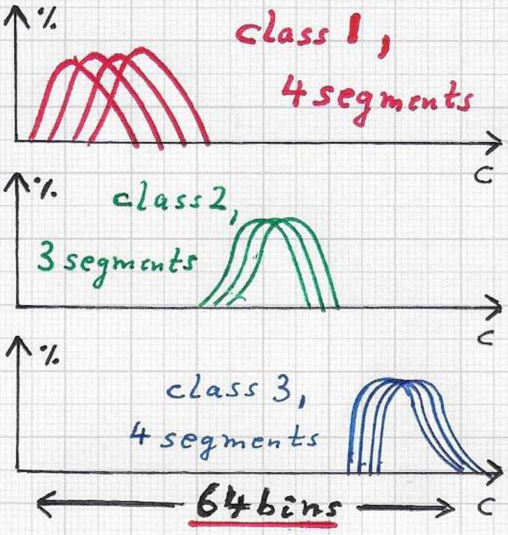
Bernd Hamann
Stratovan

7/21/2022

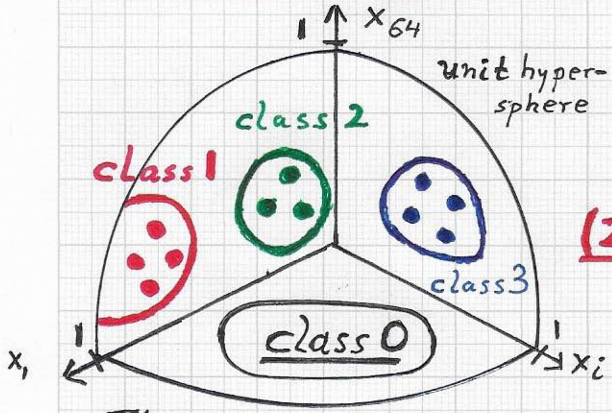
OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• *Laplacian eigenfunctions and neural networks:...*

Scenario involving three classes represente by several sample segments (only one scale considered):



As shown, one can assume that coefficient value histograms of same-class samples are close, clustered. These normalized DISCRETE, BINNED histograms define points on a unit hypersphere.



Three point clusters on the hyperspherical patch $x_1^2+...+x_{64}^2=1$, $x_1,...,x_{64} \geq 0$, correspond to classes 1, 2 and 3. The region "between" the clusters defines class 0.

Class-0 image segments are merely segments that do **NOT** belong to any of the specific (threat) material classes used for classification. Class-0 segments are not of interest; they are not relevant for the "real classification," and they should be identified at the beginning of the segment-processing pipeline via a simple and computationally efficient algorithm. In principle, such an algorithm can compute the answer to one of these questions:

(1) When can a given unclassified segment **NOT** belong to any of the classes of interest (class 1, 2, 3, ...)?

(2) When is a given unclassified segment a class-0 segment?

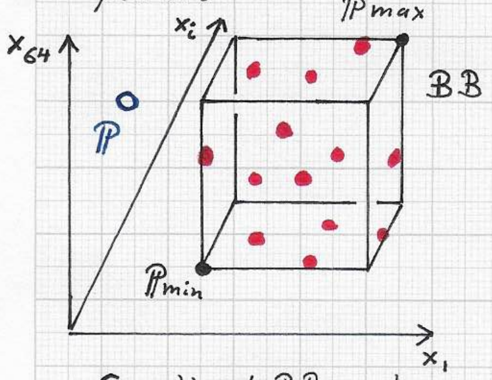
Question (1) requires one to devise a criterion for **NOT** belonging to one of the classes of interest, while (2) requires one to define a criterion for belonging to class 0.

...

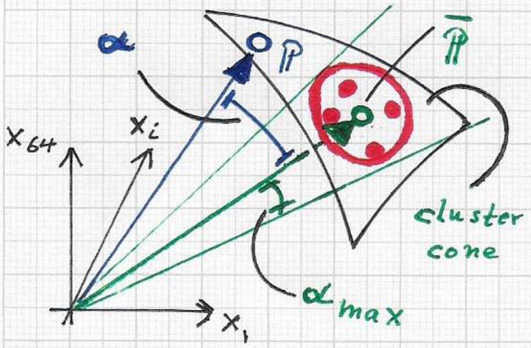
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks: ...

