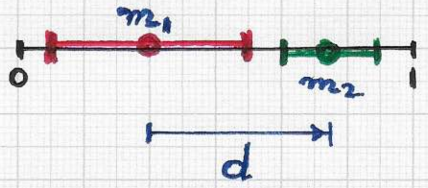
Bernd Hamann

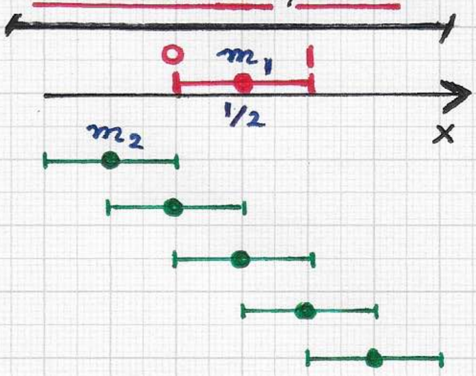
Stratovan

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

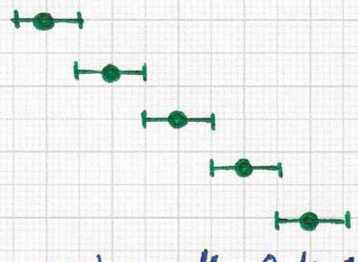
• Laplacian eigenfunctions and neural networks:...



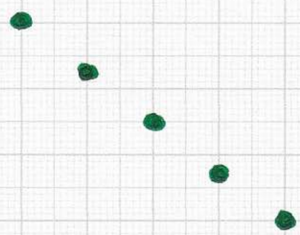
(Signed) distance d based on (oriented) difference of two interval midpoints.



m_2-values: $-1/2, 0, 1/2, 1, 3/2$



m_2-values: $-1/2, 0, 1/2, 1, 3/2$



m_2-values: $-1/2, 0, 1/2, 1, 3/2$

We have motivated the need for a better distance measure for two intervals on the previous page. We can employ the computationally simple and efficient measure explained via the top-left figure on the previous page; it merely calculates the (signed) distance of interval midpoints (multiplied by the factor 4). We can adapt this measure to our classification setting, see top-left figure. We define the distance d as $d = m_2 - m_1$. We consider

m_1	m_2	$m_2 - m_1$
$1/2$	$-1/2$	-1
$1/2$	0	$-1/2$
$1/2$	$1/2$	0
$1/2$	1	$1/2$
$1/2$	$3/2$	1

the behavior of this distance for a fixed m_1-value and a "moving" m_2-value, shown in the three interval sequences in the left figure. The m_2-values are the midpoints of intervals of lengths 1, $1/2$ and 0. The table (above) lists the distance values for the 5 m_2-midpoints.

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Stratovan

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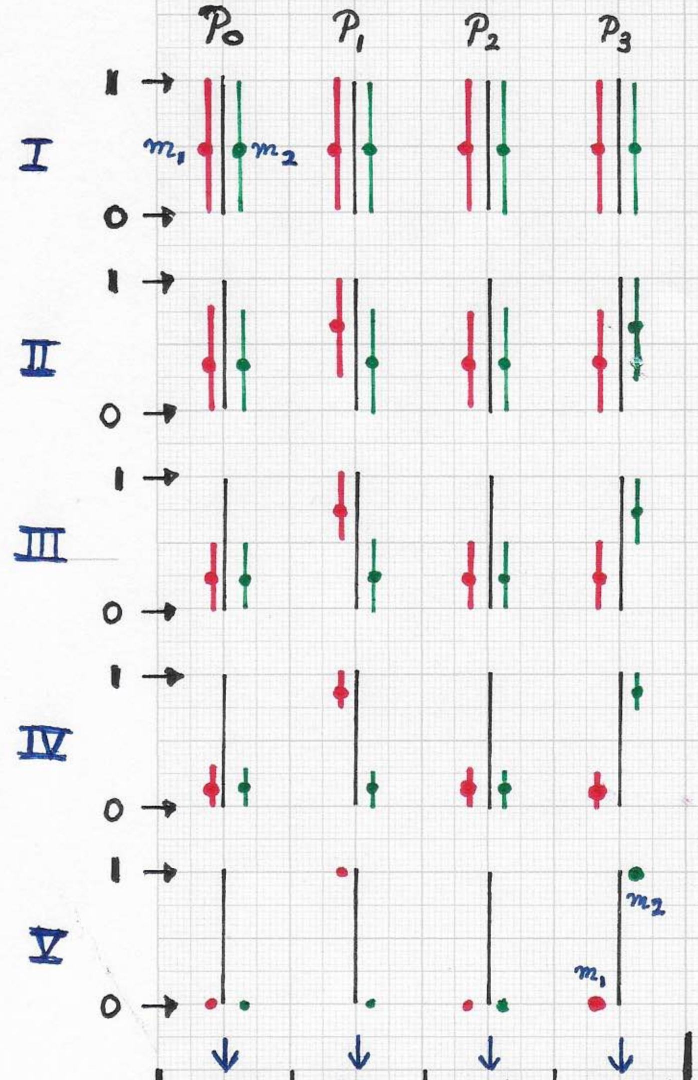

