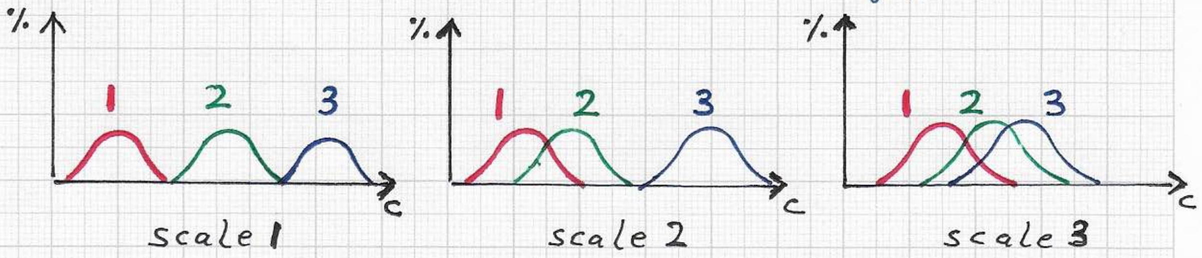
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... We consider an example in an attempt to clarify these rather complicated concepts. We assume that three material classes are of interest, and the number of scales used for classification is three as well. (The total number of scales used in a typical mask-based and eigenfunction-based image analysis is generally much larger than three; nevertheless, using only three scales for the classification process might be sufficient in some cases.) The following illustrations show a scenario for the many-samples-based coefficient value histograms for the three classes and scales at a high, abstract level:



The symbol 'c' stands for 'coefficient value' — which could be the absolute or squared value of an actual eigenfunction coefficient (a secondary issue). It must be assumed that the sketched distribution functions are all normalized. Scale 1: Distributions/histograms mutually do not overlap. When an unclassified image segment's scale-1 histogram has a high degree-of-similarity with one of the three histograms for the three material classes, the F-value should be large. ...

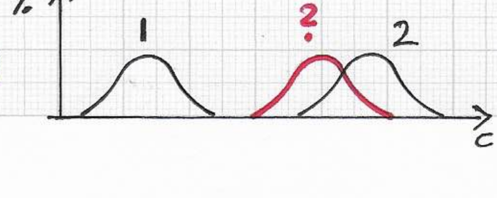
■ OBJECT AND MATERIAL EIGENFUNCTIONS — Cont'd.

• Laplacian eigenfunctions and neural networks: ... **Scale 2: Distributions/histograms for classes 1 and 2 overlap.** When an unclassified image segment's scale-2 histogram has a relatively high degree-of-similarity with the histograms for classes 1 or 2, the F-values for these two classes should be "scaled down" accordingly. **Scale 3:** All distributions/histograms for the three classes overlap. When an unclassified image segment's **scale-3 histogram** has a certain degree-of-similarity with the histograms of classes 1, 2 or 3, the F-values for these three classes should also be "scaled down" accordingly. **In summary, the "discriminative power" of these histograms decreases with increasing scale — in this example.**

In order to understand the interplay between p-values and F-function values, based on histogram overlap, we consider some possible scenarios:

i)


i) 2 classes (1,2), not overlapping; degree-of-similarity values for '?': $p^1 = 0$, $p^2 = 0$

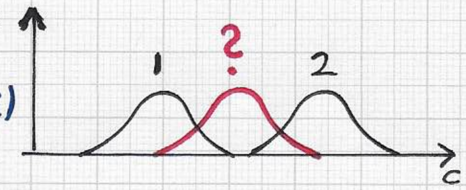
ii)


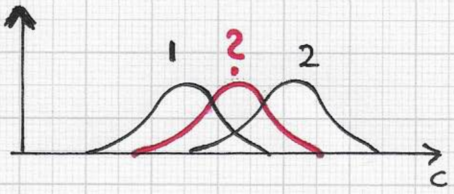
ii) 2 classes (1,2), not overlapping; degree-of-similarity values for '?': $p^1 = 0$, $p^2 > 0$

