Stratovan

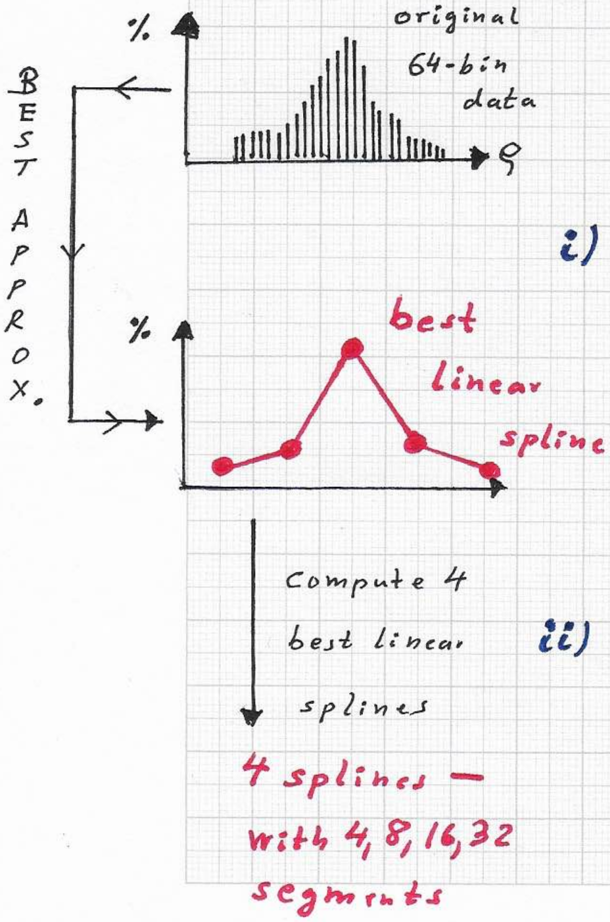
■ RICHARDSON EXTRATOLATION - Cont'd.

• **Summary:** [Using an example to summarize...]

1) Given: 64-bin histogram of distribution of material density values (ρ values) of one segment / one object of a specific material class

2) Wanted: "THE ONE UNIQUE DENSITY VALUE ρ FOR (THIS SEGMENT OF) THIS MATERIAL CLASS" [an 'ideal' goal]

3) Algorithm: [Instead of attempting to estimate a segment's unique density value ρ, estimate a near-perfect, near-unique statistical characteristic(s) of the given 64-bin histogram data.]



i) /* multi-resolution best approximation */
Compute multiple best linear spline approximations for the (noisy) 64-bin material density data. E.g., compute best linear spline approximations with 4, 8, 16 and 32 linear segments.

ii) /* statistical characteristics */
Compute statistical characteristics for all best linear spline approximations, e.g., **mean**, variance, skewness and kurtosis.

RICHARDSON EXTRAPOLATION – Cont'd.

• **Summary:** ...

4 splines ↓ compute means ↓ 4 mean values: $\mu_0^0, \mu_1^0, \mu_2^0, \mu_3^0$

ii) ... Considering only the mean of the computed best linear spline approximations (with 4, 8, 16 and 32 linear segments), use the associated mean values μ_0, μ_1, μ_2 and μ_4 as INPUT FOR RICHARDSON EXTRAPOLATION.

↓ perform repeated Richardson extrapolation ↓

$$
\begin{array}{llll}
\mu_0^0 & \mu_0^1 & \mu_0^2 & \underline{\mu_0^3} \\
\mu_1^0 & \mu_1^1 & \mu_1^2 & \\
\mu_2^0 & \mu_2^1 & & \\
\mu_3^0 & & &
\end{array}
$$

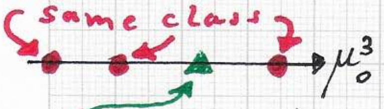
iii) /* Richardson extrapolation */

Using the four mean values μ_0, μ_1, μ_2 and μ_3 as input $(\mu_0^0, \mu_1^0, \mu_2^0, \mu_3^0)$, perform Richardson extrapolation, generating extrapolations $(\mu_0^1, \mu_1^1, \mu_2^1)$, (μ_0^2, μ_1^2) and μ_0^3. **THE VALUE μ_0^3 IS VIEWED AS 'NEAR-PERFECT' MEAN CHARACTERISTIC OF THE 64-BIN DATA!**

CLASSIFICATION:

iv) Compute this 'near-perfect' μ-value characteristic for all segments / objects known to belong to the same material class.