	$d(P_0)$	$d(P_1)$	$d(P_2)$	$d(P_3)$	$d_{1,3} = \Sigma\lvert d\rvert$
I	0	0	0	0	0
II	0	$-1/4$	0	$1/4$	$1/2$
III	0	$-1/2$	0	$1/2$	1
IV	0	$-3/4$	0	$3/4$	$3/2$
V	0	-1	0	1	2

Sequence of 5 signature pairs for classes 1 and 3 and distances.

We can now apply this interval distance measure to signature pairs of material classes. We use the signature pair for classes 1 and 3 as an example (for a 3-class classification problem). We consider the sequence of 5 signature pairs, with increasing discriminative behavior, shown in the figure (left). **Note: Case V shows the perfect class-1 and class-3 signatures where all intervals are effectively points, i.e., both signatures are binary signatures.**

The goal is to define and calculate appropriate overall distances for the signature pairs. The table (left) lists the distance values $d_{1,3}$ based on the formula

$$d_{cl1,cl2} = \sum_{cl=0}^{c} |d(P_{cl})| .$$

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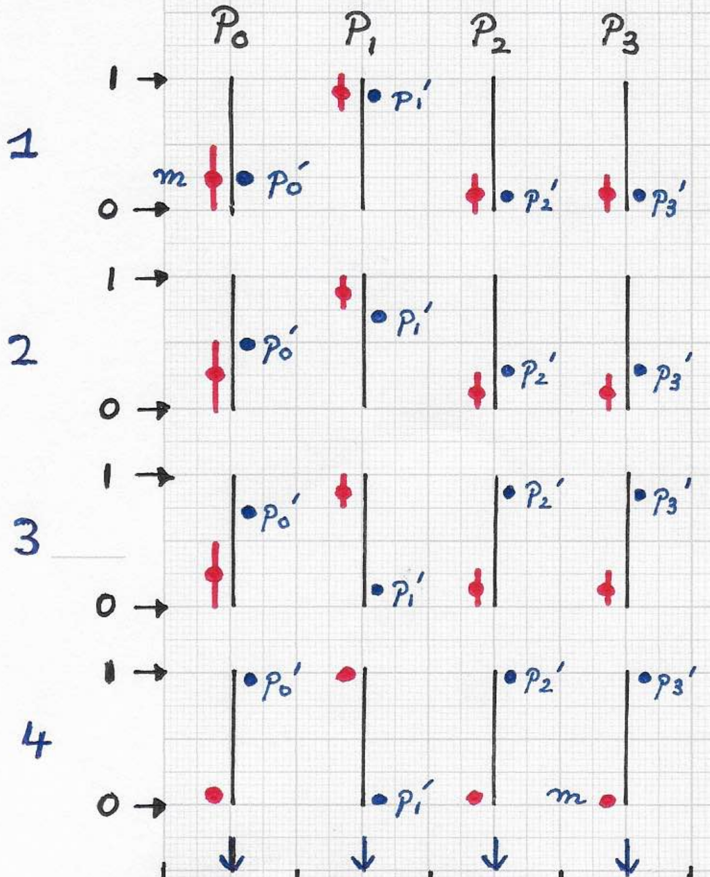

	$d(p_0')$	$d(p_1')$	$d(p_2')$	$d(p_3')$
1	0	0	0	0
2	1/4	−1/8	1/8	1/8
3	1/2	−3/4	3/4	3/4
4	1	−1	1	1

Top: The individual p_{cl}'-values of tuple (p_0', p_1', p_2', p_3') are shown next to the mid-points m of the signature intervals of material class 1. **Bottom:** Table listing the p_{cl}'-distance values using the distance measure for intervals.

• **Note.** "Perfect and ideal separation" in signature space is represented by case V (previous page). In this case, classes 1 and 3 are captured via their binary signatures – where the signatures differ maximally on the P_1- and P_3-axis: the specific interval distances on the P_1- and P_3-axis are −1 and +1, respectively (while interval distances are 0 on all other P_{cl}-axes). **Thus, the value 2 = $= \sum_{cl=0}^{3} |d(P_{cl})|$ is "ideally normalized" for this 3-class classification problem.**

The left figure and associated table show an example where the interval distance measure has been adopted to compute the dist(p_{cl}')-values of a tuple (p_0', p_1', p_2', p_3'). **By adding the $|d(p_{cl}')|$-values in each row of the table one obtains the D_{cl}-values 0 (1), 5/8 (2), 11/4 (3) and 4 (4).**

