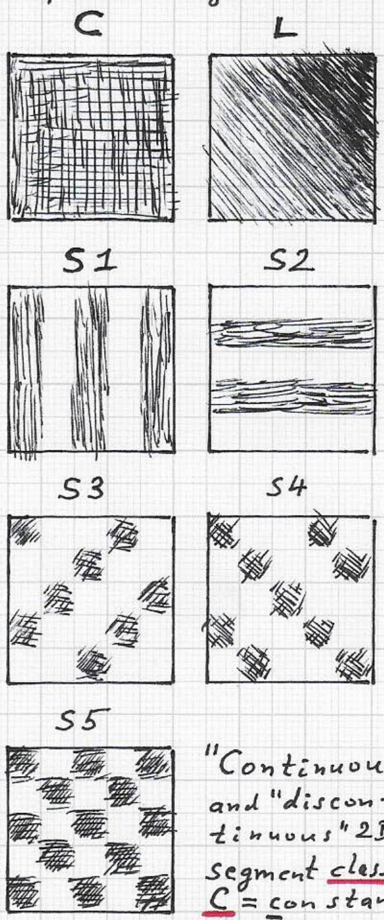
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• *Laplacian eigenfunctions:*



"Continuous" and "discontinuous" 2D segment classes: C = constant, L = linear, S1,…, S5 = stripe patterns.

We now sketch how the 5-pixel template, described on the preceeding pages, can be used to produce spectra for simple, ideal classes of 2D segments. The classes to be analyzed with the 5-pixel mask are shown (left). The simplest case is the "C" case. In this case, the pixel values used as input to the "5-pixel mask analyzer" are equal (m). The mask (left, bottom) produces the following spectral response for the eigenfunction coefficients c_i:

$$c_1 = \frac{\sqrt{2}}{4}(4m - 2m) = \frac{\sqrt{2}}{2}\,m \,,$$

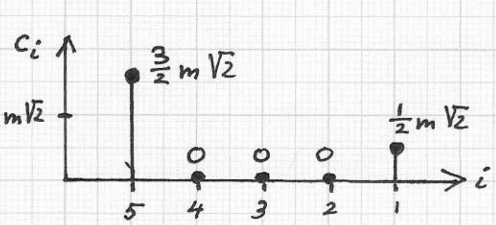
$$c_2 = \frac{\sqrt{2}}{4}(3m - 3m) = 0 \,,$$

$$c_3 = \ldots = 0 \,,$$

$$c_4 = \ldots = 0 \,,$$

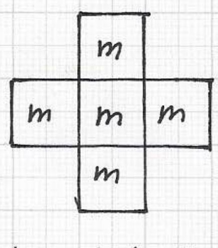
$$c_5 = \frac{\sqrt{2}}{4}(4m + 2m) = \frac{3\sqrt{2}}{2}\,m \,.$$

These spectral coefficients can be sketched:



Spectrum for C-class

• Application of 5-pixel mask to CONSTANT class



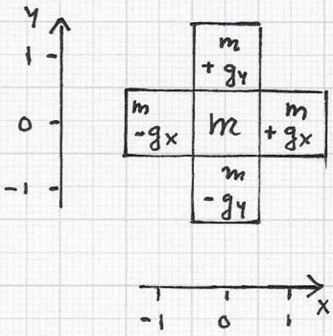
m = mass value of pixels

Input to 5-pixel mask for constant mass function.

The "L" case concerns a mass function m that varies linearly, i.e., $m(x,y) = m + g_x \cdot x + g_y \cdot y$, where m is the mass value of the center pixel of the mask and g_x, g_y are the components of the gradient. …

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

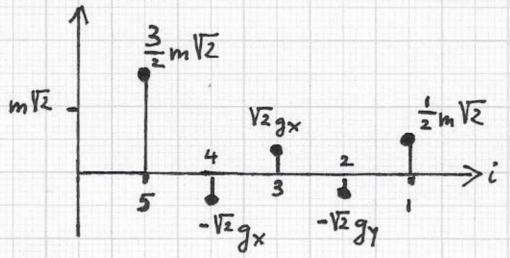
• **Laplacian eigenfunctions:** … By using 'local' x- and y-coordinate values −1, 0, 1 for the 5-pixel mask (left) a general linear function describing the mass function — $m(x,y) = m + g_x x + g_y y$ — generates the five pixel values shown in the figure. The resulting eigenfunction coefficients are:



