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OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks: ...

4) In general, we characterize image segment data for H scales. The small-mask-based convolution method generates coefficient value distributions (histograms) for the large number of local image function expansions, using H eigenfunctions as multi-scale basis functions. IT IS REASONABLE TO ASSUME THAT MATERIAL CLASSES CAN BE RECOGNIZED EVEN WHEN USING A SMALL SUBSET OF ALL AVAILABLE SCALE-SPECIFIC DATA FOR THE CLASSIFICATION OF IMAGE SEGMENTS. For example, it might be possible to classify an image segment as a segment belonging to a class 'C' when determining a high degree of similarity for only three scales, e.g., scales 1, 4 and 6. Thus, it should be design goal for an efficient data analysis and classification system/architecture/network to use a "most appropriate" – possibly a "minimal" – subset of all available multi-scale data for rapid high-probability classification of image segments. For example, when a to-be-classified image segment has a degree of similarity of .8 with class 'C', considering only scale 1, the corresponding

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OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd

• Laplacian eigenfunctions and neural networks:... probability for a 'class-C match' might be .9. When also considering scale 5, the to-be-classified image segment has a degree of similarity of .9 with class 'C', and the corresponding probability for a 'class-C match' might be .8. BUT: When viewing the two individual probabilities together (.9 and .8), the combined probability for a 'class-C match' could be .99 – sufficiently large a value to stop the classification process and declare that the image segment belongs to class 'C'. (The p-values represent degree of similarity [in terms of scale-specific coefficient value histograms], and the F-values represent class-match probabilities.)

The result that a to-be-classified image segment is a 'class-C match' should be calculated, i.e., decided, with a minimal number of processing steps and comparisons, maximally efficiently. The result that such a segment does not belong to any of the classes of interest should also be decided maximally efficiently.

5) Assuming that the available multi-scale characterizations of all classes of interest make it possible to decide whether an unclassified image ...

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... segment is a class-match for exactly one or none of the classes of interest, one wants:

IDEALLY, EACH CLASS-MATCH SHOULD BE DEFINED VIA AN OPTIMIZED, I.E., MINIMIZED BOOLEAN AND/OR FUZZY LOGIC EXPRESSION, USING, FOR EACH CLASS, THE MOST DISCRIMINATIVE AND THE MINIMAL NUMBER OF SCALES. For example, an unclassified image segment might be a 'class-C match' if: $(F_1(p_1) > .9) \vee (F_2(p_2) > .8 \wedge F_{3,4}(p_3, p_4) > .8)$. Here, F_1 and F_2 are univariate decider functions, with the degree-of-similarity for scale 1 and degree-of-similarity for scale 2, respectively, as arguments; $F_{3,4}$ is the bivariate decider function that calculates a match probability based on degree-of-similarity for the scale pair 3 and 4. When an unclassified segment is not a match for any of the classes of interest, i.e., it does not satisfy any of the class-specific conditions, it is of class 'NONE.'

6) A database of materials to be detected stores, for each material class, the eigenfunction-based multi-scale coefficient value

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■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... distributions (for H scales, represented in discrete form via binned histograms). Our task is to determine – EFFICIENTLY – whether a new, given unclassified image segment most likely belongs to one of the classes of interest (and which one) or not. For example, one can think of this task as having to define the multiple "cases" that specify the material-specific condition that must be satisfied by a new, given image segment to belong to one of the material classes of interest (or not). The condition must be expressed via a Boolean logic expression – that is different for every material class and combines values of univariate, bivariate, ..., multi-variate decider functions. The challenge is the design of the needed Boolean logic-based expressions that operate on, with minimal computational cost, the best-possible set of available p-value and F-value data, for each material class. Again, a p_i-value reflects the degree-of-similarity, for scale i, between an unclassified segment and a class of interest; an $F_i(p_i)$-value, for example, defines the probability for the unclassified segment to belong to the class for which the specific p_i-value was computed.

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■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:...

7) A small number of scales, and the corresponding scale-specific histograms and derived data, might suffice to perform the classification problem. Nevertheless, the various Boolean logic expressions defining class membership for the different classes are likely involving large numbers of logic operators and variables, based on p- and F-values. THUS, IT IS IMPORTANT TO MINIMIZE THESE EXPRESSIONS BY TRANSFORMING THEM TO EQUIVALENT EXPRESSIONS THAT CAN BE EVALUATED WITH MINIMAL COST. For example, the KARNAUGH MAP / KARNAUGH–VEITCH DIAGRAM is commonly used to find a minimal Boolean logic circuit for a given Boolean expression. While the Karnaugh map is a simple and effective tool for expression minimization, it is a viable tool only for expressions not involving "many" variables, i.e., not involving more than six variables. When the number of variables is large, one must use combinatorial optimization methods to design a (near-) minimal Boolean logic circuit. Such methods are also used for the minimization of more general electronic circuits. Simulated annealing is just one of several methods one can consider.

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