data in the domain that represent either "empty space" or material types that are not of interest for classification (NOT-A-CLASS class data). Assign the value 0 to this data – and use 0 as interpolation/approximation condition when computing the RBF expansion.



$\Rightarrow F(\mathbf{x}_j) = 0$ for all feature vector values representing a material not to be classified

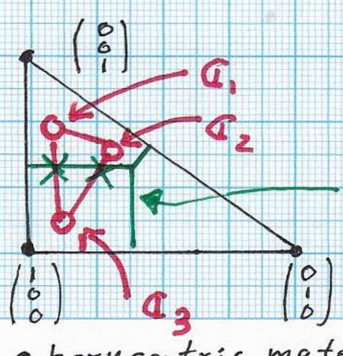
iv) MATERIAL INTERFACES:

➔ Potential relationship between certain "material boundary"/"material interface" extraction methods and establishing the boundaries of "classes in feature space"



$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$

$\mathbb{C} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$, $\sum c_i = 1$



perpendicular bisector $\Rightarrow$ VORONOI tessellation $\Rightarrow$ MAP 2 intersections × back to physical space $\Rightarrow$ interface between classes 1 and 3

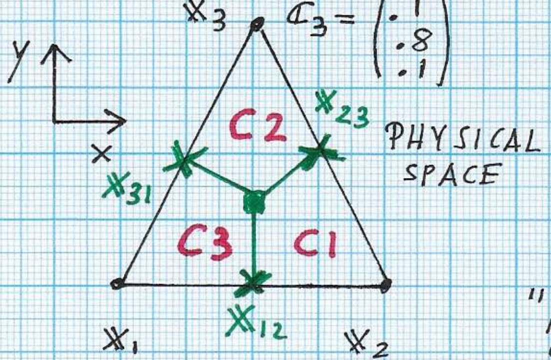
• physical 2D (or 3D) space: corners of triangle (or tet) with "material fractions" c_i

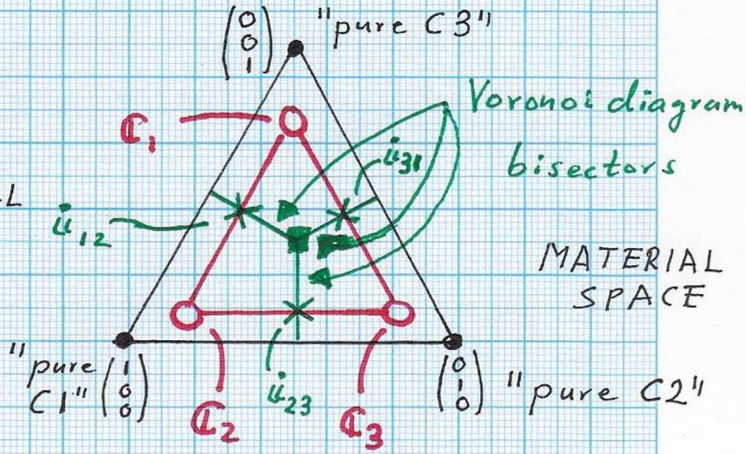
• barycentric material space for 3 material classes (use triangle)

RBFs AND CLASSIFICATION – Cont'd.

... MATERIAL INTERFACES: ...

- Ex.: 3-material case, all 3 material classes C1, C2, C3 present in one physical space triangle





$c_1 = \begin{pmatrix} .1 \\ .1 \\ .8 \end{pmatrix}$ $c_2 = \begin{pmatrix} .8 \\ .1 \\ .1 \end{pmatrix}$

e.g.: line segment $\overline{c_1 c_2}$ intersects bisector in i_{12}

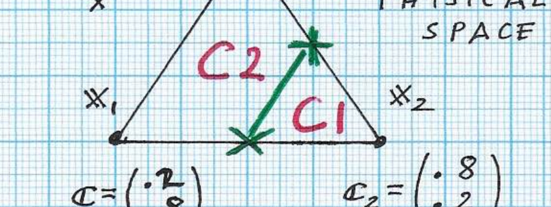
interface point x_{12} on edge $\overline{x_1 x_2}$:

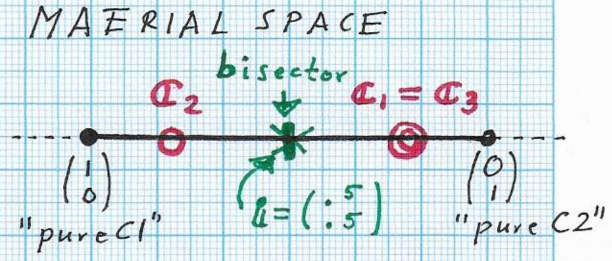
$x_{12} = (1-t)\, x_1 + t\, x_2$

$\Rightarrow$ $i_{12} = (1-t)\, c_1 + t\, c_2$ (material space)

$\Rightarrow$ $t = \ldots$

- Ex.: 2-material case, both material classes C1, C2 present in one physical space triangle





$c_1 = \begin{pmatrix} .2 \\ .8 \end{pmatrix}$ $c_2 = \begin{pmatrix} .8 \\ .2 \end{pmatrix}$

$x_{12} = \frac{1}{2} x_1 + \frac{1}{2} x_2$

$x_{23} = \frac{1}{2} x_2 + \frac{1}{2} x_3$

$\Rightarrow$ $i = \frac{1}{2} c_2 + \frac{1}{2} c_1$

$i = \frac{1}{2} c_2 + \frac{1}{2} c_3$

RBFs AND CLASSIFICATION – Cont'd.

... MATERIAL INTERFACES: ...

→ Application to 2D/3D image segmentation / "class interface" extraction: must adapt the material interface extraction method:

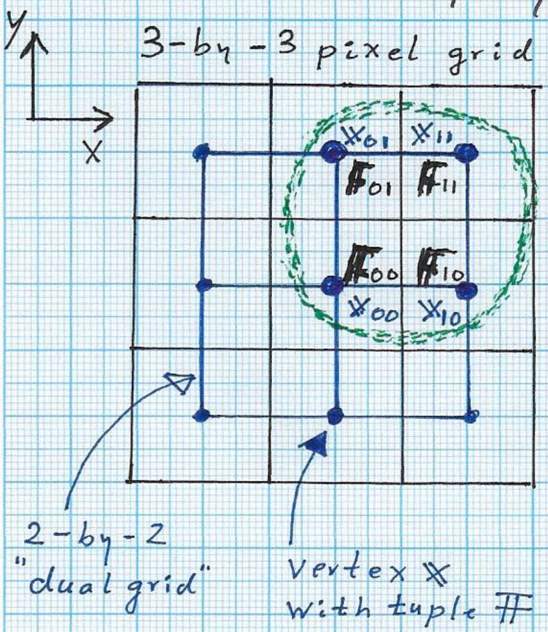
- PHYSICAL SPACE: The center points of pixels (2D) or voxels (3D) become $\mathbb{X}$ vertices; these points have $\mathbb{F}$ vectors associated with them, i.e., $\mathbb{F}(\mathbb{X}) = (F_1(\mathbb{X}), F_2(\mathbb{X}), F_3(\mathbb{X}), \ldots)^T$, where the dimension of $\mathbb{F}$ is the number of classes [redacted] used [redacted]. IDEALLY, $0 \leq F_i \leq 1$ and $F_i = 1$ (or 'F_i is nearly 1') implies that a point $\mathbb{X}$ belongs to class i.

- MATERIAL SPACE: Assuming that the class-specific RBF expansions $F_i(\mathbb{X})$ vary between 0 and 1, material space becomes a "class space" to be analyzed over a unit hyper-cube $[0,1]^d$ whose dimension is given by the number of $F_i(\mathbb{X})$ functions. The problem of determining boundaries of / interfaces between classes in physical 2D/3D image space requires one to determine in what sub-regions of the hyper-cube specific $\mathbb{F}$-tuples lie – and to use that information to define 2D/3D image space boundaries.

