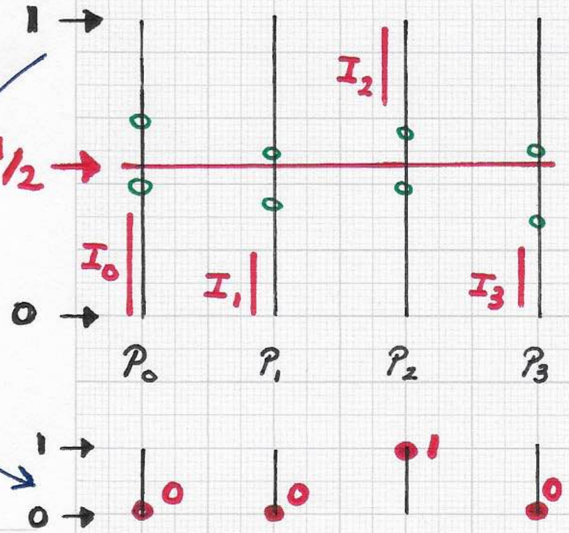
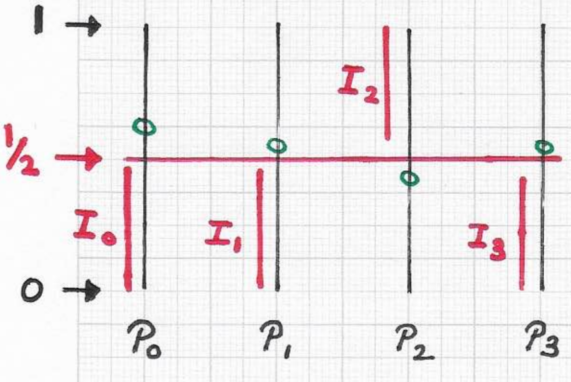
Stratovan

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:...



Mapping "confidence intervals" to binary values. Thus, whenever an unclassified material segment (of class 2) causes the system to generate P_{cl}-values that all lie inside the intervals I_{cl}, then the segment will be correctly classified as a class-2 material segment.



Increasing the confidence level from 90% to 99% yields longer intervals I_{cl} – but outlier data 'o' can remain.

The conditions **i)** and **ii)** on the previous page imply (for the specific example considered) that **I_2 is a sub-interval in $(\frac{1}{2}, 1]$ and I_{cl}, $cl \neq 2$, is a sub-interval in $[0, \frac{1}{2})$.** This implication is a consequence of the requirements **$MIN_2 > \frac{1}{2}$ and $MAX_{cl} < \frac{1}{2}$.** The left figure shows that — in this "ideal" setting — exactly one interval, I_2, is mapped to the binary value **1**, and all other intervals, I_{cl}, $cl \neq 2$, are mapped to the binary value **0**. The figure still shows "outlier data" 'o' on all P_{cl}-axes that, for practical purposes, require additional special case treatment. The second figure (left, bottom) sketches a hypothetical result for **I_{cl} "confidence intervals" after increasing the confidence level from 90% to 99%.** Generally, one must assume that outlier data 'o' can always exist.

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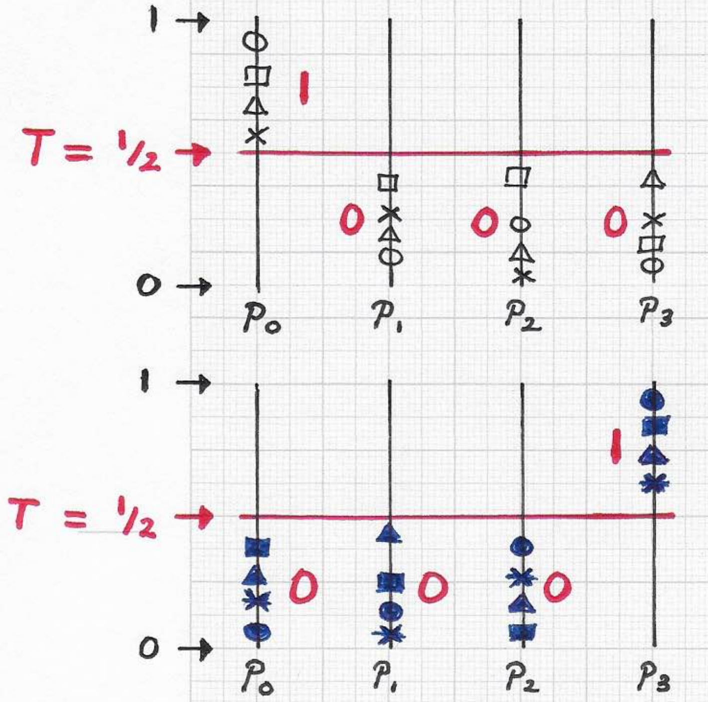

Exemplary P_{cl}-values generated by a "perfect" system. Top: P_{cl}-values for four class-0 materials; bottom: P_{cl}-values for four class-3 materials.

