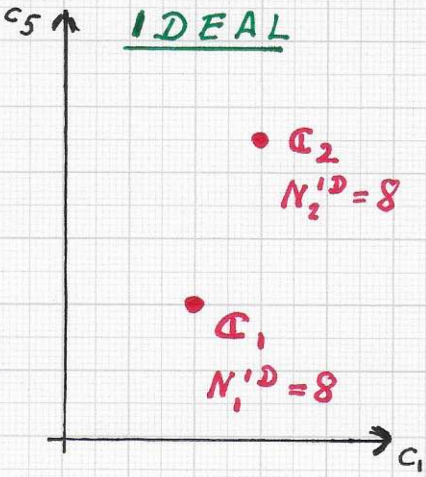
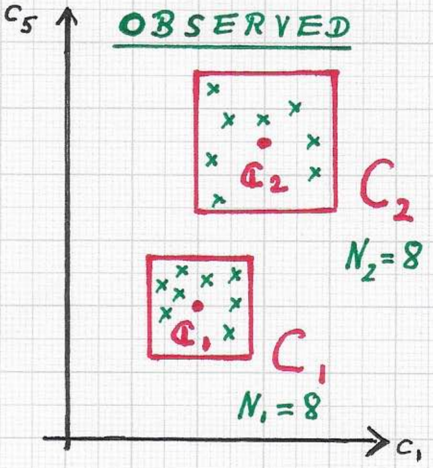
OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions:





Comparison of numbers of cases. Top: Cases 1 and 2 arise ideally eight (8) times; bottom: cases 1 and 2 are observed eight (8) times using the INSIDE-CUBOID test. Thus, when considering only cases 1 and 2 and only components c_1 and c_5, the new segment and its observed mask responses can be viewed as a "perfect STATISTICAL match."

8) THE NUMBERS OF RECOGNIZED CASES MUST BE RECORDED. THE IDEAL CHECKERBOARD-TEXTURED SEGMENT WITH $t \cdot t$ SQUARE TEXELS AND $p \cdot p$ PIXELS PER TEXEL, AND WITH ALTERNATING VALUES m AND M PER TEXEL, GENERATES t^2p^2 MASK RESPONSES – A $\mathbb{C}$-TUPLE REPRESENTING ONE OF 18 POSSIBLE MASK RESPONSES, $\mathbb{C}_1$ .... or $\mathbb{C}_{18}$. MASK RESPONSE $\mathbb{C}_i$ RESULTS EXACTLY $N^{ID}(\mathbb{C}_i) = N_i^{ID}$ TIMES, AND $N_1^{ID} + \ldots + N_{18}^{ID} = t^2p^2$. GIVEN A NEW SEGMENT TO BE ANALYZED AND CLASSIFIED, WE COUNT THE NUMBERS OF MASK RESPONSES PRODUCING $\mathbb{C}$-TUPLES INSIDE THE 18 CUBOIDS DEFINING THE 18 CASES' UNCERTAINTY REGIONS. THE FINAL CASE COUNTS ARE $N_1 = N(C_1), \ldots, N_{18} = N(C_{18})$. THUS, IT IS POSSIBLE TO COMPARE THE IDEAL CASE COUNT TUPLE $\mathbb{N}^{IDEAL} = (N_1^{ID}, \ldots, N_{18}^{ID})$ WITH THE OBSERVED TUPLE $\mathbb{N} = (N_1, \ldots, N_{18})$. !

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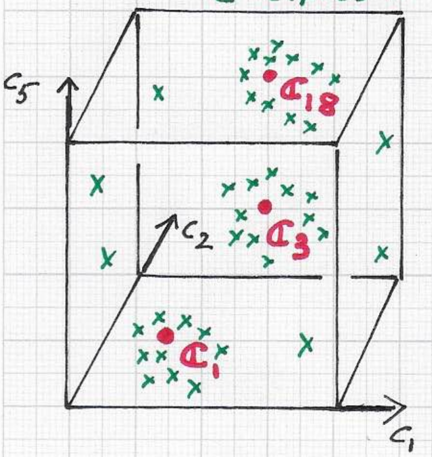
Stratovan

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions:

Distribution of $\mathbb{C}$-tuples 'x'



