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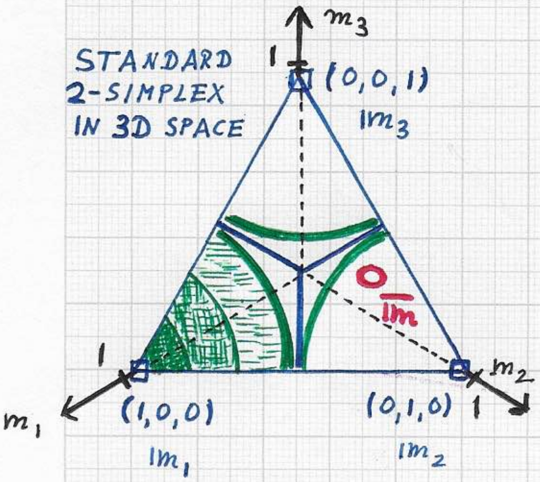
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ... Once the classified $\mathbb{m}_i$-tuples and the unclassified $\overline{\mathbb{m}}$-tuple have been mapped to the STANDARD (k-1)-SIMPLEX in k-dimensionally space, having k vertices, we can express $\overline{\mathbb{m}}$ in terms of k barycentric coordinates $m_1, \dots, m_k$. The interior and the boundary of the standard simplex is defined by its convex hull, i.e., the set of all points/k-tuples $(m_1, \dots, m_k)$ satisfying the conditions $m_1 + \dots + m_k = 1$ and $m_i \geq 0$. Using one of the described distance measures for the distances between $\overline{\mathbb{m}}$ and $\mathbb{m}_i$, $i = 1 \dots k$, one can compute a probability value p for $\overline{\mathbb{m}}$ (and indirectly for $\mathbb{m}$) that estimates to what degree one could consider the $\overline{\mathbb{m}}$ ($\mathbb{m}$) - tuple to belong to class i. It is now possible to devise a design of algorithms performing probabilistic object/material classification. Material classes are defined ("trained") by one or more class samples ("segments") per class; each segment is described by a multi-scale signature that consists of normalized histograms of coefficient values of eigenfunction expansions. ...



After mapping the classified and the one unclassified tuples to the shown STANDARD SIMPLEX configuration, one can calculate a class probability value p for $\overline{\mathbb{m}}$ based on a distance to each simplex vertex. The region shaded in green, for example, represents the fact the a p-value for class $\mathbb{m}_1$ decreases with increasing distance to $\mathbb{m}_1$. The bold green circular arcs indicate the boundaries of the regions where p-values are > 0; when an $\overline{\mathbb{m}}$-tuple lies beyond the region bounded by such an arc, then the p-value will be 0 concerning the class with its $\mathbb{m}_i$-tuple being the center of the circle.

In the figure, $\overline{\mathbb{m}}$ can at most lie in one circular arc-bounded region where a p-value is greater than 0. Thus, "class membership" is "exclusive" as $\overline{\mathbb{m}}$ can only belong to (0 or) 1 class.

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions:... Given H normalized coefficient value histograms $h_1(c), \ldots, h_H(c)$ of an unclassified segment's eigenfunction expansions at H scales, we can calculate probability values for the H scales as follows: The "database" of classified image segments contains sample training data for (i) C classes of materials, $cl_1, \ldots, cl_C$; (ii) $k_1, \ldots, k_C$ segments $sg_1, \ldots, sg_{k_{cl}}$, $cl = k_1, \ldots, k_C$, for each material class; (iii) H scales for each segment. Thus, the "database" stores a total of $(k_1 + k_2 + \ldots + k_C) \cdot H$ scale-specific normalized histograms. The table (left) summarizes at a high level the probability values generated for the classification of one new, unclassified image segment. As far as the actual size of this table is concerned, a practical classification application might involve (i) $C = 100$ material classes; (ii) $k = 100$ sample segments per class, on average; and $H = 5^3 = 125$ scales, when assuming that a $5 \times 5 \times 5$ voxel mask is used for generating coefficient value histograms. ...

