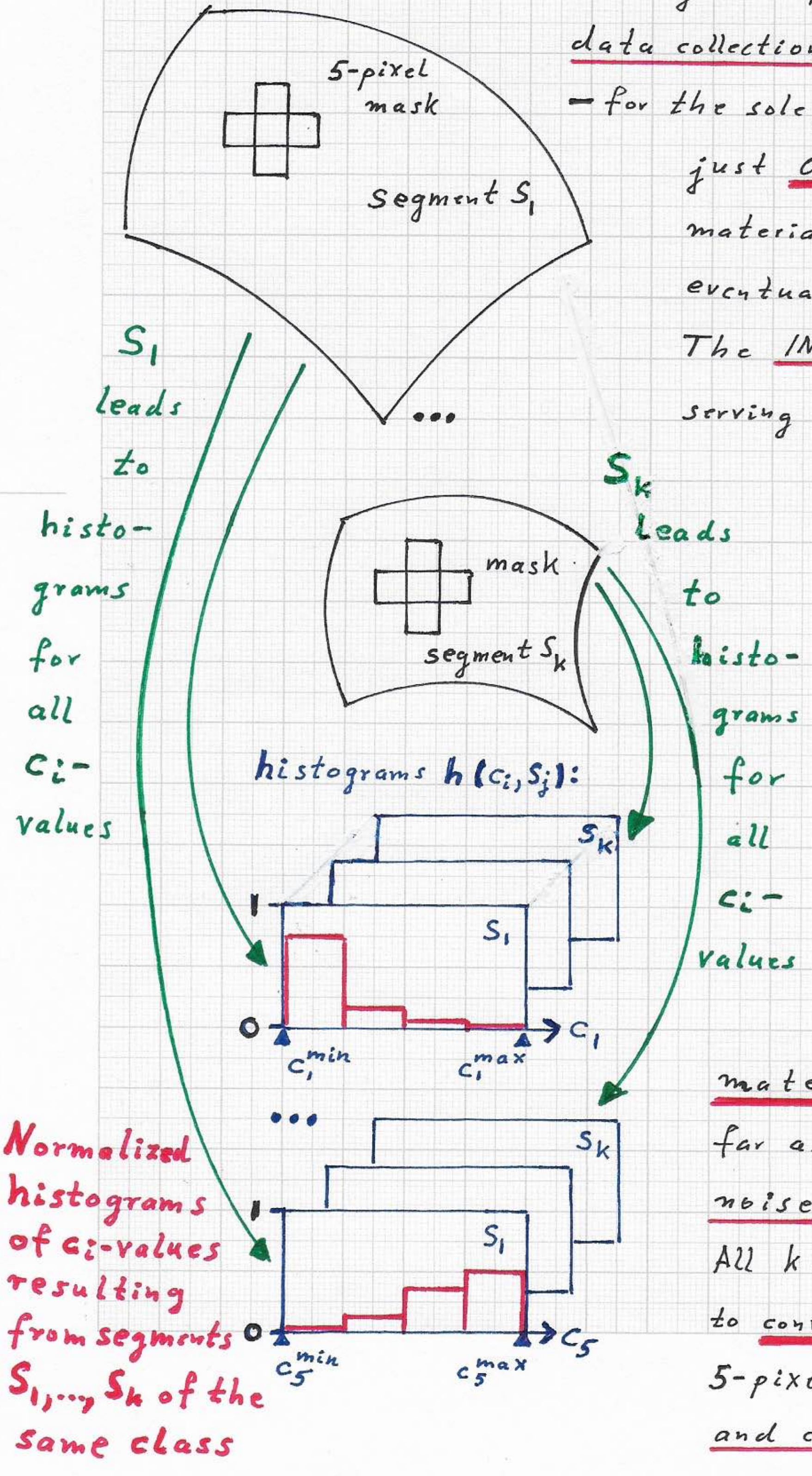
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ... The illustration on this page summarizes, at a high level, the process of initial data collection, processing and storage – for the sole purpose of charactering just ONE CLASS of multipe material classes to be used for eventual data classification. The INPUT consists of k segments serving as (training) SAMPLES of a specific material. (It is possible that a segment consists of several unconnected pieces, fragments.) Generally, the segments have different numbers of pixels (voxels), and collectively capture the "variety of the material's appearances," as far as mass/density, texture, noise etc. are concerned. All k segments are subjected to convolution: For example, a 5-pixel mask is used to analyze and characterize the material. ...