When computing the $\mathbb{C}$-tuple mask responses for all pixels of a new segment/data set to be analyzed, the $\mathbb{C}$-tuples 'x' reside in 5-dimensional space. The ideal $\mathbb{C}_i$-tuples also reside in this space, with associated perfect case counts N_i^{ID}. One can view the set of calculated $\mathbb{C}$-tuples 'x' as a discrete distribution of points in a 5-dimensional space. Further this distribution depends on the values of m, M, t and p. The case counts N_i for a specific cuboid C_i also depend on m, M, t and p, and one can therefore understand $\mathbb{N} = (N_1, \ldots, N_{18})$ as a vector-valued (18-dim.) function depending on m, M, t, p: $\mathbb{N} = \mathbb{N}(m, M, t, p)$.

• **Note:** Only in the "ideal setting" the values of m and M are precise, unique, without error, the same throughout the ideal data set of $t^2 p^2$ pixels. When analyzing a new, given data set of $t^2 p^2$ pixels, the calculated $\mathbb{C}$-tuple mask responses DO NOT MATCH PRECISELY AND ONLY THE PERFECT **18** UNIQUE $\mathbb{C}$-tuple mask responses for the ideal data set. This fact makes it necessary to use the concept of a cuboid uncertainty region. In other words a $\mathbb{C}$-tuple is the "response function" that depends not merely on two distinct values for m and M but rather on two uncertainty intervals associated with m and M: $[m-\varepsilon, m+\varepsilon]$, $[M-\varepsilon, M+\varepsilon]$. As a consequence, the case counts $N_1, \ldots, N_{18}$ calculated for a new, given data set are influenced by the errors associated with m- and M-values and the values of t^2 (no. of texels) and p^2 (no. of pixels per texel).

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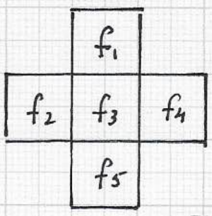
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions:

5-pixel mask



The mask performs a convolution – with the piecewise-constant function with values $f_1, \ldots, f_5$: $\mathbb{f} = (f_1, f_2, \ldots, f_5)^T$

$$\mathbb{f} = \sum_{i=1}^{5} c_i \, \mathbb{e}_i^n$$

The convolution produces a tuple $\mathbb{C} = (c_1, \ldots, c_5)$; its components c_i are the coefficients of $\mathbb{f}$'s expansion using the underlying basis $\{\mathbb{e}_i^n\}$.

THE BASIS FUNCTIONS $\mathbb{e}_i^n$ CAPTURE LOCAL MULTI-SCALE BEHAVIOR: THE EIGENFREQUENCY OF $\mathbb{e}_i^n$ IS ω_i – i.e., $\omega_i = \sqrt{|\lambda_i|}$, WITH λ_i BEING $\mathbb{e}_i^n$'s CORRESPONDING EIGENVALUE.

⋮

9) **THE MAIN ASPECTS OF THE APPROACH DISCUSSED SO FAR ARE:**

i) We apply a 5-pixel convolution mask to a set of 5 pixels.

ii) The mask returns 5 values, the 5 coefficients of a local representation of the 5 pixel-associated "mass values" in a multi-scale / multi-frequency basis consisting of 5 piecewise constant linearly independent basis functions.

iii) The 5-dimensional tuple response of the mask application step reflects the "behavior of pixel mass values" at 5 eigenfrequencies: The components of a $\mathbb{C}$-tuple $\mathbb{C} = (c_1, c_2, \ldots, c_5)$ are the coefficients of a local function expansion at 5 levels of scale. (Thus, a "relatively high absolute value of coefficient c_i" implies that the associated i^{th} basis function – and its associated frequency – is locally dominant.)

iv) The mask is applied to all 5-pixel neighborhoods in an image data set, a segment, to be analyzed and classified. •••

