OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ... THE INPUT SEGMENT CAN BE RECOGNIZED – at least in a statistical sense – AS CHECKBOARD-CLASS OBJECT, WITH m-VALUE AND M-VALUE TEXELS, WHEN THE CONVOLUTION GENERATES THE EXPECTED 18 VALUES IN 5-DIMENSIONAL FEATURE SPACE WITH THE EXPECTED MULTIPLICITIES $N_1, N_2, \ldots, N_{18}$ FOR THE RESPECTIVE SPECTRAL RESPONSES. Otherwise, the input segment is not recognized as a material/object of this "checkerboard class."

• **Note:** The "ideal scenario" discussed here is used for explaining fundamental aspects of the approach. When adapting the fundamental principles to a "real-world, applied setting" one must adopt concepts including "being very close, nearly the same, approximately equal, within a limited distance, ..." In other words, the approach must be generalized further to be useful when one must handle more general distributions and probabilities.

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: 5) IF ONE WERE GIVEN A "PERFECT COPY" OF THE IDEAL CHECKERBOARD-TEXTURED SEGMENT AND IF "NO NUMERICAL ERRORS" WERE POSSIBLE, THEN ONE WOULD BE ABLE TO COMPUTE ONLY THE EXACT $\mathbb{C}$-TUPLE VALUES OF THE IDEAL CHECKERBOARD-TEXTURED SEGMENT. FURTHER, A "PERFECT COPY" WOULD ALSO LEAD TO THE SAME NUMBERS OF CASE INSTANCES, I.E., THE NUMBERS OF THE 18 POSSIBLE SPECTRAL MASK RESPONSES: $N_1, N_2, \ldots, N_{18}$.

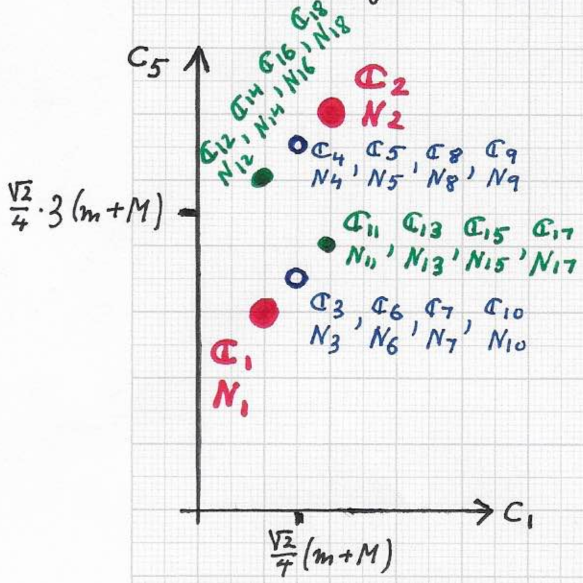
• **Note:** When analyzing a given segment that is NOT a "perfect copy" of the ideal segment, applying the convolution mask to all t^2p^2 pixels of the given segment, one can still obtain the exact (nearly exact) values for the stored ideal $\mathbb{C}_i$-tuples AND their exact numbers $N_1, N_2, \ldots, N_{18}$. This is a consequence of the fact that the analysis and characterization of the given segment is still "merely a statistical characterization."

...

Bernd Hamann
Stratovan

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ... As far as "case counts"/"numbers of observed case instances" are concerned, one could keep track of recognized cases in a given segment being analyzed, producing a "count tuple $\mathbb{N}$," i.e., $\mathbb{N} = (N_1, \ldots, N_{18})$. One could then determine whether this tuple $\mathbb{N}$ is "very similar" to a tuple $\mathbb{N}^{IDEAL}$ of a class to be recognized. In this case one would compare $\|\mathbb{N}\|$ and $\|\mathbb{N}^{IDEAL}\|$ and consider the tuples' directional difference, i.e., $(\mathbb{N} \bullet \mathbb{N}^{IDEAL}) / (\|\mathbb{N}\| \cdot \|\mathbb{N}^{IDEAL}\|)$.



The 18 unique $\mathbb{C}$-tuples $\mathbb{C}_1, \ldots, \mathbb{C}_{18}$ with their associated counts $N_1, \ldots, N_{18}$. Feature points $\mathbb{C}_i$ have been projected here to 2D (c_1, c_5)-space.

6) The $\mathbb{C}$-tuples defining the 18 unique spectral responses of the convolution mask determine 18 points/positional vectors in 5-dimensional feature space. Again, only in the ideal setting precisely 18 points exist. One could view this definition of the perfect characterization of the checkerboard segment as a "Dirac function," where 18 "pulses $\mathbb{C}_i$" with associated numbers N_i exist in the 5-dimensional feature space.

The corresponding DIRAC function defining this spectral analysis function can be written as

$$\underline{N(\mathbb{C})} = \begin{cases} N_i, & \text{if } \mathbb{C} = \mathbb{C}_i \\ 0, & \text{otherwise} \end{cases}.$$

