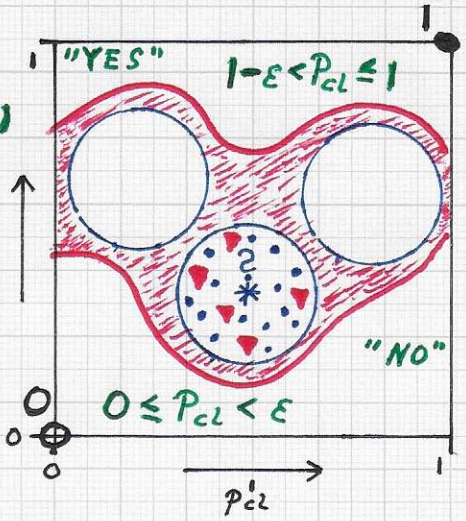
OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• Laplacian eigenfunctions and neural networks:...

Use of multiple local RBF-based models for the computation of a needed overall classification probability value:

$P_{cl} = P_{cl}(p^1_{cl}, p^2_{cl})$



The shaded region (red) in the domain is the most crucial region for carefully calculating a value of P_{cl}. In this shaded region, one would use a locally defined RBF best approximation. In this example, given the tuple (p^1_{cl}, p^2_{cl}) indicated as '?' and '*', one would rapidly construct the needed local RBF over the shown hyperball with '?' as its center. The quickly computed "high-quality estimate" for $P_{cl} = P_{cl}(p^1_{cl}, p^2_{cl})$ is ultimately used to potentially turn the real value of P_{cl} — $0 \leq P_{cl} \leq 1$ — into a binary "NO" or "YES" value for classification.

Once again, it is important to relate RBFs, e.g., Hardy's multiquadric or thin-plate splines, to the classification of a specific material/object extracted from a 3D, volumetric image and represented as a set-of-voxels segment. At this stage of image data processing and analysis, we assume that we have calculated and avaible C·H class- and scale-specific probability values p^{sc}_{cl}, $cl = 1 \ldots C$, $sc = 1 \ldots H$, and we "know" at this point that the segment to be classified has not (yet) been identified as a "non-threat" segment. Classes (cl) are indexed from 1 to C, and scales (sc) are indexed from 1 to H. The value of p^{sc}_{cl} is viewed/defined as the probability of the unclassified segment's belonging to class cl when only considering similarity at scale sc. Generally, a single p^{sc}_{cl}-value does not suffice for classification; multiple p^{sc}_{cl}-values define P_{cl}'s probability value.

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■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• **Laplacian eigenfunctions and neural networks:...**

Simplified view of the computation and meaning of "overall and final" probabilities $P_1, \ldots, P_C$:

• A material segment to be classified is called '*' and is identified by C·H individual class- and scale-specific probabilities p_{cl}^{sc}, i.e.,

$$* = (p_1^1, \ldots, p_1^H, \ldots, p_C^1, \ldots, p_C^H).$$

The p_{cl}^{sc} probabilities have real values in the interval $[0,1]$; p_{cl}^{sc} only considers similarity of * with class cl at scale sc and its value therefore represents a probability that merely uses the minimal amount of data for defining a class-match probability.

Each probability P_{cl} is based on a subset of all available p_{cl}^{sc}-probabilities. The value of P_{cl} is (generally) also a real value in the interval $[0,1]$. One must understand P_{cl} as a function of '*', i.e., as a multivariate function

$$P_{cl} = P_{cl}(*) = P_{cl}(p_1^1, \ldots, p_1^H, \ldots, p_C^1, \ldots p_C^H).$$

The final result is a vector-valued multivariate function:

$$(P_1, \ldots, P_C) = (P_1(*), \ldots, P_C(*)).$$

• Note. $\sum_{cl=1}^{C} P_{cl} \neq 1$.

Thus, based on "classification difficulty" and number of different materials to be recognized by the classification method, the probability function P_{cl}, $cl \in \{1, \ldots, C\}$, which "combines" certain individual p_{cl}^{sc}-values, should consider more or less of the known p_{cl}^{sc}-data – ideally an optimal number of p_{cl}^{sc}-data necessary to satisfy the classification performance requirements.

Maximally, P_{cl} would consider all available individual probabilities, i.e., $P_{cl} = P_{cl}(p_1^1, \ldots, p_1^H, \ldots, p_C^1, \ldots p_C^H)$.