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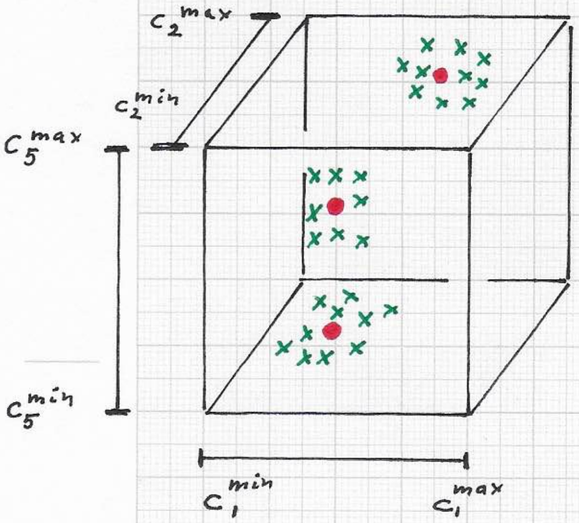
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ... The individual $\mathbb{C}$-tuple responses of the mask are recorded in 5-dimensional $\mathbb{C}$-space, "feature space." Thus, the result is a "**point** cloud" in $\mathbb{C}$-space, and the distribution of the points reflects how strongly certain basis functions are "present and expressed" in the data set being analyzed. For example, should there be a high-density cluster of "$\mathbb{C}$-points" in $\mathbb{C}$-space, the cluster indicates that (nearly) the same function expansion can be used to represent the given data set in many regions.

⋮

The mask produces "$\mathbb{C}$-tuple point clouds" in $\mathbb{C}$-space.



In the case of a "near-perfect detection," the point clouds {x} form dense point clusters that have at their centers an "ideal tuples" ● that represent ideal, expected perfect local expansions.

(For example, one can think of the "ideal tuples" ● as averages obtained via an initial TRAINING process that generates the needed training $\mathbb{C}$-samples for defining the "ideal" ● values to be detected.)

⋮

v) "Classical spectral analysis" determines whether certain elements are present in a probe being analyzed by identifying specific "bright and intense spectral lines" that indicate the existence of elements. Instead of spectral lines our approach uses "spectral $\mathbb{C}$-tuples," i.e., points in 5-dimensional feature space that indicate the existence of a specific type of function expansion. Thus, multiple clusters of $\mathbb{C}$-tuples reflect distinct types of different function expansions.

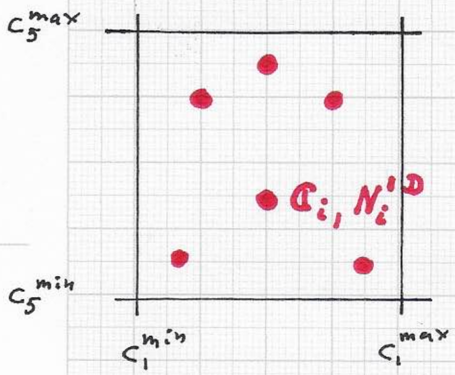
...

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: vi) One can also interpret the $\mathbb{C}$-tuple point cloud as a discrete distribution (function) of $\mathbb{C}$-values. If it is necessary, one will be able to define and calculate distribution / density values by binning $\mathbb{C}$-tuples in 5-dimensional space, or using kernels to compute local distribution/density values by counting the number of $\mathbb{C}$-tuples in certain regions.

⋮

↓



Sketch of the "ideal tuples" • in 5-dimensional $\mathbb{C}$-tuple, feature, space. Each tuple has an associated value N_i^{ID}, the number that defines how many times an "ideal" tuple $\mathbb{C}_i$ must exist / be detected when analyzing a new, given data set.

⇒ A GIVEN DATA SET CAN BE CALLED "PERFECT MATCH" WHEN ITS CALCULATED $\mathbb{C}$-TUPLE VALUES ALL (NEARLY) AGREE WITH THE "IDEAL" $\mathbb{C}$-TUPLE VALUES STORED AS PROFILES – AND WHEN THERE IS ALSO AGREEMENT IN THE N_i^{ID} VALUES.

vii) FOR THE PURPOSE OF DETECTION / RECOGNITION ONE MUST COMPARE THE COMPUTED SET OF $\mathbb{C}$-TUPLE RESPONSES WITH THE SPECIFIC KNOWN "FINGERPRINT $\mathbb{C}$-TUPLE PROFILES" OF MATERIALS TO BE FOUND. THESE PROFILES ARE EITHER KNOWN PRECISELY A PRIORI OR ARE ESTABLISHED VIA "TRAINING" A PROFILE DATABASE WITH SAMPLES.

Considering the "perfect 2-material segment with ideal values of m and M," the $\mathbb{C}$-tuple responses of a new, given segment to be analyzed, the calculated responses define points "very close to" the perfect material whenever "nearly error-free" values m and M exist in the segment.