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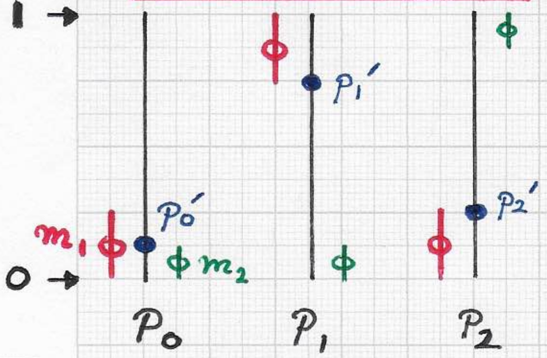
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks: ...

Simple example of a 2-class classification problem:

$(p_0', p_1', p_2') = (1/8, 3/4, 1/4)$



– The intervals on the P_{cl}-axes are indicated by the red (left) and green (right) line segments shown next to the three P_{cl}-axes. The interval midpoints are shown as circles. The values of the tuple (p_0', p_1', p_2') are shown as solid bullets on the central P_{cl}-axes.

– Signature distance:

$$d_{1,2} = |1/16 - 1/8| + |1/16 - 7/8| + |15/16 - 1/8| = 27/16$$

– Tuple distance:

$$D_1 = |1/8 - 1/8| + |3/4 - 7/8| + |1/4 - 1/8| = 1/4$$

$$D_2 = |1/8 - 1/16| + |3/4 - 1/16| + |1/4 - 15/16| = 23/16$$

⇒ Class-1 tuple

• **Note.** Case 4 (previous page) shows a "dual binary pair": The perfect binary signature for material class 1 is the tuple $(0,1,0,0)$. Thus, the dual (and unclassified) tuple $(1,0,1,1) = (p_0', p_1', p_2', p_3')$ has maximal distance. The D_{cl}-value 4, as calculated on the previous page, is a proper value, as it represents 4 times the maximal absolute distance per P_{cl}-axis, $cl = 0 \ldots 3$. Thus, we can consider the following definition as another option for D_{cl}:

$$D_{cl} = w_{cl} \cdot |dist(p_{cl}')| + \sum_{\substack{class=0 \\ class \neq cl}}^{class=c} |dist(p_{class}')| .$$

Here, w_{cl} represents a weight for class cl; the "dist" measure in this case uses distance values between a p_{cl}'-value and a signature interval.

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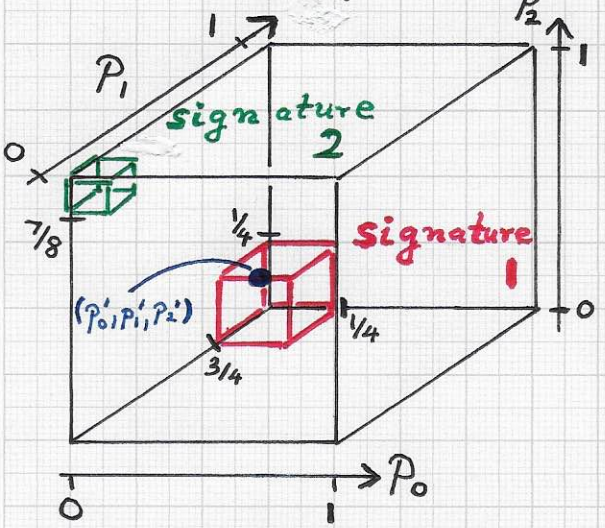
Geometrical interpretation of signatures 1 and 2 from figure on previous page:



The two cuboids represent the signatures when a Cartesian product of the intervals on the P_{cl}-axes is considered.

The signature-1 cuboid is in the corner (0, 1, 0), and signature-2 cuboid is in the corner (0, 0, 1). Generally, this "placement" of the signature cuboids makes possible a high-quality classification due to the clear separation of the cuboids in the embedding cube $[0,1]^{C+1}$, which is the cube $[0,1]^3$ in this example.

It is important to note that the class signatures fully consider all scales.

• The distance value $d_{1,2} = 27/16$ is close to 2 and thus indicates that the two signatures are "well separated" for these two material classes.

• The tuple distance $D_1 = 1/4$ is substantially smaller than $D_2 = 23/16$; further, 1/4 is a small value per se. Thus, one can classify the given tuple as a class-1 tuple.

The figure (left) provides us with a geometrical view of class signatures. When adopting a Cartesian product view of the intervals on the P_{cl}-axes of the class signatures, the resulting Cartesian products define sub-cuboids in the embedding cube $[0,1]^{C+1}$. Class 1 should define a sub-cuboid in the "corner" (0, 1, 0, ..., 0), ..., and Class C in the "corner" (0, 0, ..., 0, 1).

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