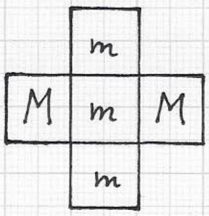
Definition of linear $m(x,y)$ function using local coordinates.

$$c_1 = \frac{\sqrt{2}}{4}\,(4m + g_x - g_x + g_y - g_y - 2m) = \frac{\sqrt{2}}{2}\,m\,,$$
$$c_2 = \frac{\sqrt{2}}{4}\,(3m - 3g_y - m + g_x - m - g_x - m - g_y) = -\sqrt{2}\,g_y\,,$$
$$c_3 = \frac{\sqrt{2}}{4}\,(3m + 3g_x - m + g_x - m - g_y - m + g_y) = \sqrt{2}\,g_x\,,$$
$$c_4 = \frac{\sqrt{2}}{4}\,(3m - 3g_x - m - g_x - m - g_y - m + g_y) = -\sqrt{2}\,g_x\,,$$
$$c_5 = \frac{\sqrt{2}}{4}\,(4m + g_x - g_x + g_y - g_y + 2m) = \frac{3\sqrt{2}}{2}\,m\,.$$

(These coefficients are equal to those of the "C" case – when $g_x = g_y = 0$.)



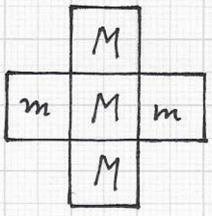
Spectrum for L-class



Stripe pattern S1

Type 1

$m \ll M$



Stripe pattern S1

Type 2

$m \ll M$

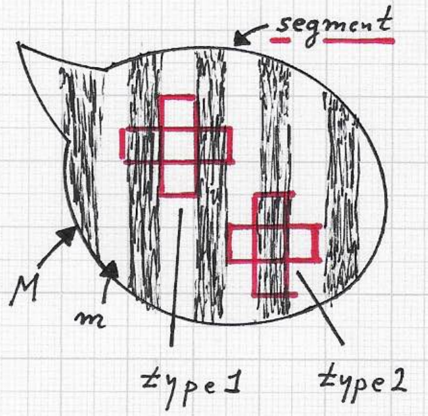
The two types of 'sampling' that can result when one applies the 5-pixel mask to the vertical stripe pattern S1 – where stripe width is exactly 1 pixel. The values of m and M are assumed to be very different, indicating discontinuous behavior.

When applying the mask repeatedly to many 5-pixel segment subsets of such a vertically striped segment, one obtains instances of type 1 and type 2.

When $g_x \ll m$ and $g_y \ll m$, the linear function is "nearly constant," and coefficients c_2, c_3, c_4 could be neglected. The situation is different when the mass values in the mask are "very different and indicate discontinuous mass function behavior." The discontinuous stripe pattern S1 is shown left. …

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ... Stripe patterns S1, type 1, and S1, type 2, generate the following coefficient values c_i:



type 1

$c_1 = \frac{\sqrt{2}}{2} M$,
$c_2 = \frac{\sqrt{2}}{2} (m-M)$,
$c_3 = \frac{\sqrt{2}}{2} (M-m)$,
$c_4 = \frac{\sqrt{2}}{2} (M-m)$,
$c_5 = \frac{\sqrt{2}}{2} (M+2m)$,

type 2

$c_1 = \frac{\sqrt{2}}{2} m$,
$c_2 = \frac{\sqrt{2}}{2} (M-m)$,
$c_3 = \frac{\sqrt{2}}{2} (m-M)$,
$c_4 = \frac{\sqrt{2}}{2} (m-M)$,
$c_5 = \frac{\sqrt{2}}{2} (m+2M)$.

The figure (left, top) shows the application of the mask to a larger segment with stripes having the width of one pixel. For completeness, we also analyze the horizontal stripe pattern S2, with types 1 and 2 (left).

	M	
m	m	m
	M	

Stripe pattern S2
Type 1
$m \ll M$

	m	
M	M	M
	m	

Stripe pattern S2
Type 2
$m \ll M$

The two types of discontinuous horizontal stripes.