Bounding box (BB) of cluster of "histogram points":



Smallest BB containing all "histogram points" ● of all samples of a specific class at a certain scale. To determine whether a "histogram point" $\mathbb{P}$ (○) is NOT inside the BB, one must consider all 64 dimensions.



Alternative test: All sample "histogram points" of a class lie inside a cluster cone. A cluster center point $\overline{\mathbb{P}}$ (with normalized positional vector) defines the cone's axis, and a maximal angle α_{max} defines the cone's boundary. Since $\alpha > \alpha_{max}$, point $\mathbb{P}$ is outside the cone.

● The simplest way to define a region in (64-dimensional) space is the calculation of a (minimal) bounding box (BB) – as the region that bounds the set of "histogram points" of the samples of a class at a certain scale. The BB is defined by its two extremal corner points $\mathbb{P}_{min}$ and $\mathbb{P}_{max}$, see left figure, where $\mathbb{P}_{min} = (x_1^{min}, \ldots, x_{64}^{min})^T$ and $\mathbb{P}_{max} = (x_1^{max}, \ldots, x_{64}^{max})^T$. Here, x_i^{min} (x_i^{max}) is the minimal (maximal) value of all x_i-values of the "histogram points" in a class cluster.

To make the final decision whether a "histogram point" $\mathbb{P}$ is NOT inside a BB, one must determine the value of $(x_1 \notin [x_1^{min}, x_1^{max}] \wedge \ldots \wedge x_i \notin \wedge \ldots \wedge x_{64} \notin [x_{64}^{min}, x_{64}^{max}])$. Further, if one must consider BBs of C classes, one must perform the NOT-inside test for all BBs, i.e., $BB_1, \ldots, BB_C$. **!**

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks: ...

• Calculations involved in the angle test:

i) Compute the average point (centroid) of the class-specific sample "histogram points" and normalize the positional vector of this average point; call the resulting point $\overline{\mathbb{P}}$.

ii) Denoting the sample "histogram points" as $\mathbb{P}_j$ (with normalized positional vectors $\mathbb{P}_j - \mathbb{O}$), compute a minimal inner product (cos) value, i.e.,

$$\text{minCOS} = \min\{\langle \overline{\mathbb{P}}, \mathbb{P}_j \rangle\}.$$

This cos value defines the minimally required angle necessary to ensure that all sample "histogram points" are inside the cone with its axis defined by $\overline{\mathbb{P}}$ and with opening angle

$$\cos^{-1}(\text{minCOS}).$$

iii) An unclassified "histogram point" $\mathbb{P}$ is NOT inside the (slightly enlarged) cone if

$$\langle \mathbb{P}, \overline{\mathbb{P}} \rangle < \text{minCOS} - \text{epsilon}.$$

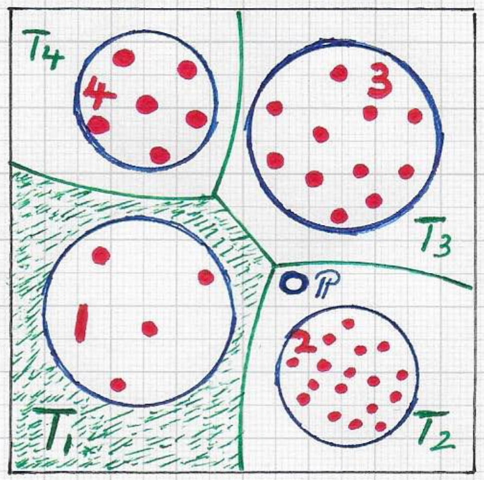
In summary, the "NOT-belonging" test makes it necessary – in principle – to perform the "NOT-inside" BB test for all classes and all scales, considering all, for example, 64 dimensions used to represent histograms discretely via 64 bins. An alternative test is illustrated on the previous page (Left, bottom figure). Instead of using a BB, one can use an angle test: One can define a center/average point $\overline{\mathbb{P}}$ for all "histogram points" based on the samples of one class. By interpreting $\overline{\mathbb{P}}$ as a vector $(\overline{\mathbb{P}} - \mathbb{O})$ and normalizing it, one obtains a unit direction vector – for the axis of a (hyper-) cone that must contain all sample "histogram" points. Based on the axis, one can calculate a (minimal) value for the opening angle of the cone, α_{max}, that guarantees that all sample "histogram points" are clearly inside the cone region.

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks: ...

Generalized Voronoi tessellation in the plane for point clusters with circles bounding the regions of the various clusters:



Abstract illustration of a test to determine whether a "histogram point" $\mathbb{P}$ (O) is inside the class-O space.

