■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ... Table of all 5-pixel mask responses...

	Case	$\mathbb{C} = (c_1, c_2, c_3, c_4, c_5)$	Average $\mathbb{C}$-tuple of dual cases
HORIZONTAL EDGE	⑦ m m m m M	$\sqrt{2}/4\,(m+M,\ 3(M-m),\ m-M,\ m-M,\ 5m+M)$	$\sqrt{2}/4\,(m+M,\ 0,0,0,\ 3(m+M))$
	⑧ M M M M m	$\sqrt{2}/4\,(m+M,\ 3(m-M),\ M-m,\ M-m,\ 5M+m)$	
	⑨ m M M M M	$\sqrt{2}/4\,(m+M,\ M-m,\ M-m,\ M-m,\ 5M+m)$	$\sqrt{2}/4\,(m+M,\ 0,0,0,\ 3(m+M))$
	⑩ M m m m m	$\sqrt{2}/4\,(m+M,\ m-M,\ m-M,\ m-M,\ 5m+M)$	
EIGHT CORNER CASES ...	⑪ m m m M M	$\sqrt{2}/2\,(M,\ M-m,\ M-m,\ m-M,\ 2m+M)$	$\sqrt{2}/4\,(m+M,\ 0,0,0,\ 3(m+M))$
	⑫ M M M m m	$\sqrt{2}/2\,(m,\ m-M,\ m-M,\ M-m,\ 2M+m)$	
	⑬ m M m m M	$\sqrt{2}/2\,(M,\ M-m,\ m-M,\ M-m,\ 2m+M)$	$\sqrt{2}/4\,(m+M,\ 0,0,0,\ 3(m+M))$
	⑭ M m M M m	$\sqrt{2}/2\,(m,\ m-M,\ M-m,\ m-M,\ 2M+m)$	
	⑮ M m m M m	$\sqrt{2}/2\,(M,\ m-M,\ M-m,\ m-M,\ 2m+M)$	$\sqrt{2}/4\,(m+M,\ 0,0,0,\ 3(m+M))$
	⑯ m M M m M	$\sqrt{2}/2\,(m,\ M-m,\ m-M,\ M-m,\ 2M+m)$	

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ... Table of all 5-pixel mask responses ...

... CORNER CASES

Case	$\mathbb{C} = (c_1, c_2, c_3, c_4, c_5)$	Average $\mathbb{C}$-tuple of dual cases
17 M M m m m	$\sqrt{2}/2\,(M, m-M, m-M, M-m, 2m+M)$	$\sqrt{2}/4\,(m+M, 0, 0, 0, 3(m+M))$
18 m m M M M	$\sqrt{2}/2\,(m, M-m, M-m, m-M, 2M+m)$	

The 18 cases can also be described as follows – from the perspective of the location of the center pixel of the 5-pixel mask:

Case	The center is in a texel with value	and it is	of an(a)
1	m	interior •	——
2	M	interior •	——
3	m	W	edge (V) •
4	M	W	"
5	M	E	"
6	m	E	"
7	m	N	edge (H) •
8	M	N	"
9	M	S	"
10	m	S	"
11	m	NW	corner •
12	M	NW	"
13	m	NE	"
14	M	NE	"
15	m	SW	"
16	M	SW	"
17	m	SE	"
18	M	SE	"

V - vertical

H - horizontal

W - west NW - northwest
E - east NE - northeast
N - north SW - southwest
S - south SE - southeast

• Concerning the checkerboard example of a square segment with $t \cdot t$ texels, with $p \cdot p$ pixels per texel, the mask produces one of these 18 responses per pixel.

