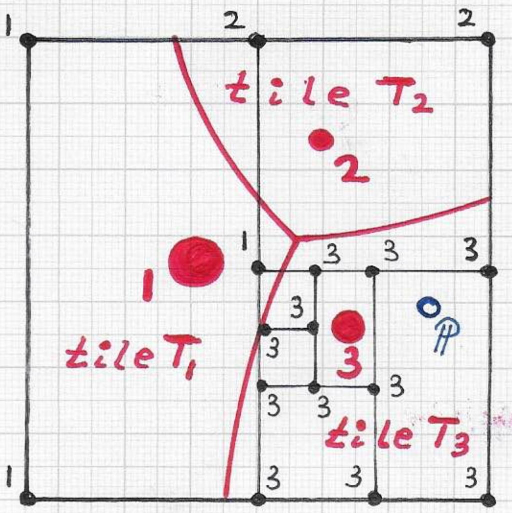
Stratovan

■ OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• Laplacian eigenfunctions and neural networks:...

Superposition of generalized Voronoi tessellation of three disks and binary space partition tree (with axis-aligned edges):



The corner points of the cells of the spatial subdivision inherit the index of the tile T_i they lie in. This example involves three disks/classes, **1,2,3**, and the resulting Voronoi tessellation. **It is not necessary to compute the actual tile boundaries; one must only determine the index of the closest class disk for each cell corner point.** "Histogram point" $\mathbb{P}$ lies in a cell with corner points with index **3**. Therefore, one concludes that $\mathbb{P}$ resides in the Voronoi tile of sample class **3**.

In our application, we can use a hierarchical spatial data structure to determine the tile that contains a given unclassified point $\mathbb{P}$. This criterion can be adopted for decision-making: **If the cell of the spatial data structure that contains the point $\mathbb{P}$ is a cell that is a 'class-i cell,' then $\mathbb{P}$ will be assumed to lie in the tile of the disk containing the class-i samples.** We must recall that the goal is to have a spatial data structure in place, pre-computed and stored in the material database, so that we can do the following:

(i) determine the tile that contains $\mathbb{P}$ in logarithmic search time via a tree data structure, and

(ii) determine whether $\mathbb{P}$ is indeed a class-0 point, by checking whether $\mathbb{P}$ is outside the disk at the tile's center.

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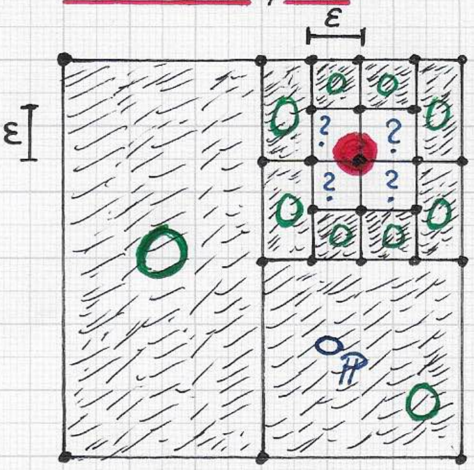
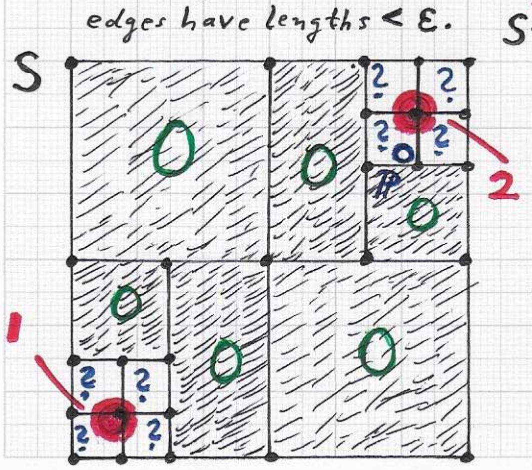
Simplest case considering only one material class – implying that the spatial domain only includes one disk for class-1 samples:

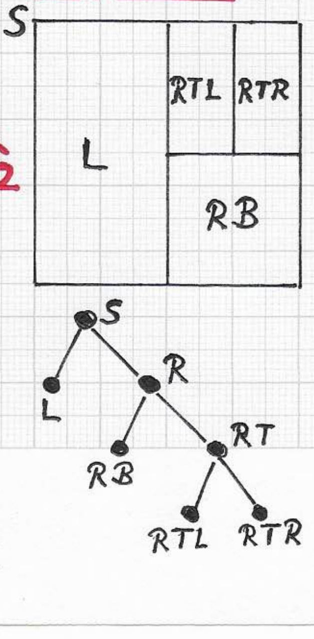


Point 𝒫 is a class-0 point when it lies inside one of the shaded cells. Point 𝒫 potentially lies inside the disk ● when it resides in one of the cells with '?'. Cells are no longer subdivided when all their edges have lengths < ε.