→ Four classes exist: 1, 2, 3, 4.

→ The class-regions defined by the sample "histogram points" are bounded by circles.

→ Tile T_i consists of all the points closer to class i^{th} bounding circle than to any other circle(s).

→ All points (not just sample points) inside the circle-bounded regions do **NOT** belong to tiles.

→ The union of all tiles, i.e., $\bigcup_{i=1}^{4} T_i$, defines the class-O region – and the tessellation.

If one must consider C classes (per scale) for an angle-based NOT-inside-cone test, then one will have to compute C inner products $\langle \mathbb{P}, \overline{\mathbb{P}} \rangle$, with class-specific $\overline{\mathbb{P}}$-values. (**Note:** The values of all class- and scale-specific $\overline{\mathbb{P}}$ and **minCOS** parameters can be pre-computed and stored in the material **database**.) Thus, the computational cost of this "NOT-belonging-to-a-class" test depends on the number of classes (C) and the number of histogram bins (B, e.g., B = 64).

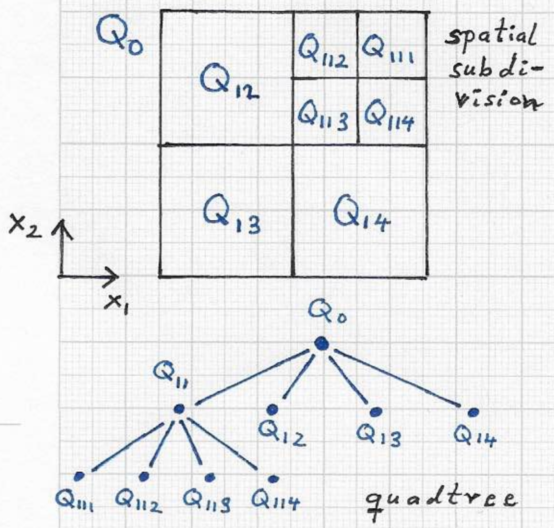
As discussed earlier, an alternative test type can be based on a criterion defining when a "histogram point" $\mathbb{P}$ belongs to class O, i.e., a test that determines whether $\mathbb{P}$ lies inside the region corresponding to class-O image segments. The figure (Left) shows a generalized Voronoi tessellation of four disk regions in the plane. A "histogram point" $\mathbb{P}$ is a class-O point when it is outside all disks.

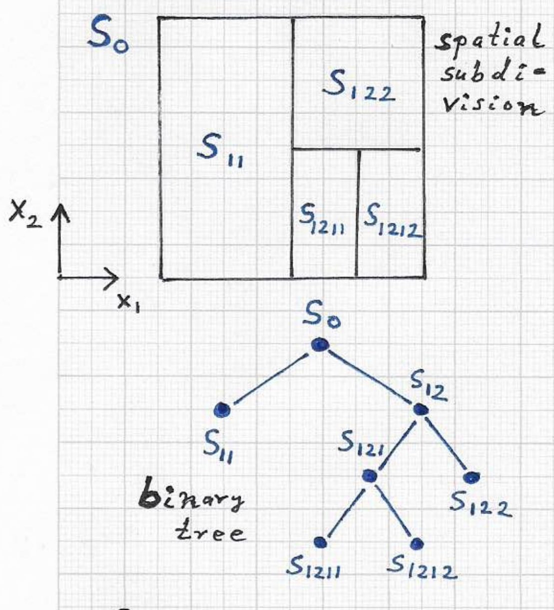
...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:...

Examples of efficient **2D** spatial data structures:





Possible ways to repeatedly subdivide space into hierarchically nested regions with implied tree data structures. **Superposition of such structures onto a Voronoi diagram is desirable.**

Since $\bigcup T_i$, the union of all tiles, defines a "meaningful" and precise region that represents class O, one must determine whether an unclassified "histogram point" p resides inside one of the tiles (and therefore NOT inside or on the boundary of any disk regions). Specifically, an efficient **spatial data structure** is necessary to quickly determine the tile an unclassified point p lies in. Many spatial data structures are known to represent hierarchical, nested spatial subdivisions to break down regions in space into smaller and smaller sub-regions – for discrete representations of space. Typical data structures are **quadtrees, octrees, ..., binary space partition (BSP) trees and k-dimensional (k-d) trees**. These tree structures make it possible to perform point queries rapidly.

...