• Note. The rationale and need for considering several individual probabilities as arguments for the "overall and final" probability function P_{cl} is this: With an increasing number of "scale fingerprints" of an increasing number of classes, the ability to distinguish between classes improves.

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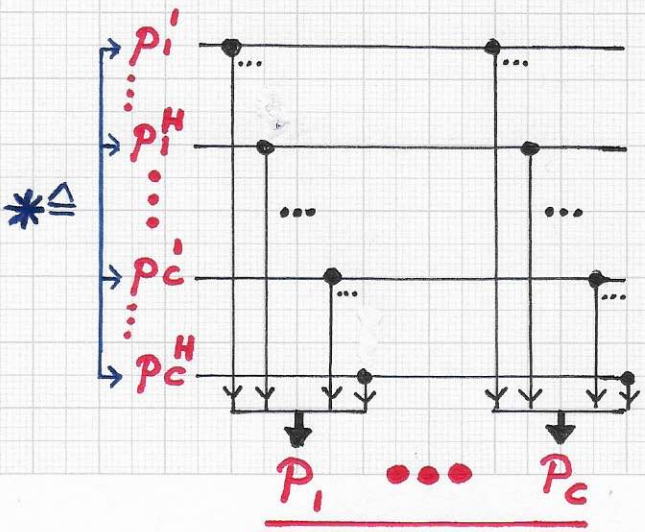
OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks: ...

The described computations ultimately produce a "vector of final real probabilities $P_1, \ldots, P_c$": $\mathbb{P} = (P_1, \ldots, P_c)$. Here, $0 \leq P_{cl} \leq 1$ and $\sum_{cl=1}^{c} P_{cl} \neq 1$. The value of P_{cl} is the probability of the unclassified material segment's belonging to class cl. Depending on the needs of a classification process, the real value of P_{cl} might have to be mapped to a binary value (0 or 1, FALSE or TRUE). The threshold value required for P_{cl} for "optimal classification decision-making" should be determined via a process that, for example, maximizes the number of true positives and true negatives and minimizes the number of false positives and false negatives.



The left figure provides an abstract, high-level view of the classification probability computations for a to-be-classified '*' represented by the tuple $(p_1^1, \ldots, p_1^H, \ldots, p_c^1, \ldots, p_c^H)$, where the p_{cl}^{sc} components are the individual match probabilities for a class cl when only considering scale-sc-based similarity. ...

■ OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• Laplacian eigenfunctions and neural networks:... At the bottom, the figure on the previous page shows the resulting REAL probabilities $P_1, \ldots, P_C$, being the match probabilities for class 1, ..., class C, respectively. If it is desired to perform binary classification for the unclassified '*', then one will have to (experimentally) determine C class-specific, optimized thresholds $t_1, \ldots, t_C$. These thresholds can therefore be used to compute the Boolean values of all "$P_{cl} > t_{cl}$". The result TRUE corresponds to "'*' identified as class-cl material"; FALSE corresponds to "'*' not identified as class-cl material". In principle, it is possible that '*' is generating TRUE for more than one class. For example, based on ALL data available for the "* material" it might indeed be impossible to associate '*' with at most one class. Whenever such conflicting "simultaneous classifications" arise — a material identified as a material simultaneously belonging to more than one class — a final decision must be made by an expert. The "discriminative feature" used by the expert should be incorporated into the classification system, if at all possible.

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■ OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• Laplacian eigenfunctions and neural networks:... The entire progression of computations and the intermediate results data, ultimately producing the probabilities P_{cl}, are involved and complex. This is a good place in the discussion to summarize the most important aspects of the materials classification process.

- A to-be-classified material is given as a set of 3D image voxels that define the material segment (possibly fragmented into several unconnected components).
- The "density function" — a piecewise constant function — is locally analyzed and represented via eigenfunction-based expansions at "H scales." The value of H is defined by the size (number of voxels) of a locally applied convolution mask. THE H SCALES TOGETHER CAPTURE THE ENTIRE INFORMATION THAT EXISTS (LOCALLY). SINCE THE EIGENFUNCTION BASIS IS ORTHOGONAL, THE SCALES ARE MUTUALLY INDEPENDENT; each scale captures unique information.
- The convolution mask is applied to each local voxel subset of a segment where the mask can be used for convolution. Each mask application generates H coefficients, for the H orthogonal multi-scale eigenfunctions. For each scale, all mask applications together define H scale-specific coefficient value histograms.

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