OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks: ...

8) Generally, we can consider the use of the following variables/functions to involve in the determination of an image segment's class: (i) For an unclassified segment, we know its degree-of-similarity values – for H scales, for all classes of interest ("p-values"). (ii) For an unclassified segment, and for all classes and scales, we know/have specified univariate, bivariate, ..., multivariate decider functions ("F-functions") – that define the probability of belonging to a specific class for one or several known p-values. For example, the meaning of "$F_{1,3}(p_1,p_3)=.95$" is the following: "Given two degree-of-similarity values for the unclassified segment for scales 1 and 3, and the definition of the bivariate decider function $F_{1,3}$, the probability of the unclassified segment of belonging to the class being compared against is .95." Of course, the value .95 is the result returned by $F_{1,3}$ for a specific value pair for p_1 and p_3, e.g., $p_1 = .9$ and $p_3 = .9$. (iii) One can consider the use of weights (optimized) that could combine all or some of the multi-scale data

...

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... available for the segment's classification: p-values p_1, $p_2, p_3, \ldots, p_H$ and decision function values, F-values, depending on one or multiple p-values, i.e., $F_1, F_2, F_3, \ldots, F_H$, $F_{1,2}, F_{1,3}, F_{1,4}, \ldots, F_{1,H}, \ldots, F_{1,2,3,\ldots,H}$. There are H p-values and $2^H - 1$ F-values (assuming that all possible decider functions F are indeed known). When involving weights to determine a "final classification probability," one needs to calculate/define (and optimize) the values of weights $w_1, w_2, w_3, \ldots, w_H$ for the p-values and of weights $W_1, W_2, W_3, \ldots, W_H, W_{1,2}, W_{1,3}, W_{1,4}, \ldots, W_{1,H}, \ldots, W_{1,2,3,\ldots,H}$ for all possible F-values.

9) A practical method based on this concept must be designed with a keen awareness of the computational cost defined by the number of scales being used in classification calculations. THE NUMBER OF SCALES AND THEREFORE THE NUMBER OF p-VALUES AND THE NUMBER OF DECIDER FUNCTIONS, I.E., F-VALUES, SHOULD BE MINIMAL – MINIMIZED FOR EACH CLASS – TO SATISFY EFFICIENCY REQUIREMENTS.

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... **10) One can adopt and design a "class-centric" approach for performing the classification of an unclassified image segment. A "standard" classification approach might compute/define ALL possible p- and F-values for the unclassified segment by comparing it with each class of interest, each represented by its H scale-specific coefficient histograms. An alternative approach would only consider one material class at a time and compare its coefficient histograms with those of the unclassified segment. Algorithmically, one could implement this alternative as follows:**

- **Input:** Eigenfunction coefficient histograms for one unclassified image segment and for multiple material classes of interest, where each class' histograms have been established via many sample image segments
- **Output:** Classification result for the unclassified segment
- **Algorithm:**

```
/* Perform iterative comparision process */
/* between a specific class of interest */
/* and the unclassified image segment */
/* until a first "class-match" is de- */
/* termined or no "class-match" can be found. */
```

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:...

```
/* Assuming that it is possible */
/* and allowed that an unclassi- */
/* fied image segment could be */
/* a "match" for several classes of interest, one */
/* could use a WHILE loop implementation: */
• noMatches = 0;
• WHILE (additional classes are available for comparison) DO
      - compere unclassified segment with specific class;
      - IF (there is a "class-match")
            - store the class-ID with the unclassified segment;
            - noMatches += 1;
• IF (the noMatches is 0)
      - store the class-ID "NONE" with the unclassified segment;
/* One can also terminate the WHILE loop */
/* when a first "match" has been found. */
/* Note. i) p-values (degree-of-similarity) */
/*          would be computed in the loop. */
/*       ii) F-values/-functions can be pre- */
/*          defined/-computed. */
```

Point ii) of the Note in the comment above is important: The possible, maximal number of F-functions is $2^H - 1$, for image segment data characterized with H scales. Thus, the fact that these F-functions can be pre-computed is essential for efficiency.

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... **Pre-computation** of **F-functions** is possible for the following reason: For each material class of interest, a "large" number of representative samples is used to establish its H eigenfunction-based coefficient value histograms. For example, the scale-1 histograms of the materials can be similar or dissimilar. If one histogram **h** is very dissimilar to all other histograms, then the implication will be this: When an unclassified image segment has a histogram that is very similar to **h** (for the same scale), the associated F-function value should be large, indicating a high probability for a class-match. In other words, scale-specific histogram data for material sample images must return relatively larger F-function values when they are rather distintive, i.e., very different from other histogram data. **Point i)** of the Note emphasizes that only degree-of-similarity values, p-values, must be calculated in "real time" for the analysis and classification of an unclassified image segment. Further, high-probability classification might often be possible with just a small number of p-values. ...