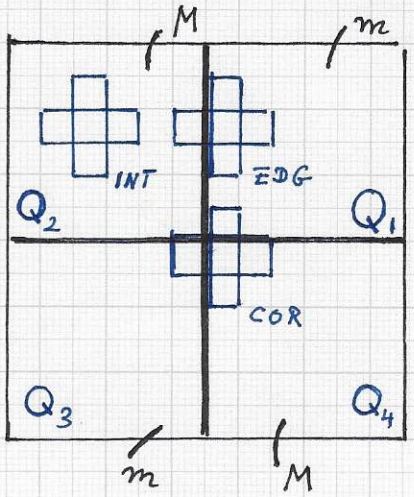
OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• Laplacian eigenfunctions:



The possible cases when applying the 5-pixel mask to the 12×12 image segment: INTERIOR, EDGE and CORNER.

We denote the total number of masks inside the image segment by N; the numbers of interior, edge and corner masks are N_I, N_E and N_C, respectively; $N = N_I + N_E + N_C$.

The ratio $N_I : N_E : N_C$ has impact on the "combined or averaged spectral response" when aggregating all individual 5-pixel mask responses.

We consider the case of the 2×2 checkerboard texture represented on a 12×12 2D image segment (Left). Three cases arise when applying the 5-pixel mask to all possible pixel subsets lying entirely inside the segment. The three cases are:

1) INTERIOR – The mask called "INT" consists only of pixels entirely inside one quadrant, Q_2; this is an interior case.

2) EDGE – The mask called "EDG" consists only of pixels from quadrants Q_1 and Q_2, making this mask an edge mask (vertical edge mask); this is an edge (vertical edge) case.

3) CORNER – The mask called "COR" consists of pixels from more than two quadrants, i.e., pixels from quadrants Q_1, Q_3 and Q_4, representing an example of the corner case.

We briefly summarize the resulting eigenfunction coefficient values c_i for these cases.

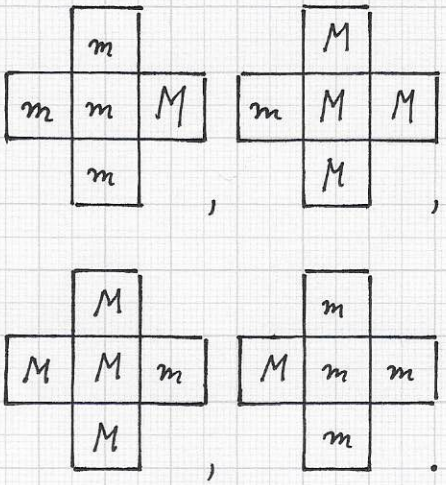
1) INTERIOR case: [2 sub-cases]

$$\mathbf{c} = \sqrt{2}/2\ (m, 0, 0, 0, 3m) \quad \text{or}$$

$$\mathbf{c} = \sqrt{2}/2\ (M, 0, 0, 0, 3M).$$

■ OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• Laplacian eigenfunctions: 2) (Vertical) EDGE case: [4 sub-cases]

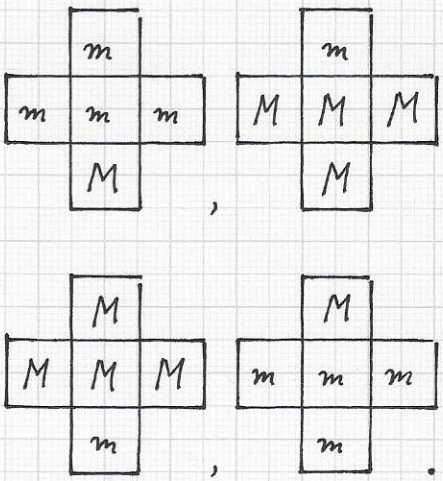


The 4 possible cases of a "vertical edge" passing through the 5-pixel mask.

The figure (left, top) shows the 4 sub-cases that arise when a "perfectly vertical edge" (a "large discontinuity" in mass function value) is encountered by the 5-pixel mask. Assuming that the shown 4 sub-cases occur with the same probability, one is interested in the AVERAGE SPECTRAL RESPONSE of the mask. The average coefficient vector is

$$\mathbf{c} = \sqrt{2}/4\,(m+M,\ 0,\ 0,\ 0,\ 3(m+M)).$$

(Horizontal) EDGE case: [4 sub-cases]



The 4 possible cases of a "horizontal edge" passing through the 5-pixel mask.