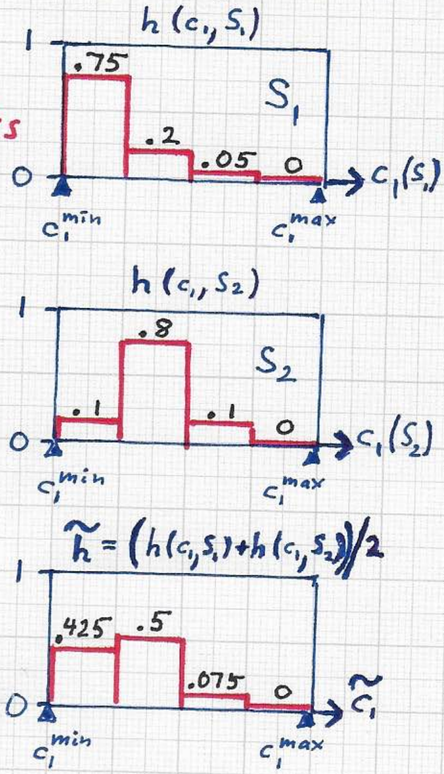


■ OBJECT AND MATERIAL EIGENFUNCTIONS — Cont'd.

• Laplacian eigenfunctions: ... Each application of the 5-pixel mask generates five values, i.e., values for c_1, c_2, c_3, c_4 and c_5. These c_i-values are the coefficient values needed to expand "the mass function" locally at multiple (three) levels of scale/detail. Once all segments have been processed, one can meaningfully define the extremal values for the five coefficients: $[c_1^{min}, c_1^{max}], \ldots, [c_5^{min}, c_5^{max}]$. Further, by dividing these $[c_i^{min}, c_i^{max}]$ intervals into bins, one obtains k binned histograms, more precisely k binned histograms for each of the five coefficients. When all binned histograms have been determined, a final normalization step ensures that the normalized bin-specific values are in the interval $[0,1]$; this is achieved for a specific histogram by dividing the bin-specific counts (integers) by the total number, the sum, of these bin-specific counts. The bottom part of the illustration on the previous page provides a sketch of the OUTPUT of this process: $5 \times k$ normalized binned histograms.

...

Segments S_1 and S_2 belong to the same class!



Result of averaging normalized histograms for c_1-values resulting from two segments, S_1 and S_2.

The variable $\tilde{h}$ represents the average of c_1-value histograms of segments S_1 and S_2.

It is assumed that $c_1(S_1)$ and $c_1(S_2)$ reflect two possible, "allowable" types of c_1-value histogram behaviors. These two behaviors are quite different — BUT S_1 and S_2 are of the same class! Thus, averaging histograms is, generally, inappropriate.

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ... The 5 × k normalized binned histograms resulting from k segments (k samples) of the same material class characterize this class at MULTIPLE SCALES OF MATERIAL MASS FUNCTION BEHAVIOR. The possibility to characterize segments at **low**-frequency, **medium**-frequency and **high**-frequency behavior, for example, and the ability to use frequency-specific histogram data for material classification makes the presented approach very powerful: The approach makes possible the comparison of a segment to be classified and representative samples of specific material classes by COMPARING HISTOGRAM DATA FOR ALL CAPTURED EIGENFUNCTION-BASED FREQUENCY BANDS. In other words, it is possible to perform multiple frequency-/band-specific comparisons for segment characterization and classification. In our 2D example, the **5-pixel** mask supports analysis, characterization, comparison and classification for three distinct frequencies $(\omega_1, \omega_2=\omega_3=\omega_4, \omega_5)$, based on the three distinct eigenvalues $(\lambda_1, \lambda_2=\lambda_3=\lambda_4, \lambda_5)$. ...

• 5 × k histograms:

$h_{i,j}$, $i=1\ldots5$, $j=1\ldots k$,

$h_{i,j} = h(c_i, S_j)$

$$\begin{bmatrix} h_{1,1} & \cdots & h_{1,k} \\ \vdots & & \vdots \\ h_{5,1} & \cdots & h_{5,k} \end{bmatrix}$$

Abstract view/data structure of 5×k sample histograms to be used for segment classification.

• 5 histograms of segment to be classified:

h_i, $i=1\ldots5$,

$h_i = h(c_i, S)$

$$\begin{bmatrix} h_1 \\ \vdots \\ h_5 \end{bmatrix}$$

Column vector representation of the five histograms h_i characterizing a segment S to be classified.