If simple rounding of P_{cl}-values were done, the top four P_{cl}-quadruples would map to (1,0,0,0) and the bottom four would map to (0,0,0,1).

Instead of rounding P_{cl}-values, i.e., using 1/2 as threshold, one could map a P_{cl}-value to a b_{cl}-value via

$$b_{cl} = \begin{cases} 1, & \text{if } P_{cl} > T \\ 0, & \text{otherwise} \end{cases}.$$

! **The value of T is an optimization parameter.**

A "perfect" classification system would always classify a class-cl material segment as a class-cl segment. The figure (left) illustrates the necessary behavior of such a "perfect" system for four classes (0, 1, 2, 3). Top: Using four class-0 segments (unclassified) as system input (○, □, △, ×), the P_0-values for all segments would be greater than 1/2; and the P_{cl}-values, $cl \neq 0$, for all segments would be smaller than 1/2. Bottom: Using four class-3 segments (unclassified) as input (●, ■, ▲, ✱), the P_3-values for all segments would be greater than 1/2; and the P_{cl}-values, $cl \neq 3$, for all segments would be smaller than 1/2. **If one used the threshold value 1/2 to map a P_{cl}-value to a binary value b_{cl}, i.e., "b_{cl} = (INT)(ROUND (P_{cl}))," the data shown in the figure would be mapped to the quadruples (1,0,0,0) and (0,0,0,1), respectively.** ...

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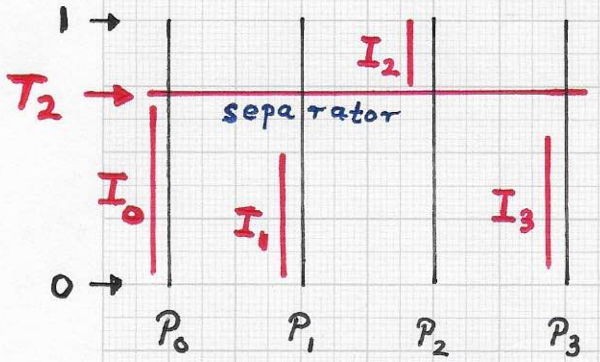
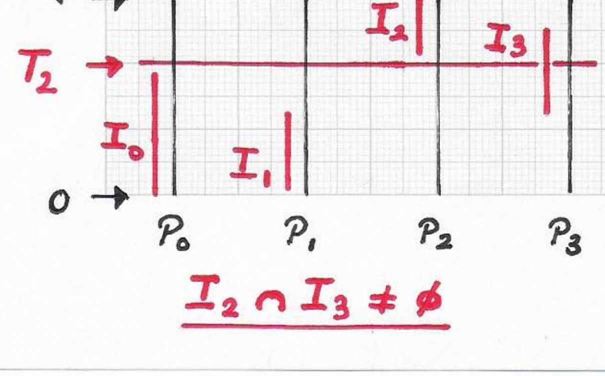


Scenario where 99% of the class-2 material segments produce a classification system response where

$$I_2 \cap I_{cl} = \emptyset,\ cl \neq 2.$$

In this situation, one can use one T_2-value as a separator that defines a "dividing line" between I_2 and all other I_{cl}-intervals, as shown in the figure.

Of course, depending on the "degree of separability" of the material classes, I_2 can potentially have a non-empty intersection with one or more of the other I_{cl}-intervals:



$$I_2 \cap I_3 \neq \emptyset$$

One can — and should — adopt a "highly refined" approach that takes into account known and/or observed classification system behavior as much as possible. The figure (left) should be understood as follows:

- When testing/observing the trained system concerning the classification of (unclassified) class-2 material segments, using a 99% "confidence interval" level, the sketched intervals I_{cl} result.
- The lower bound of I_2, I_2^{min}, is larger than all upper bounds I_{cl}^{max}, $cl \neq 2$, i.e., $I_2^{min} > I_{cl}^{max}$, $cl = 0 \ldots 3$, $cl \neq 2$.
- One can calculate a value of an adaptive threshold T_2 that satisfies the conditions
  - i) $T_2 < I_2^{min}$ and
  - ii) $T_2 > I_{cl}^{max}$, $cl = 0 \ldots 3$, $cl \neq 2$.
- "Statistically, the system response is correct 99% of the time when having class-2 input, using threshold T_2."

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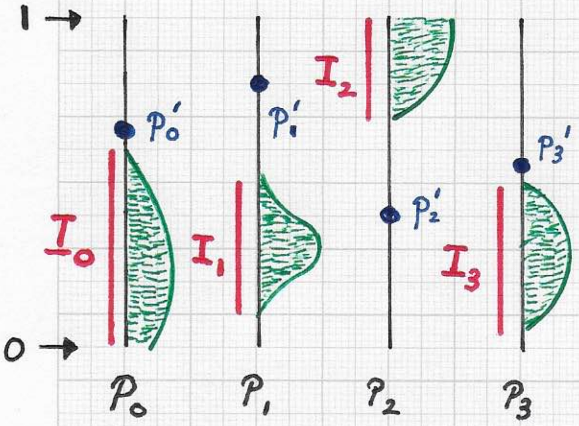

"Confidence intervals" I_{cl} and (green) sketch of the associated 99% P_{cl}-value distributions.