μ_0^3 values of 3 segments of same class



μ_0^3 value of 1 new segment; not belonging to this class: $|\blacktriangle - \bullet| > \varepsilon$

v) Compute this 'near-perfect' μ-value characteristic for a NEW segment/object to be classified. The new segment's/object's μ-value must be VERY CLOSE to a μ-value of one of the segments (defining class samples) to identify the new segment/object as segment/object of the same class. ∴ BH

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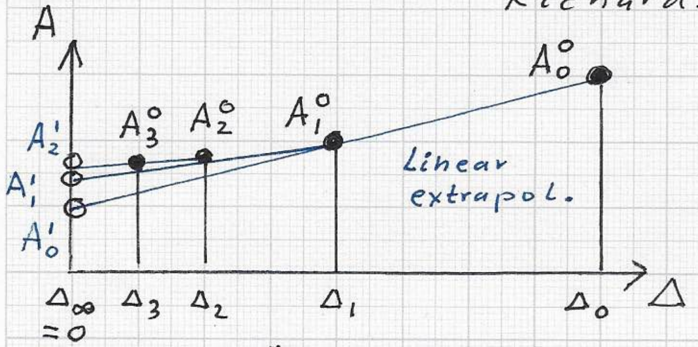
RICHARDSON EXTRAPOLATION – Cont'd.

- **Note:** Polynomial Extrapolation and Richardson Extrapolation



given: $\mu_0, \mu_1, \mu_2, \mu_3$

define: $A_i^0 = \mu_i$, $i = 0 \ldots$

and linear polynomials

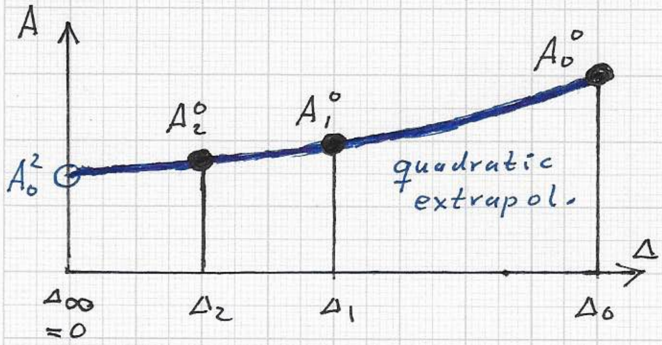
$$A_i^1(\Delta) = \frac{\Delta_{i+1} - \Delta}{\Delta_{i+1} - \Delta_i} A_i^0 + \frac{\Delta - \Delta_i}{\Delta_{i+1} - \Delta_i} A_{i+1}^0$$

$\Downarrow$ compute A_i^1 via extrapolation of $A_i^1(\Delta)$ for $\Delta = \Delta_\infty = 0$:

$$A_i^1(0) = \frac{\Delta_{i+1}}{\Delta_{i+1} - \Delta_i} A_i^0 + \frac{-\Delta_i}{\Delta_{i+1} - \Delta_i} A_{i+1}^0 \quad \Big| \text{ use } \Delta_{i+1} = \frac{\Delta_i}{2}$$

$$= \frac{\Delta_i/2}{\Delta_i/2 - \Delta_i} A_i^0 + \frac{-\Delta_i}{\Delta_i/2 - \Delta_i} A_{i+1}^0$$

$$= -A_i^0 + 2A_{i+1}^0$$



$\Rightarrow$ iterate process and construct quadratic polynomial extrapolation:

$$A_i^2(\Delta) = \frac{\Delta_{i+2} - \Delta}{\Delta_{i+2} - \Delta_i} A_i^1(\Delta) + \frac{\Delta - \Delta_i}{\Delta_{i+2} - \Delta_i} A_{i+1}^1(\Delta)$$

$$\overset{\Delta=0}{=} \frac{\Delta_{i+2}}{\Delta_{i+2} - \Delta_i}\left(-A_i^0 + 2A_{i+1}^0\right) + \frac{-\Delta_i}{\Delta_{i+2} - \Delta_i}\left(-A_{i+1}^0 + 2A_{i+2}^0\right)$$

$$\overset{\Delta_{i+2} = \Delta_i/4}{=} \frac{\Delta_i/4}{-3\Delta_i/4}\left(-A_i^0 + 2A_{i+1}^0\right) + \frac{-\Delta_i}{-3\Delta_i/4}\left(-A_{i+1}^0 + 2A_{i+2}^0\right)$$

$$= \frac{1}{3}\left(A_i^0 - 6A_{i+1}^0 + 8A_{i+2}^0\right) = A_i^2(0)$$

$\Rightarrow$ construct more higher-order polynom. extrapol.

~BH