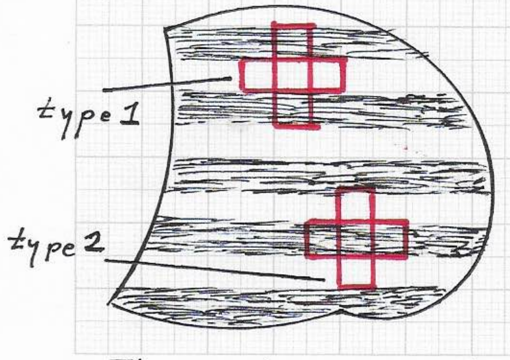
The resulting coefficient values are:

type 1

$c_1 = \frac{\sqrt{2}}{2} M$,
$c_2 = \frac{\sqrt{2}}{2} (M-m)$,
$c_3 = \frac{\sqrt{2}}{2} (m-M)$,
$c_4 = \frac{\sqrt{2}}{2} (m-M)$,
$c_5 = \frac{\sqrt{2}}{2} (M+2m)$,

type 2

$c_1 = \frac{\sqrt{2}}{2} m$,
$c_2 = \frac{\sqrt{2}}{2} (m-M)$,
$c_3 = \frac{\sqrt{2}}{2} (M-m)$,
$c_4 = \frac{\sqrt{2}}{2} (M-m)$,
$c_5 = \frac{\sqrt{2}}{2} (m+2M)$.



The mask is applied to multiple 5-pixel subsets in a segment, producing average coefficient values c_i.

Note: Given a "large" 2D pixel segment to be analyzed, having either the one-pixel-wide vertical or horizontal stripe pattern, the

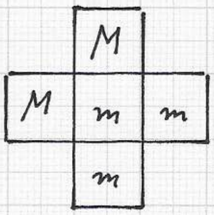
AVERAGE COEFFICIENT VALUES c_i are:

$$\mathbb{C} = \left(\frac{\sqrt{2}}{4}(m+M), 0, 0, 0, \frac{3\sqrt{2}}{4}(m+M) \right).$$

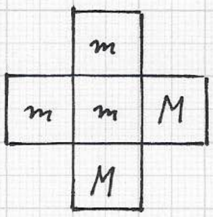
■ OBJECT AND MATERIAL EIGENFUNCTIONS — Cont'd.

• Laplacian eigenfunctions: Next, we consider diagonal stripe pattern S3. In this case, three types arise; the coefficient values c_i for the three types are:

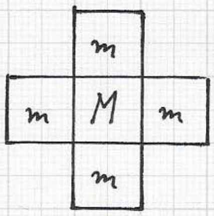
type 1	type 2	type 3
$c_1 = \frac{\sqrt{2}}{2} M$,	$c_1 = \frac{\sqrt{2}}{2} M$,	$c_1 = \frac{\sqrt{2}}{2}(2m - M)$,
$c_2 = \frac{\sqrt{2}}{2}(m-M)$,	$c_2 = \frac{\sqrt{2}}{2}(M-m)$,	$c_2 = 0$,
$c_3 = \frac{\sqrt{2}}{2}(m-M)$,	$c_3 = \frac{\sqrt{2}}{2}(M-m)$,	$c_3 = 0$,
$c_4 = \frac{\sqrt{2}}{2}(M-m)$,	$c_4 = \frac{\sqrt{2}}{2}(m-M)$,	$c_4 = 0$,
$c_5 = \frac{\sqrt{2}}{2}(2m+M)$,	$c_5 = \frac{\sqrt{2}}{2}(M+2m)$,	$c_5 = \frac{\sqrt{2}}{2}(2m+M)$.



Stripe pattern S3 Type 1 $m \ll M$



Stripe pattern S3 Type 2 $m \ll M$

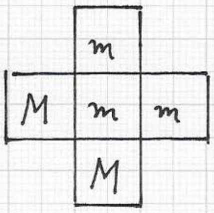


Stripe pattern S3 Type 3 $m \ll M$

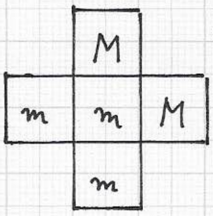
The three types for diagonal stripe pattern S3.