An unclassified material segment causes the system to generate the P_{cl}-value output shown as '•' data. The material tuple (p_0', p_1', p_2', p_3') can be compared with the signature of class 2.

In this case, all p_{cl}'-values lie outside the I_{cl}-"confidence intervals," i.e., $p_{cl}' \notin I_{cl}$, $cl = 0 \ldots 3$. Thus, one can assume with a high degree of certainty that the quadruple (p_0', p_1', p_2', p_3') does **NOT** represent a material of class 2.

While the distance of p_2' from the interval I_2 might be most indicative for concluding — in this case — that the unclassified material most likely does not belong to class 2, one should consider behavior on all P_{cl}-axes.

The left figure emphasizes the fact that a value distribution exists for each P_{cl}-axis, for each I_{cl}-interval — in this case based on the system response for 99% of only class-2 material segments (for which the system calculates these P_{cl}-values). **One can understand the data visualized in this figure — or the visualization itself — as the prototypical PROFILE, FINGERPRINT or SIGNATURE of class 2.** This signature can therefore be used to determine whether a given unclassified material segment with $\mathbb{P}$-tuple (p_0', p_1', p_2', p_3') is likely a class-2 segment or not. The class-2 "confidence intervals" I_{cl} have midpoints $\bar{x}_{cl}$. A simple first method for determining the distance between the $\mathbb{P}$-tuple and the class-2 signature might consider $\sum_{cl=0}^{3} |\bar{x}_{cl} - p_{cl}'|$ as a measure.

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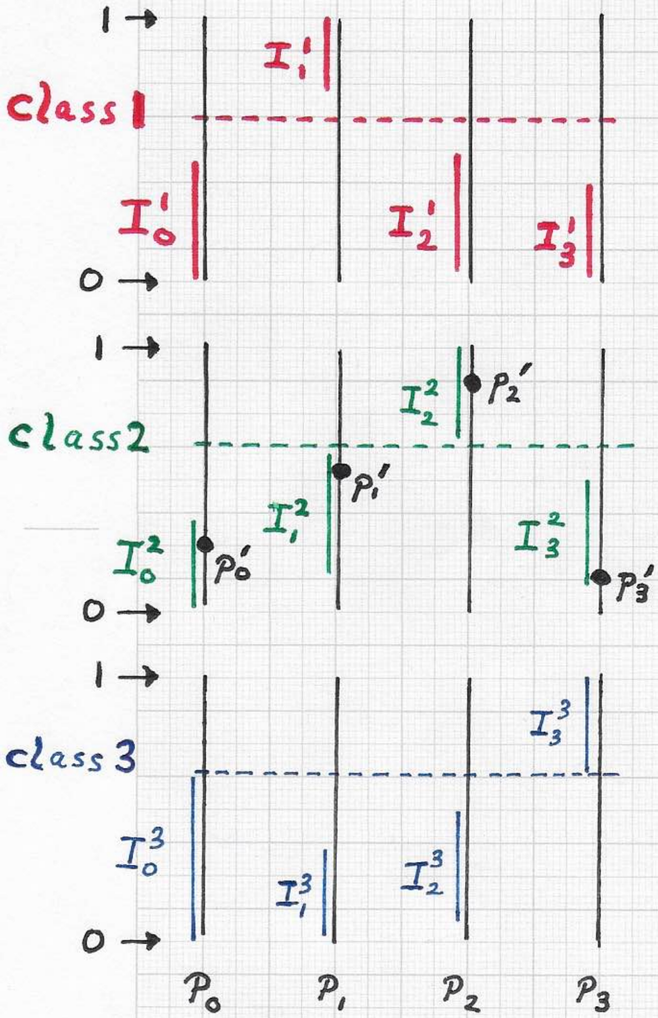

Response of trained classification system for a three-class scenario. In this ideal, synthetic case, the three class-indicative intervals I_1^1, I_2^2 and I_3^3 are perfectly separated from the other intervals for their respective classes.

Here, the tuple (p_0', p_1', p_2', p_3') is recognizable as a class-2 tuple, with high certainty.

The left figure illustrates a more general scenario. The scenario considers three material classes (classes 1, 2 and 3), with "class 0" consisting of all those materials that do not belong to classes 1, 2 or 3. We assume that the classification system (for these three/four classes) is fully trained and produces the "confidence intervals" I_{cl}^{CL} as shown in the figure – requiring, as before, that the intervals are based on "99% system behavior." When recording the system's response for a "large set" of exclusively class-1 material segments to be classified, the class-1 intervals I_{cl}^1, $cl = 0 \ldots 3$, are obtained. Using the same approach for class-2 and class-3 segments, one obtains class-2 intervals I_{cl}^2, $cl = 0 \ldots 3$, and class-3 intervals I_{cl}^3, $cl = 0 \ldots 3$.

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