~ BH

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: • Note: For completeness, it should be noted that averaging the

- 4 vertical edge cases (3, 4, 5, 6),
- 4 horizontal edge cases (7, 8, 9, 10),
- 8 corner cases (11, 12, 13, 14, 15, 16, 17, 18)

generates the same $\mathbb{C}^*$-tuple, i.e.,

$$\mathbb{C}^* = \sqrt{2}/4\,(m+M,\ 0,\ 0,\ 0,\ 3(m+M)).$$

As a consequence, the overall average $\mathbb{C}$-tuple of the entire "checkerboard" is

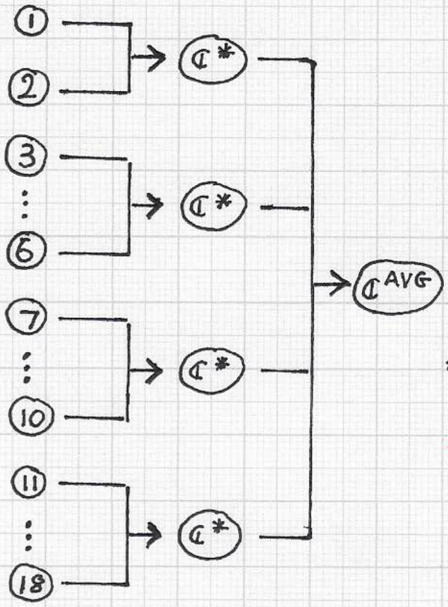
$$\mathbb{C}^{AVG} = \mathbb{C}^*.$$

The illustration (left, top) shows how it is possible to view the averaging process of $\mathbb{C}$-tuples from the highest level of resolution (pixel-level) to the lowest level of resolution (segment-level) as a "multi-layer data analysis" process.

– "Data features are captured at multiple scales, from fine- to coarse-graind behavior."

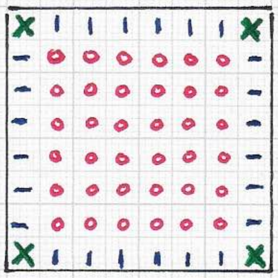
For a pixel-level (statistical) analysis of the probabilities of the cases 1 to 18, we summarize the respective numbers:

Cases	no. for each case	sum of nos. for all cases
1, 2	$\frac{1}{2}\,t^2\,(p-2)^2$	$t^2\,(p-2)^2 = n_I$
3,…, 6	$\frac{1}{2}\,t^2\,(p-2)$	$2\,t^2\,(p-2) = \frac{1}{2} n_E$
7,…, 10	$\frac{1}{2}\,t^2\,(p-2)$	$2\,t^2\,(p-2) = \frac{1}{2} n_E$
11,…, 18	$\frac{1}{2}\,t^2$	$4\,t^2 = n_C$



Multi-layer/multi-scale data analysis. Initially, spectral responses are computed at the pixel scale (left, cases 1 – 18); Finally, an overall average spectral response is calculated at the level of the entire segment (right, $\mathbb{C}^{AVG}$).

1 of t×t texels



One texel of type 'm' or 'M'. The figure shows the p×p pixels of type interior (o), edge (–, |) and corner (×).

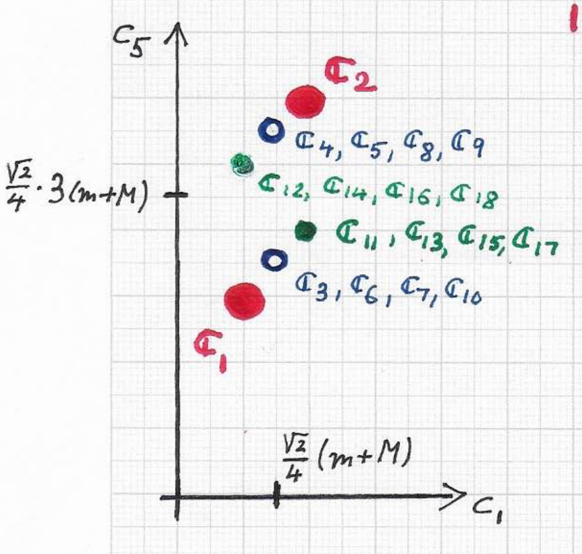
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: At this point, it is possible to consider some of the main aspects of the discussed eigenfunction-based, spectral, multi-scale analysis approach. In the following, a high-level summary is provided, using the simple "checkerboard" segment scenario with $t \times t$ texels, with alternating type 'm' and 'M' and $p \times p$ pixels per texel, and the 5-pixel convolution mask as the specific scenario. Having an understanding of the principles applied to this simple scenario is beneficial when considering more general cases.