The figure (left, bottom) shows the 4 sub-cases possible for a "perfectly horizontal edge. In this situation, the AVERAGE SPECTRAL RESPONSE of the mask for the coefficient vector is

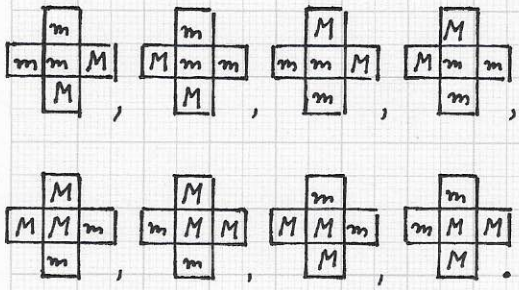
$$\mathbf{c} = \sqrt{2}/4\,(m+M,\ 0,\ 0,\ 0,\ 3(m+M)).$$

3) CORNER case: [8 sub-cases]

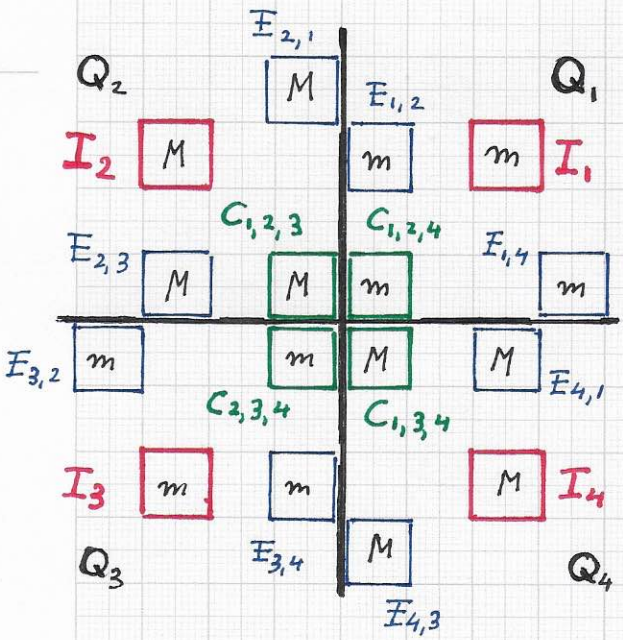
In this case, the pixels of the 5-pixel mask lie in 3 of the 4 quadrants, see figure on p. 21 (previous page). The pixels of the mask lie in Q_1, Q_2, Q_4 or Q_1, Q_2, Q_3 or Q_2, Q_3, Q_4 or Q_2, Q_3, Q_4 ....

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions:



The 8 possible cases of a "corner" j pixels of the 5-pixel mask lie in 3 quadrants.



Depiction of combinatorially possible cases of the location of the center pixel of the 5-pixel mask. Only the center pixel location of the mask is shown. This example depicts the idealized case of a checker board pattern of resolution 2x2, where mass values m and M alternate for neighbor quadrants Q.

... The 8 possible corner sub-cases are sketched in the figure (left). The 8 sub-cases also happen to belong to the class "stripe pattern", S3, types 1&2, and S4, types 1&2, see p. 19 (11/2/2021). The 4 sub-cases in the bottom row of the figure (left) are the "duals" of the 4 sub-cases in the top row. (Values m and M are exchanged.) The AVERAGE SPECTRAL RESPONSE of the mask for the coefficient vector is

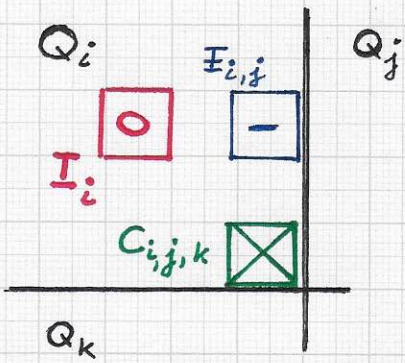
$$\mathbb{C} = \sqrt{2}/4\,(m+M,\ 0,\ 0,\ 0,\ 3(m+M)).$$

• Considering the example of a "perfect" checker board pattern of alternating values m and M (associated with quadrants Q_1, Q_2, Q_3, Q_4 defining a pattern of resolution 2x2), we summarize the possible spectral response behavior of the mask, see left figure (bottom). Center pixels of a 5-pixel mask are "I" (interior), "E" (edge) or "C" (corner) pixels. The indices of an "I", "E" or "C" mask center pixel refer to the quadrants that contain the mask's pixels.