Bernd Hamann
Stratovan

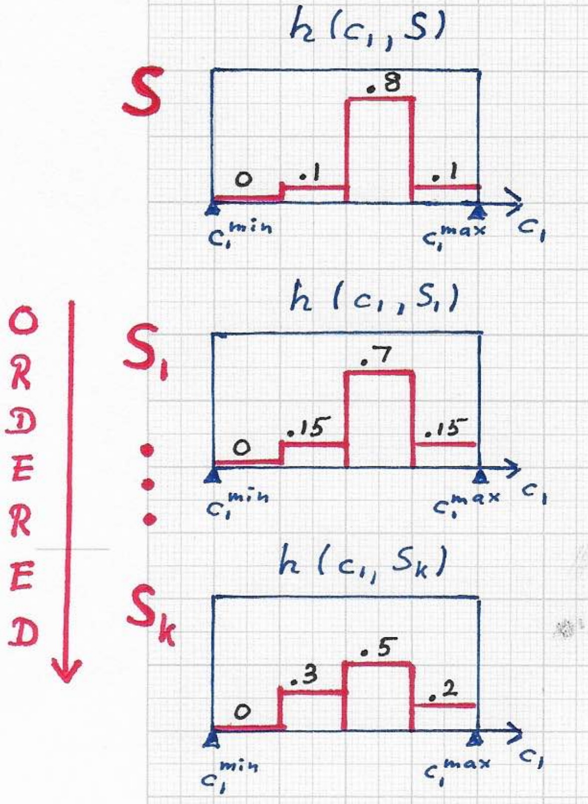
12/23/2021

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ... Once one has determined the kind distance measure to be used for two histograms h_1 and h_2, one can calculate distances $d_{i,j} = dist\left(h(c_i, S), h(c_i, S_j) \right)$. In our example, distances are computed for five scales ($i = 1 \ldots 5$) and by comparing, for a specific scale, the histogram of the segment S to be **compared** with the sample histograms S_j. For example, the following measure could serve as default measure:

$$d_{i,j} = \left[\int_{\Omega_i} \left(h(c_i, S) - h(c_i, S_j) \right)^2 dc_i \right]^{1/2}$$

$$j = 1 \ldots k, \quad i = 1 \ldots 5.$$



Example of scale-specific comparison of given segment S and sample segments $S_1, \ldots, S_k$. Scale 1 has been used for comparison, generating distance values $d_{1,j}$, $j = 1 \ldots k$. The $d_{1,j}$-values induce an order for segments S_j. In this figure the sample segments have already been ordered: S_1 has minimal and S_k maximal distance to S.

Based on the minimal distance value $d_{1,1}$ and a "threshold" one can decide whether S is of the class represented by the k samples – as far as scale 1 is concerned.

The domain Ω_i is the interval on the c_i-axis over which integration must be done, i.e., $\Omega_i = [c_i^{min}, c_i^{max}]$.

For a specific scale i, the $d_{i,j}$-values define an "**order**" for the sample segments S_j – defining a "**closest**" and a "**furthest-away segment**." (In addition or as an alternative, one could also calculate a representation/approximation of $h(c_i, S)$ given as a "**best**" linear combination of $h(c_i, S_j)$: $h(c_i, S) = \sum_j \alpha_j h(c_i, S_j)$.)

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS — Cont'd.

• Laplacian eigenfunctions: ... Determining a given segment's "class membership" via the purely histogram-distance approach presented is just one possibility: If there are distance-thresholded values $d_{i,j}$ of a given segment, then it will be possible to associate the segment with the class that is exemplified by the stored k sample segments S_k — when these $d_{i,j}$-values are smaller than some tolerance/threshold; further, since multiple scales i are considered, class association can be based on one or multiple scales.

An alternative approach for deciding class membership is based on computing an expansion of an unclassified segment's histogram by using the stored sample histograms as "basis elements." In this case, one must consider two numbers: (i) the number of bins (B) used to subdivide a histogram's c_i-value domain into equal intervals and (ii) the number of segment samples (k). Representing $h(c_i, S)$ as $\sum_j \alpha_j h(c_i, S_j)$ can lead to an under- or over-determined linear system for the unkown α_j-values. ...

• "Ideal," exact representation/identification of a given histogram $h(c_i, S)$:

$$h(c_i, S) = h(c_i, S_j)$$

holds for one value $j \in \{1, \ldots, k\}$. In this case, the given segment's histogram $h(c_i, S)$ is identical to one of the k stored sample histograms $h(c_i, S_j)$; in this case, bin and sample segment numbers agree: $B = k$. In other words, the expansion $\sum_j \alpha_j h(c_i, S_j)$ has an α-coefficient tuple $\alpha = (0, \ldots, 0, 1, 0, \ldots, 0)$, where one α_j-value is 1 and all other values are 0.