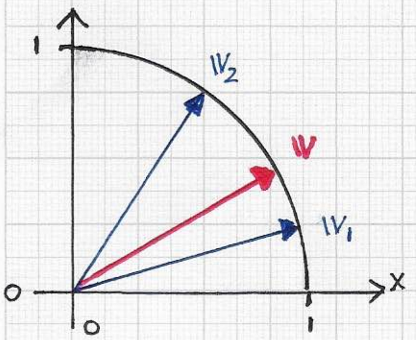
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ...

• Representations with respect to normalized non-orthogonal basis vectors (histogram functions):



A normalized vector $\mathbb{V}$ (or a "histogram function) is expanded via normalized basis vectors $\mathbb{V}_i$. The basis vectors represent a specific class' characteristics (feature vector, for example). A "meaningful" and "robustly computable" similarity measure is needed – to determine class membership of $\mathbb{V}$.

Computed expansion:

$$\mathbb{V} = \alpha_1 \mathbb{V}_1 + \alpha_2 \mathbb{V}_2$$

⇒ Use this expansion, i.e., coefficients α_1 and α_2, to determine whether $\mathbb{V}$ belongs to class $\mathbb{V}_1$, class $\mathbb{V}_2$ or to "another class."

⇒ **Classification via non-orthogonal basis expansion**

We must establish the relationship between our specific object/material classification problem and the expansion of a vector ("histogram function") in terms of a non-orthogonal basis. We described a technique to characterize a material segment in a 2D/3D image by locally expanding the segment's density (or mass) function with eigenfunctions. These expansions in turn generate coefficient value distributions – "spectra" – that capture material characteristics at multiple scales, from coarse to detail characteristics. Multiple, different material classes produce different coefficient value distributions. (In principle, two different materials can lead to the same, or nearly the same, distributions – when recorded image data do not capture material differences.) **Thus: A new material scan produces coefficient value "histogram functions" to be expanded in terms of stored "histogram functions." The expansion(s) can be used for classification.** ...

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■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ... The representation/expansion of a "histogram function" in terms of a basis consisting of linearly independent but not orthogonal "histogram sample functions" must be viewed in the context of the overall problem and the main steps of the solution approach:

- A specific material/object class is represented by k segments, i.e., sets of 2D/3D pixels/voxels with associated density (or mass) values.
- Each segment serves as a material sample for "training"; it is characterized in a spectral fashion by expanding the segment's associated density (or mass) function via locally computed eigenfunction expansions.
- Each local segment eigenfunction expansion generates a coefficient tuple; the set of all such local expansions generates distributions for the computed values for the components of coefficient tuples.

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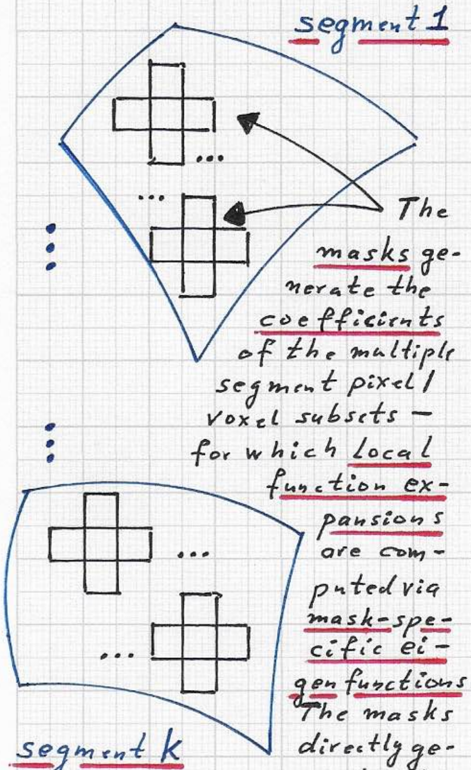


The masks generate the coefficients of the multiple segment pixel/voxel subsets – for which local function expansions are computed via mask-specific eigenfunctions. The masks directly generate the coefficients by computing the involved convolutions with weights used for pixel/voxel values.

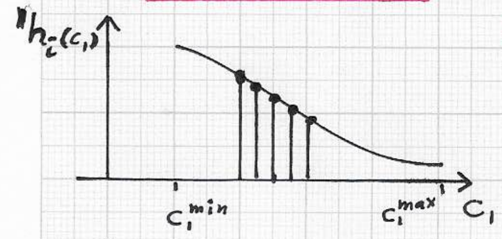
The k segments – all being training representatives of the same material class – generate coefficient value histograms based on sets of many local eigenfunction-based expansions.

Each segment i has a set of associated expansion coefficient value "histogram functions." These "histogram functions" of expansion coefficient values describe the segment's low-, medium- and high-frequency characteristics.

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ...

Considering the **5-pixel mask** example (last page), each segment i, $i=1\ldots k$, generates **5** coefficient value histograms / "histogram functions." These **5** functions are written as ${}^{1}h_i(c_1), \ldots, {}^{5}h_i(c_5)$.