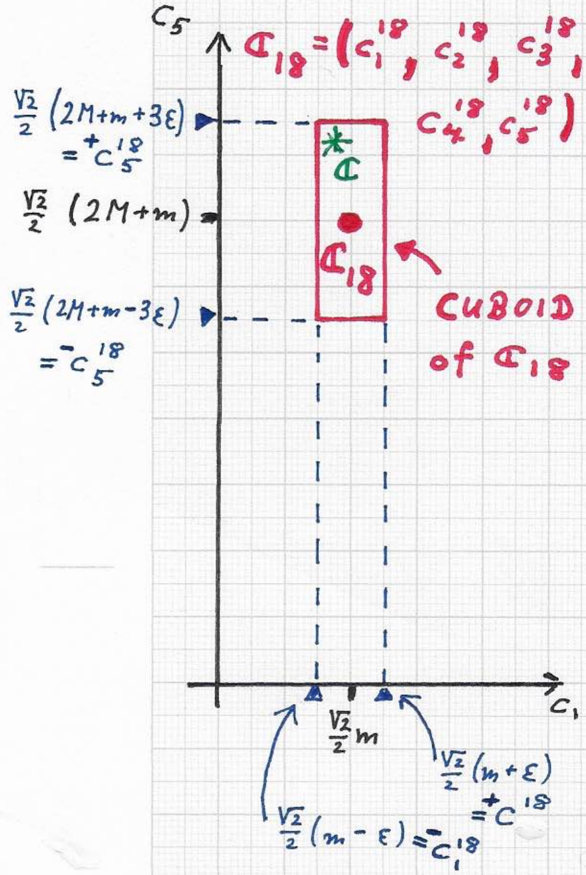
$N(\mathbb{C})$ can serve as detector for the (m, M)-checkerboard segment class.

!

■ OBJECT AND MATERIAL EIGENFUNCTIONS — Cont'd.

• Laplacian eigenfunctions: 7) **IN THE IDEAL SITUATION CONSIDERED HERE, IT IS POSSIBLE TO DEFINE THE EXACT 18 $\mathbb{C}_i$-VALUES PERFECTLY, TOGETHER WITH THEIR COUNTS N_i.** (It will be necessary to use DISTRIBUTIONS of "slightly different $\mathbb{C}$-values" to determine a region in 5-dimensional $\mathbb{C}$-space that is viewed as region associated with one of the 18 cases.) **WHEN A NEW SEGMENT MUST BE ANALYZED, ONE MUST CONSIDER ERROR (OR UNCERTAINTY): "m" SHOULD BE "$m \pm \varepsilon$"; "M" SHOULD BE "$M \pm \varepsilon$."** Specifically, the values of the components of a $\mathbb{C}$-tuple of one of the 18 possible cases involve terms like these: $m, M, 3m, 3M, m+M, m-M, 3(m-M), 5m+M, 2m+M$ etc. One must use INTERVAL ARITHMETIC when determining whether a $\mathbb{C}$-tuple computed for a new segment to be classified represents one of the specific 18 $\mathbb{C}_i$-tuple cases or not. The figure (left) sketches this issue for case 18. ...

!



Interval arithmetic and case recognition. Case 18 has the associated tuple $\sqrt{2}/2 \cdot (m, M-m, M-m, m-M, 2M+m) = \mathbb{C}18$. Considering the first and fifth tuple components, m and $2M+m$, and and replacing m by $m \pm \varepsilon$ and M by $M \pm \varepsilon$, the point ●, $\mathbb{C}_{18}$, is "expanded" to the cuboid

$$[{}^{-}c_1^{18}, {}^{+}c_1^{18}] \times [{}^{-}c_5^{18}, {}^{+}c_5^{18}].$$

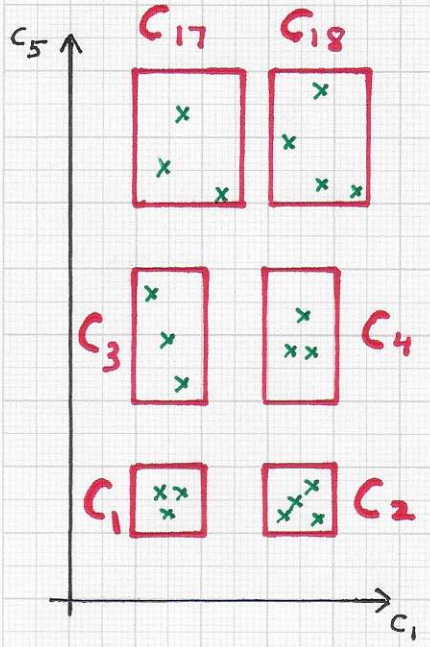
The point/tuple $*$, $\mathbb{C}$, lies inside this cuboid and thus represents case 18 — when only considering c_1 and c_5.

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ... The exact 18 $\mathbb{C}_i$-tuples are assumed to be known, i.e., $\mathbb{C}_i = (c_1^i, c_2^i, c_3^i, c_4^i, c_5^i)$, $i = 1 \dots 18$. For the purpose of analyzing and classifying a new given segment, we associate "uncertainty intervals" with each component of all $\mathbb{C}_i$-tuples, using the principle described on the previous page for case 18. We view $\mathbb{C}_i$ as the center point of a hyper-cuboid in 5-dimensional space defining the uncertainty region C_i for case i:

$$C_i = [{}^{-}c_1^i, {}^{+}c_1^i] \times [{}^{-}c_2^i, {}^{+}c_2^i] \times \dots \times [{}^{-}c_5^i, {}^{+}c_5^i],$$

i.e., C_i is the Cartesian product of the five uncertainty intervals associated with $\mathbb{C}_i$'s exact component values. For example, $[{}^{-}c_1^i, {}^{+}c_1^i]$ is the uncertainty interval associated with c_1^i. When classifying a $\mathbb{C}$-tuple of a new segment the TUPLE $\mathbb{C}$ REPRESENTS A "CASE $\mathbb{C}_i$" WHEN $\mathbb{C} \in C_i$, i.e., when $\mathbb{C}$ lies inside the cuboid associated with $\mathbb{C}_i$: C_i.



Sketch of cuboids of six of the 18 possible cases in our example. The cuboids are projected into (c_1, c_5)-space. The points 'x' represent $\mathbb{C}$-tuples of pixels of a new segment that have passed the "INSIDE-CUBOID" test.

Whenever a pixel of a new segment being analyzed has an associated $\mathbb{C}$-tuple inside cuboid C_i, the COUNTER for C_i is incremented.