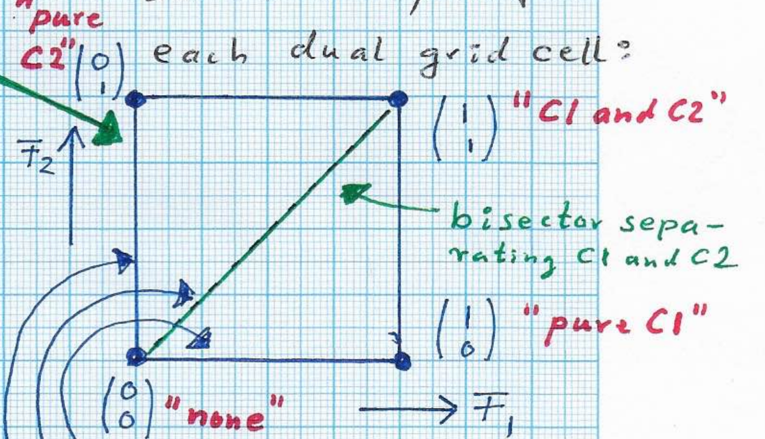
RBFs AND CLASSIFICATION – Cont'd.

... MATERIAL (= CLASS) INTERFACES: ...

- Ex.: 2D image (= set of pixels) with 2-dim. [redacted] vector $\mathbb{F} = (F_1(x), F_2(x))^T$ for each pixel ⇒ Extract boundaries of / interfaces between class C1 and class C2 in physical image!



⇒ 2-dim. CLASS SPACE to be analyzed for each dual grid cell:

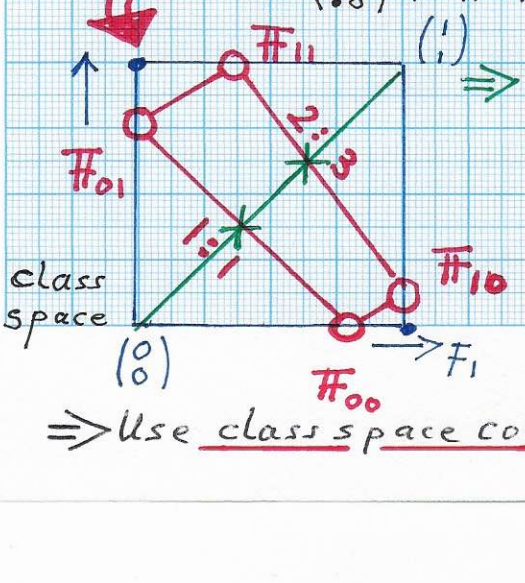


$F2=0,\ 0<F_1$ ⇒ "only C1"

$F1=F2,\ 0<F_1, F_2$ ⇒ "equal mix"

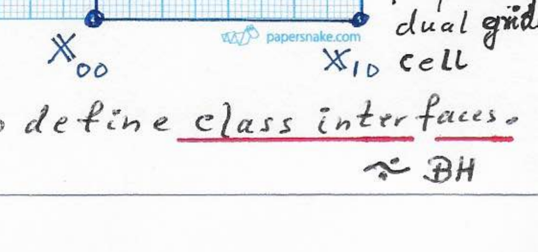
$F1=0,\ 0<F_2$ ⇒ "only C2"

e.g., $\mathbb{F}_{00} = \binom{.8}{0}$, $\mathbb{F}_{10} = \binom{1}{.1}$, $\mathbb{F}_{01} = \binom{0}{.8}$, $\mathbb{F}_{11} = \binom{.3}{1}$



⇒ 2 interface points x, given by intersections between bisector $F_1 = F_2$ and $\overline{\mathbb{F}_{00}\mathbb{F}_{01}}$ and $\overline{\mathbb{F}_{10}\mathbb{F}_{11}}$

⇒



⇒ Use class space configuration to define class interfaces.

~BH