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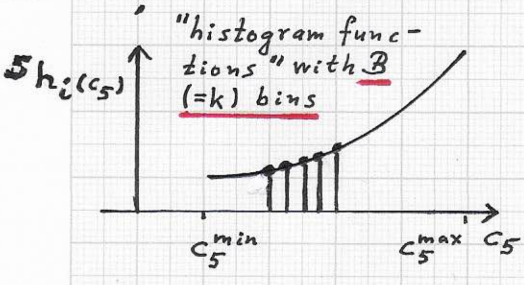

For compatibility the value of c_1^{min} (c_1^{max}) is the same for all functions, i.e., for all ${}^{1}h_i(c_1)$, $i=1\ldots k$, **AND** the "unclassified function" $h(c_1)$. The same holds for all other coefficient value "histogram funtions," i.e., for $c_2, c_3, c_4, \ldots$

The **k segments** have different numbers of pixels/voxels. Thus, the number of data for these coefficient value histograms varies from segment to segment; this is assumed to be the maximal number of masks that "fit" into a segment.

• These coefficient value distributions are (initially) represented via histograms – piecewise constant "histogram functions" – dividing the value domain into B bins.

• **NOTE.** The eigenfunctions (and their associated coefficient values) inherently define a multi-scale, multi-frequency characterization of a segment's density (or mass) function behavior. This fact can therefore be used to analyze a segment's "coarse-scale," "medium-scale" and "fine-scale behavior." Thus, one can eventually perform scale-based, scale-specific comparisons between an unclassified segment and those stored as training samples of a specific class.

• The "histogram function" (distribution) of a specific coefficient of a segment's expansion, via multiple local eigenfunction expansions, is represented as a linear combination of the "histogram sample functions."

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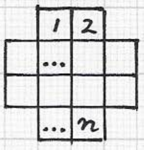
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ...

A mask consisting of n pixels/voxels produces the n coefficient values of a local eigenfunction expansion:



$$f = \sum_{i=1}^{n} c_i \, e_i ,$$

$e_i = i^{th}$ eigenfunction

The eigenfunctions e_i correspond to n eigenvalues/-frequencies. As seen before, some eigenvalues can have multiplicities > 1. Nevertheless, a term $c_i \, e_i$ captures local behavior "at scale i." The number of scales one can consider for classification is n.

AN UNCLASSIFIED IMAGE/SCAN SEGMENT IS ANALYZED AND DESCRIBED VIA n COEFFICIENT VALUE "HISTOGRAM FUNCTIONS" $h(c_1), \ldots, h(c_n)$. THEY SUPPORT MULTI-SCALE CLASSIFICATION.

- The stored "histogram sample functions" of all material/object segment samples (and of all their eigenfunction associated coefficients) capture the distinct spectral signatures, "fingerprints," of all classes considered for the classification of a new unclassified segment.
- The new unclassified segment's eigenfunction coefficient value distributions ("histogram functions") produce several linear combinations that combine the stored "histogram sample functions" to reproduce the unclassified "histogram functions."
- The problem of classifying a new segment is therefore a problem of comparing the coefficients of those linear combinations – that "represent the unclassified data in terms of classified data." DO THESE COEFFICIENTS INDICATE THAT THE UNCLASSIFIED SEGMENT REFLECTS THE CHARACTERISTICS OF A KNOWN CLASS(ES)?

...

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ...

The classification of an unclassified segment can use the "histogram functions" in the following table:

$h(c_1), \ldots, h(c_n)$
${}^1h_1(c_1), \ldots, {}^nh_1(c_n)$
$\vdots \qquad \vdots$
${}^1h_k(c_1), \ldots, {}^nh_k(c_n)$

same class

The number of samples (k in this case) varies from class to class.

Available coefficient value "histogram functions". The unclassified segment is described via n functions $h(c_1), \ldots, h(c_n)$, each one characterizing the segment's behavior at a specific scale. The stored and classified "histogram functions" ${}^jh_i(c_j)$, $i = 1 \ldots k$, also describe the behavior of the k training samples at n scales ($j = 1 \ldots n$).

These functions make possible the comparison and classification of a segment by CONSIDERING JUST ONE OR MULTIPLE SCALES.

• Thus, the classification decision to be made for an unclassified segment is made based on the coefficients of linear combinations of known, stored "histogram sample functions": Each such linear combination is of the form $h(c) = \sum_{i=1}^{k} \alpha_i h_i(c)$. The function $h(c)$ is represented exactly, without error, in the basis $\{h_i(c)\}_{i=1}^{k}$, e.g., by enforcing that $B = k$. The coefficient tuple $(\alpha_1, \ldots, \alpha_k)$ must serve the purpose of determining whether $h(c)$ can be viewed as function "representative" of material class 1, 2, ... or k – or not.

• More precisely, the use of an n-pixel mask, i.e., the use of n eigenfunctions to locally expand segment function values via an n-term linear combination, produces n "histogram functions" $h(c_j)$ for the unclassified segment and the k classified sample functions for a class, called ${}^jh_i(c_j)$, $j = 1 \ldots n$, $i = 1 \ldots k$.

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