m	M	m	M
M	m	M	m
m	M	m	M
M	m	M	m

Perfect, ideal checkerboard segment with 4x4 texels of alternating type 'm' and 'M', with 10x10 pixels per texel.

compute associated $\mathbb{C}_i$-values in 5-dimensional feature space



Visualization of all 18 possible $\mathbb{C}_i$-tuples via a projection from 5-dim. $\mathbb{C}$-space to (c_1, c_5)-space.

1) Given a perfect checkerboard segment as input, one knows exactly how often each of the cases ①, ..., ⑱ must result when applying the mask to all pixels, i.e., when using each pixel as center pixel of the 5-pixel mask. The numbers of instances of cases ①, ..., ⑱ are called $N_1, \ldots, N_{18}$.

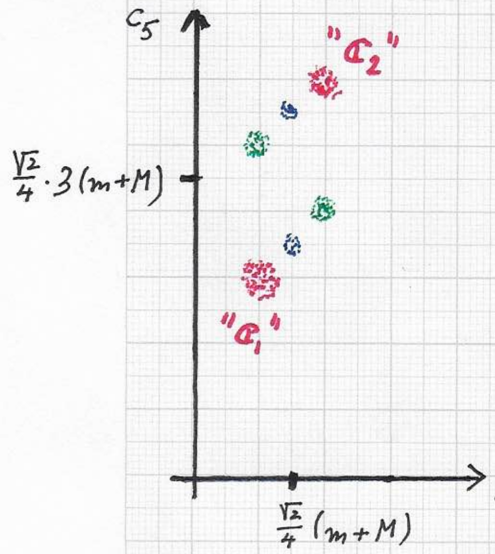
2) It is important to realize that a response of the mask convolution generates a tuple $(c_1, c_2, c_3, c_4, c_5) = \mathbb{C}$, ...

OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• Laplacian eigenfunctions: ... i.e., the mask generates a local multi-scale, 5-dimensional texture descriptor — a "feature vector" — that is unique for each of the 18 cases.

3) One can think of these feature vectors (points) in a geometrical way: A 5-dimensional feature vector $\mathbb{C}$ must take on the value that represents one of the unique 18 cases. We denote the feature vector for case i as $\mathbb{C}_i$, $i=1\ldots18$, i.e., $\mathbb{C}_i = (c_1^i, c_2^i, c_3^i, c_4^i, c_5^i)$. Each $\mathbb{C}_i$-value is a point in 5-dimensional space. For the considered checkerboard scenario, these points have specific multiplicities: $\mathbb{C}_1$ and $\mathbb{C}_2$ have multiplicity $\frac{1}{2}t^2(p-2)^2$; $\mathbb{C}_3, \ldots, \mathbb{C}_{10}$ have multiplicity $\frac{1}{2}t^2(p-1)$; and $\mathbb{C}_{11}, \ldots, \mathbb{C}_{18}$ have multiplicity $\frac{1}{2}t^2$.

4) Considering material/object recognition, the input is an "unknown" segment — of perfect resolution t^2p^2 — and the mask is applied to each of the t^2p^2 pixels.
...



Visualization of CLUSTERS of resulting $\mathbb{C}$-tuples — when applying the mask to a given segment that is "not perfectly identical" to the stored $\mathbb{C}$-tuple "signature" of the ideal, completely error-/noise-free checkerboard segment.

MUST DEFINE AND USE A METRIC FOR DECIDING WHETHER $\mathbb{C}$-TUPLE CLUSTERS OF NEW SEGMENT "AGREE WITH" THE STORED $\mathbb{C}$-TUPLE SIGNATURE OF THE CHECKERBOARD, THE IDEAL TEXTURE.