Class-0 regions, class-1 and class-2 regions (disks) and '?' regions; 𝒫 inside '?'.

The computation and representation of a generalized Voronoi diagram/tessellation is **NOT** our goal. Our primary goal at this point is the design of an effective and efficient way to determine whether a new, unclassified "histogram point" 𝒫 is a class-0 point, i.e., a **NON-THREAT** point. The structure of a Voronoi tessellation merely serves as a very helpful concept for the design of an algorithm for our classification problem. The figures (left) illustrate, at an abstract level, the type of hierarchical spatial data structure that is sufficient for addressing our problem.



This structure results from repeatedly splitting regions into left (L), right (R), bottom (B) and top (T) subregions. This simple scenario shows splitting used three times and the resulting tree.

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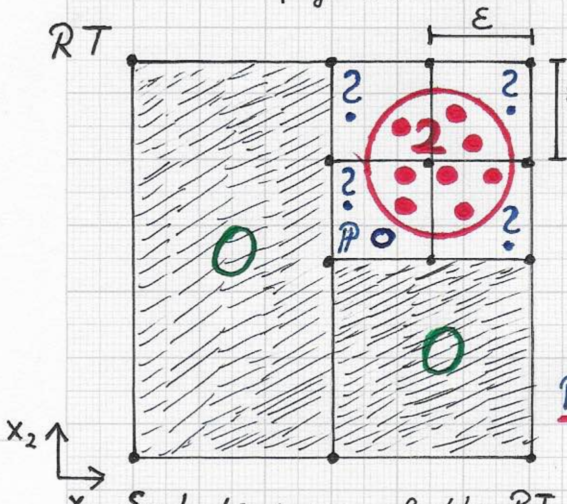
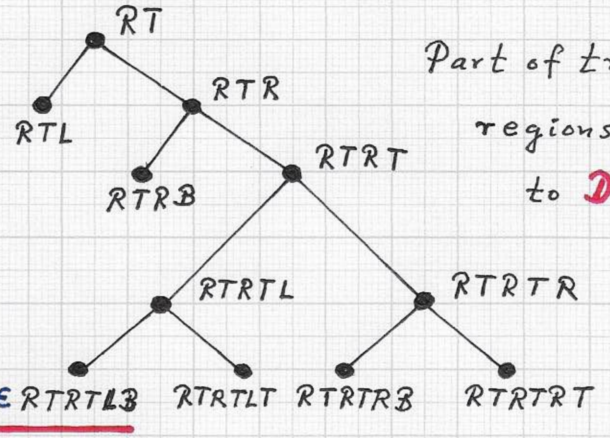
Upper-right quadrant of figure from previous page (bottom figure):



Subdivision of the RT quadrant of space **S** shown on previous page (left, bottom). **Binary subdivision is performed by always alternating splitting in x_1- and x_2--direction.**

The binary subdivision (sub-)tree for the RT quadrant/subregion shown in the figure (left) is:



Part of tree for regions close to **DISK 2**

In this scenario, "histogram points" of **class-2** samples lie inside the sketched red disk, **DISK 2**. Here, spatial subdivision has terminated (ε-limit reached), and **DISK 2** lies entirely in region RTRT and partially in each of this region's four grandchildren, i.e., RTRTLB, RTRTLT, RTRTRB and RTRTRT.

When searching for $\mathbf{p}$ in the tree, $\mathbf{p}$ does not satisfy any of the class-0 subregions' spatial conditions; the search yields the result $\mathbf{p}\in$ RTRTLB. One must now determine whether $\mathbf{p}\in$ DISK 2 or not.

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• BINARY SEARCH FOR $\mathbf{p}$:

1.	$\mathbf{p}\in S$?	yes
2.	$\mathbf{p}\in L$?	no
3.	$\mathbf{p}\in RB$?	no
4.	$\mathbf{p}\in RTL$?	no
5.	$\mathbf{p}\in RTRB$?	no
6.	$\mathbf{p}\in RTRTL$?	yes
7.	$\mathbf{p}\in RTRTLB$?	yes

!

No. of comparisons ≡ $7 = 1 + \log_2 64$.