The pattern S4 also has three types, and the associated coefficient values are:

type 1	type 2	type 3
$c_1 = \frac{\sqrt{2}}{2} M$,	$c_1 = \frac{\sqrt{2}}{2} M$,	$c_1 = \frac{\sqrt{2}}{2}(2m - M)$,
$c_2 = \frac{\sqrt{2}}{2}(M-m)$,	$c_2 = \frac{\sqrt{2}}{2}(m-M)$,	$c_2 = 0$,
$c_3 = \frac{\sqrt{2}}{2}(m-M)$,	$c_3 = \frac{\sqrt{2}}{2}(M-m)$,	$c_3 = 0$,
$c_4 = \frac{\sqrt{2}}{2}(M-m)$,	$c_4 = \frac{\sqrt{2}}{2}(m-M)$,	$c_4 = 0$,
$c_5 = \frac{\sqrt{2}}{2}(2m+M)$,	$c_5 = \frac{\sqrt{2}}{2}(M+2m)$,	$c_5 = \frac{\sqrt{2}}{2}(2m+M)$.



Stripe pattern S4 Type 1 $m \ll M$



Stripe pattern S4 Type 2 $m \ll M$

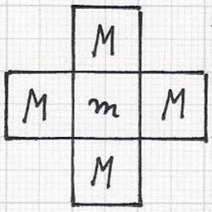
Types 1 and 2 of stripe pattern S4. Stripe pattern S4, Type 3 = Stripe pattern S3, Type 3.

NOTE: Given a "large" 2D pixel segment to be analyzed, having either stripe pattern S3 or S4, the segment's **AVERAGE COEFFICIENT VALUES c_i** are:

$$\mathbb{C} = \left(\frac{\sqrt{2}}{6}(2m+M), 0, 0, 0, \frac{\sqrt{2}}{2}(2m+M) \right).$$

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

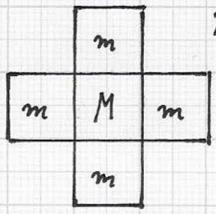
• Laplacian eigenfunctions: We now analyze pattern S5, where pixel values alternate (values m and M), defining a simple "checker board texture." The resulting coefficient values for the two possible types are:



Pattern S5

Type 1

$m \ll M$



Pattern S5

Type 2

$m \ll M$

Two types of "checkerboard" textures.

type 1

$c_1 = \frac{\sqrt{2}}{2}(2M-m)$,

$c_2 = 0$,

$c_3 = 0$,

$c_4 = 0$,

$c_5 = \frac{\sqrt{2}}{2}(2M+m)$,

type 2

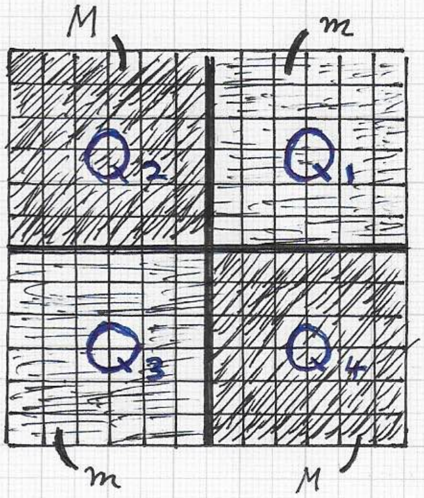
$c_1 = \frac{\sqrt{2}}{2}(2m-M)$,

$c_2 = 0$,

$c_3 = 0$,

$c_4 = 0$,

$c_5 = \frac{\sqrt{2}}{2}(2m+M)$.



Example of a 2D segment with a checkerboard texture of alternating values m and M. The 2D segment consists of 12×12 pixels, while the "texture resolution" is 2×2. The 5-pixel mask is applied to all 5-pixel subsets of the segment – with all mask pixels lying inside the segment.

IT IS NOT REALISTIC TO ASSUME THAT IMPORTANT SEGMENT TEXTURE BEHAVIOR 'OCCURS AT PIXEL SCALE', THE RESOLUTION LIMIT. FOR EXAMPLE, CONSIDER THE CHECKERBOARD SEGMENT TEXTURE SHOWN IN THE LEFT IMAGE. THE 2D SEGMENT (IMAGE) RESOLUTION IS 12×12 PIXELS; THE RESOLUTION OF THE TEXTURE IS 2×2. WHAT "SPECTRAL RESPONSE" IS OBTAINED WHEN APPLYING THE 5-PIXEL MASK TO THIS SEGMENT? ...

~ BH