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

•Laplacian eigenfunctions: ... The indices of the types of mask center pixels have the following meaning:



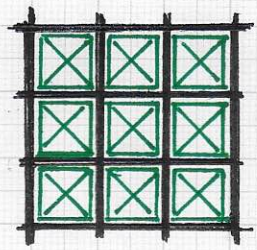
Interior, edge and corner mask center pixels in Q_i.

I_i : interior mask center pixel in Q_i

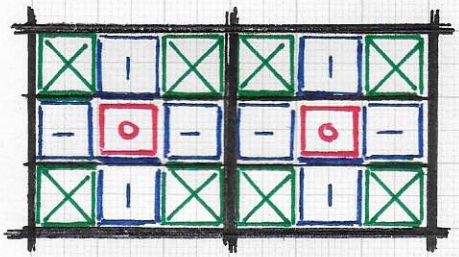
$E_{i,j}$: edge mask center pixel in Q_i, with pixels also in Q_j

$C_{i,j,k}$: corner mask center pixel in Q_i, with pixels also in Q_j and Q_k

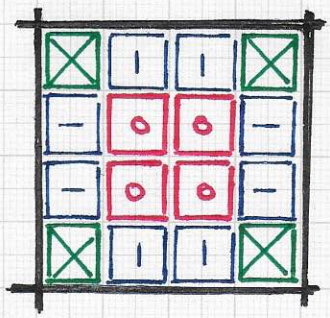
The figure on the previous page also shows the mass function value associated with the mask center pixels, m or M. By exchanging m and M one obtains the DUAL depiction of combinatorially possible cases.



corner case only



interior, edge and corner cases



interior, edge and corner cases

Checker board pattern elements:
Top: 9 elements, each of resolution 1×1;
Middle: 2 elements, each of resolution 3×3;
Bottom: 1 element of resolution 4×4.

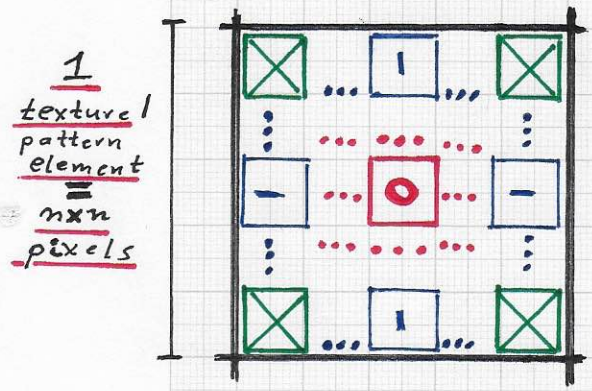
ARE ALL THE CASES IMPORTANT? HOW MANY OF THESE CASES ARE RELEVANT FOR CLASSIFICATION? ARE SOME OF THE CASES EQUIVALENT IN A QUALITATIVE SENSE, AND CAN ONE "MERGE" CERTAIN CASES WITHOUT IMPACTING THE DESIRED ANALYSIS OUTCOME USED FOR EVENTUAL CLASSIFICATION?

To be able to discuss and answer these questions, one must consider issues related to image/segment resolution, texture/pattern resolution, mask resolution, relationships between these resolutions and limit behavior.

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

•Laplacian eigenfunctions: ... The figures shown on this page (left, top) and the previous page (Left, bottom) illustrate the general and three specific cases one must consider when analyzing the behavior of the spectral response of the mask under different "resolution conditions." Specifically, one must understand how the location of the mask's center pixel influences the spectral response. The mask center pixel can have one of these three types:



Single checkerboard texture/pattern element. Shown are some mask center pixels and their types when applying the mask throughout the element. Corner (X), edge (–, |) and interior (o) types are shown. This element represents an "idealized" case for which an exact analysis of mask behavior is possible.

n	#C	#E	#I
1	1	0	0
2	4	0	0
3	4	4	1
4	4	8	4
5	4	12	9
6	4	16	16
7	4	20	25
8	4	24	36

Numbers of corner (C), edge (E) and interior (I) pixels (= mask center pixels for varying n values.

The numbers of mask center pixels of type "interior," type "edge" and type "corner" define the final (averaged, combined, histogram-characterized) spectral response for an individual texture/pattern element like the one shown here (left, top). These numbers are:

$$\#C = \begin{cases} 1 & \text{if } n = 1 \\ 4 & \text{if } n > 1 \end{cases},$$

$$\#E = \begin{cases} 0 & \text{if } n = 1 \text{ or } n = 2 \\ 4(n-2) & \text{if } n > 2 \end{cases},$$

$$\#I = \begin{cases} 0 & \text{if } n = 1 \text{ or } n = 2 \\ (n-2)^2 & \text{if } n > 2 \end{cases}.$$