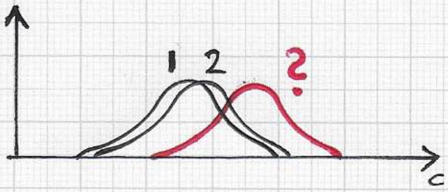
...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks: ...

iii)


iv)


v)


iii) 2 classes (1, 2), not overlapping; degree-of-similarity values for '?': $p^1 > 0$, $p^2 > 0$

iv) 2 classes (1,2), overlapping; degree-of-similarity values for '?': $p^1 > 0$, $p^2 > 0$

v) 2 classes (1, 2), "nearly identical"; degree-of-similarity values for '?': $p^1 > 0$, $p^2 > 0$ and $|p_1 - p_2| \ll \varepsilon$

Case v) is most relevant in the context of defining proper F-function values for specific p-values: In this case, the coefficient histograms for class 1 and class 2 are "nearly identical" (for this specific scale) – and thus the values of the degree-of-similarity of '?' relative to class 1 and class 2 are also "nearly identical." (The variable p^{cl} is the degree-of-similarity for class cl.) **THE OVERLAP OF CLASS HISTOGRAMS MUST BE CONSIDERED FOR THE DEFINITION OF F-FUNCTION VALUES: LET US ASSUME THAT THE COEFFICIENT HISTOGRAMS, FOR A CERTAIN SCALE, ARE IDENTICAL FOR TWO DIFFERENT CLASSES.** **!**

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:...

• If a coefficient histogram of some class has a degree-of-similarity value p when compared with the histogram for the same scale of the unclassified image segment, with $0<p<1$, then one will be able to consider two prototypical extreme cases:

– The coefficient histogram of the known class used for comparison DOES NOT overlap with another known-class histogram. A simple default F-function will be

$$F(p) = p \quad , \text{ with } 0<F<1 .$$

– The coefficient histogram of the known class used for comparison IS IDENTICAL TO EXACTLY ONE OTHER HISTOGRAM of some other known class. A simple default F-function FOR BOTH KNOWN CLASSES will be

$$F(p) = p/2 \quad , \text{ with } 0<F<1/2 .$$

Essentially, such a "scaled-down" value of an F-function emphasizes the decrease in "discriminative power" of known class histograms when they increase in their overlapping regions, thereby making it increasingly difficult to decide to what class the unclassified image segment belongs.

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... One can perform a gedankenexperiment to contemplate the effects one might desire when the coefficient histograms (for a certain scale) overlap or do not overlap. Considering once again the two-class scenario, the values of p^1 and p^2 represent degree-of-similarity values of an unclassified segment's coefficient histogram and the two histograms defining the known two classes, **1** and **2**. One can employ the same degree-of-similarity histogram measure to calculate the similarity of the two histograms of the two known classes – which we call $s_{1,2}$. The goal is to define the decider functions F^1 and F^2 – where $F^1 = F^1(p^1, s_{1,2})$ and $F^2 = F^2(p^2, s_{1,2})$. **Ultimately, it is an algorithm design decision whether $s_{1,2}$ should or should not directly impact the values of F^1 and F^2.** The following table provides an example:

$s_{1,2}$	p^1	p^2	F^1	F^2
0	0	0	0	0
0	1	0	1	0
0	0	1	0	1
0	1	1	?	?
1	0	0	0	0
1	1	0	?	?
1	0	1	?	?
1	1	1	1	1

Table of F-values not scaled down (two classes, **1** and **2**).

(Subscripts are not needed as only one scale is considered; superscripts indicate the class, **1** or **2**.) First, the rows including '?' values concern cases that are logically not possible, as far as the $(s_{1,2}, p^1, p^2)$ triples are concerned. Second, one must decide whether the values in the F^1- and F^2-columns should be divided by **2** or not.

...