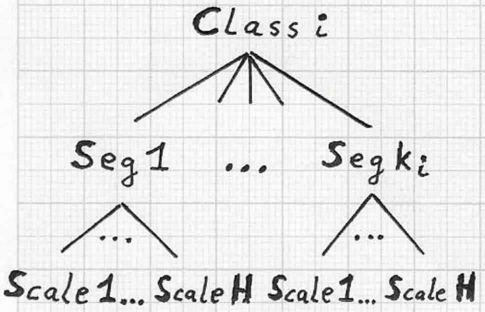
			scales 1	...	H
CLASS 1	SEG	1	$p_{1,1,1}$		$p_{1,1,H}$
		⋮		⋱	
		k_1	$p_{1,k_1,1}$		$p_{1,k_1,H}$
CLASS 2	SEG	1	$p_{2,1,1}$		$p_{2,1,H}$
		⋮		⋱	
		k_2	$p_{2,k_2,1}$		$p_{2,k_2,H}$
⋮	⋮	⋮	⋮	⋮	⋮
CLASS C	SEG	1	$p_{C,1,1}$		$p_{C,1,H}$
		⋮		⋱	
		k_C	$p_{C,k_C,1}$		$p_{C,k_C,H}$

Table of all probability values that can be computed for an unclassified image segment characterized by H normalized coefficient value histograms. The classified data is given by C classes $1 \ldots C$, exemplified by $k_1, \ldots, k_C$ image segments, respectively.

The probabilities $p_{cl,sg,sc}$ define the "degree of match" between a specific scale of the unclassified segment and the same scale of one of the segments of the of one of the classes. (sc: scale, sg: segment, cl: class)

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: … Thus, it is reasonable to assume that a realistic application would generate on the order of 1,000,000 probability values one can use for the classification of an unclassified image segment.



Material class i is represented in the sample database by k_i image segments. Each segment is hierarchically described via H scale-specific histograms. The H scales make possible a multi-resolution characterization and comparison of "textures at lower and higher frequencies."

"Scale 1" and "Scale H" refer to the coefficient value histograms that capture lowest-frequency and highest-frequency behavior, 1 and H, respectively.

h_1	$\stackrel{?}{=}$	H_1	$\Rightarrow$	P_1
h_2	$\stackrel{?}{=}$	H_2	$\Rightarrow$	P_2
$\vdots$		$\vdots$		$\vdots$
h_H	$\stackrel{?}{=}$	H_H	$\Rightarrow$	P_H

Computational building block. Classified histograms $h_1, \dots, h_H$ are compared with unclassified histograms $H_1, \dots, H_H$ producing probabilities P.