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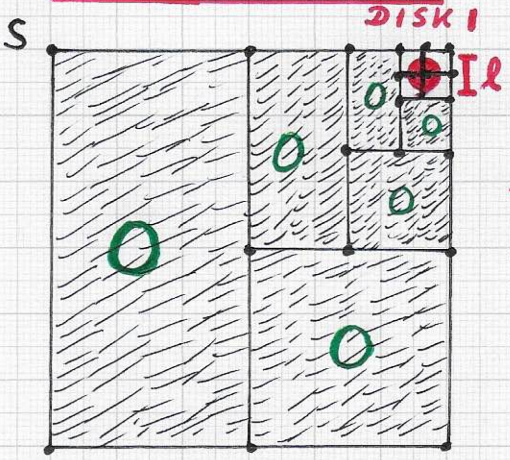
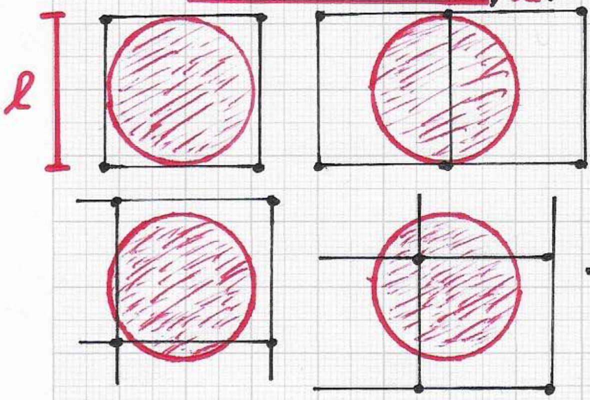
General spatial subdivision goal: Subdivide the domain space in such a way that class-0 cells are as large as possible and are "very close" to all cells containing the disks for classes 1, 2, 3, ...



Repeated binary subdivision of cells containing a disk, e.g., **DISK 1** in the shown example, should end when a cell containing a part of the disk has an edge with a length value smaller than the disk's diameter, l.



Possible terminal subdivision configurations for a disk.

• **NOTE.** By creating a binary tree structure for a binary spatial subdivision method for generating a finite, discrete representation of space, ONE CAN PERFORM A POINT SEARCH IN LOGARITHMIC TIME. **!**

Several important aspects of the tree construction and the tree's representation have not yet been discussed in detail:

→ Design of a space-/memory-efficient data structure to store the tree

→ Definition of a proper termination condition to be used for stopping spatial subdivision – considering issues like sizes of disks; overlapping disks; required and minimal value of ε (minimal cell edge length)

→ Complexity of an update scheme that one must use whenever class sample and disk data change.

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Stratovan

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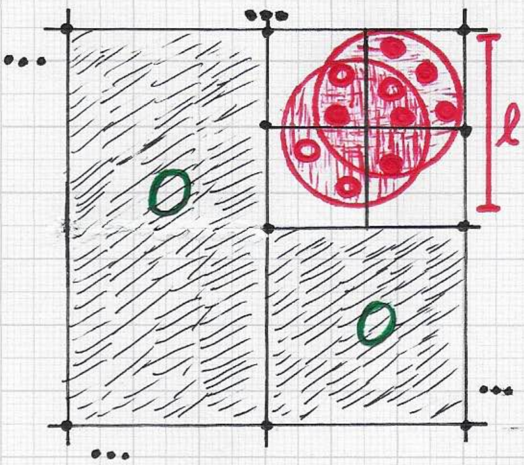
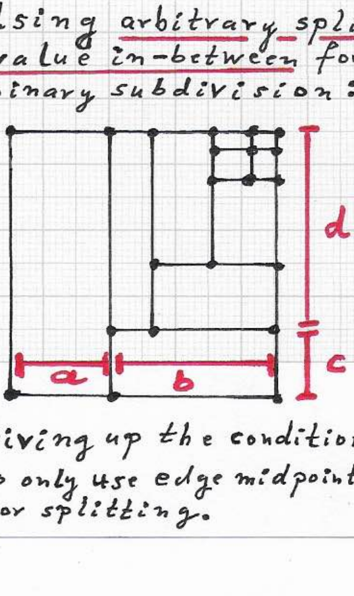
In the case of classes that are hard to classify, hard to separate, their "histogram point" disks can potentially intersect, overlap:



Scenario where two disks containing samples of type ● and type ⊙ intersect. A meaningful termination condition for stopping subdivision can be defined by the minimal edge length of the disks' bounding box, for example – l.

Using arbitrary split value in-between for binary subdivision:



Giving up the condition to only use edge midpoints for splitting.

The example shown in the left figure involves two classes with associated sample sets, "histogram point" sets, with bounding disks that intersect. Thus, the class separation / classification problem is hard – at least at this specific scale – and one must use a termination criterion for spatial cell subdivision that applies to unions of disks that intersect. (With increasing number of material classes included in the classification problem and with increasingly similar scale-specific coefficient histograms of certain materials, cases with intersecting disks will increase.)

• **NOTE.** The dimension of the (spatial) domain space we are concerned with is given by the number of histogram bins, e.g., 64. At the same time, the number of material classes is "small." An efficient, more general subdivision method and structure are needed.

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