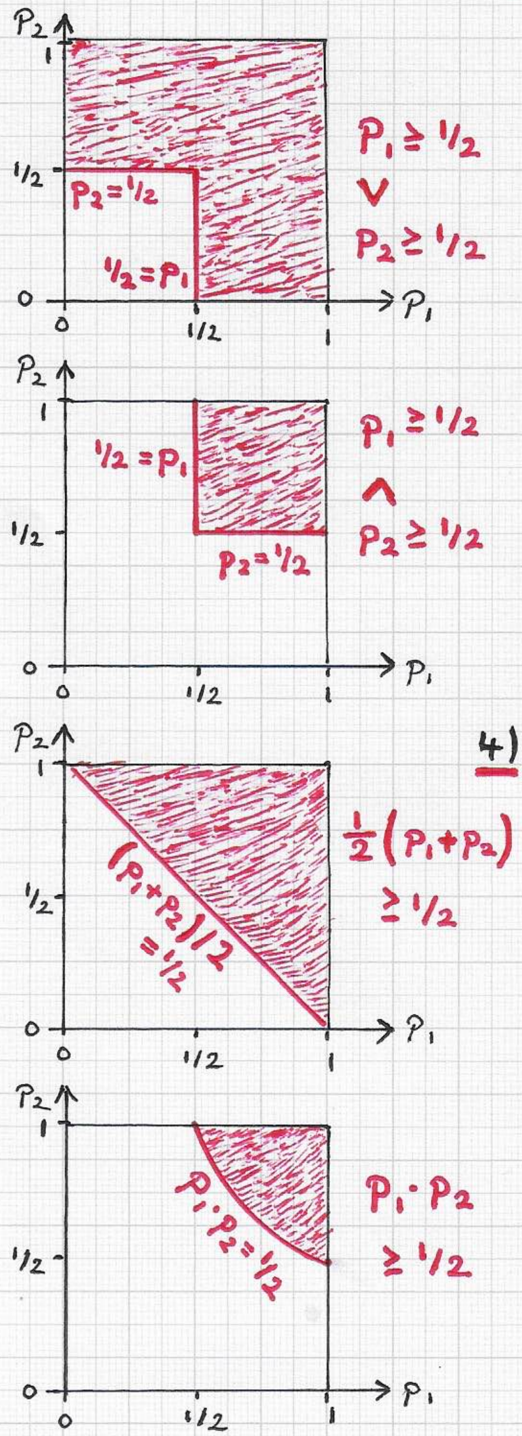
Considering the illustration of a material class i as a tree-like structure (left figure), one must keep the following possibilities in mind when using probability values for material classification:

1) Two segments belonging to the same class each have H associated coefficient value histograms; the histogram pairs for the same scale can sometimes be (nearly) identical. This behavior indicates that such segment pairs exhibit (nearly) the same texture signature for the respective scale(s).

2) Pairs of segments can be "completely identical" or "completely and substantially different" with respect to their H scale histograms, with pairs belonging to the same class. …

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: ... **3)** Generally, it must be assumed that the "feature space" of a specific material class i is often not sampled sufficiently, i.e., not sampled densely enough, to properly describe and cover the allowed variation of scale behaviors. Often, the number of material samples is extremely limited or a material class simply permits a high degree of variation of texture, at many scales.



Examples of using Boolean ($\vee, \wedge$) and arithmetic operators ($+, \cdot$) to combine probabilities for classification.

4) Given two different material classes, each represented via several sample segments and their associated scale histograms, it is possible that several histograms belonging to one class are (nearly) identical with histograms belonging to the other class. The implication is that materials from different classes **can** have the same texture signatures for certain scales.

One must carefully design a probabilistic classification system architecture that considers these facts and possibilities.

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■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions: … The figures on the previous page provide simple examples for potentially combining probabilily values $p_1, p_2, \ldots, p_H$ that result when comparing **H** normalized coefficient value histograms of an unclassified image segment with those of a classified image segment. Of course, since **p**-values reflect the degree, the strength, of a match of a specific **scale** of two image segments, one might consider and use them only individually. Nevertheless, when having to calculate a "meaningful" integrative measure of degree of similarity of two image segments at several scales, or even across all computed H scales, one must establish a **function F $(p_1, \ldots, p_H)$ and a threshold P to determine when two image segments match.** Clearly, the design of a function F must be done very carefully – considering all aspects defining a match. !

$$\sum_{i=0}^{4} \binom{4}{i} = 16 = 2^4$$

	p_1	p_2	p_3	p_4	function
1					F_{0000}
	•				F_{1000}
4		•			F_{0100}
			•		F_{0010}
				•	F_{0001}
	•	•			F_{1100}
	•		•		F_{1010}
6	•			•	F_{1001}
		•	•		F_{0110}
		•		•	F_{0101}
			•	•	F_{0011}
	•	•	•		F_{1110}
4	•	•		•	F_{1101}
	•		•	•	F_{1011}
		•	•	•	F_{0111}
1	•	•	•	•	F_{1111}

Abstract view of a function **F** used to operate on a subset of available probability values **p** to calculate a "final match probability" for two image segments. The bitstring index of **F** defines those probabilities **p** that are the function's arguments.

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