■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks: ...

The following table considers three classes, 0, 1 and 2, using two objects to be classified. The true classifications are T_0, T_1 and T_2, and the false classifications are F_0, F_1 and F_2.

truth		classification		type	number of T_0	T_1	T_2	F_0	F_1	F_2	ratio [%] of TRUE	FALSE	
0	0	0	0	T_0 T_0	2	0	0	0	0	0	100	0	C
0	0	0	1	T_0 F_1	1	0	0	0	1	0	50	50	
0	0	0	2	T_0 F_2	1	0	0	0	0	1	50	50	
0	0	1	0	F_1 T_0	1	0	0	0	1	0	50	50	
0	0	1	1	F_1 F_1	0	0	0	0	2	0	0	100	
0	0	1	2	F_1 F_2	0	0	0	0	1	1	0	100	
0	0	2	0	F_2 T_0	1	0	0	0	0	1	50	50	
0	0	2	1	F_2 F_1	0	0	0	0	1	1	0	100	
0	0	2	2	F_2 F_2	0	0	0	0	0	2	0	100	
0	1	0	0	T_0 F_0	1	0	0	1	0	0	50	50	
0	1	0	1	T_0 T_1	1	1	0	0	0	0	100	0	C
0	1	0	2	T_0 F_2	1	0	0	0	0	1	50	50	
0	1	1	0	F_1 F_0	0	0	0	1	1	0	0	100	
0	1	1	1	F_1 T_1	0	1	0	0	1	0	50	50	
0	1	1	2	F_1 F_2	0	0	0	0	1	1	0	100	
0	1	2	0	F_2 F_0	0	0	0	1	0	1	0	100	
0	1	2	1	F_2 T_1	0	1	0	0	0	1	50	50	
0	1	2	2	F_2 F_2	0	0	0	0	0	2	0	100	
2	2	0	0	F_0 F_0	0	0	0	2	0	0	0	100	
2	2	0	1	F_0 F_1	0	0	0	1	1	0	0	100	
2	2	0	2	F_0 T_2	0	0	1	1	0	0	50	50	
2	2	1	0	F_1 F_0	0	0	0	1	1	0	0	100	
2	2	1	1	F_1 F_1	0	0	0	0	2	0	0	100	
2	2	1	2	F_1 T_2	0	0	1	0	1	0	50	50	
2	2	2	0	T_2 F_0	0	0	1	1	0	0	50	50	
2	2	2	1	T_2 F_1	0	0	1	0	1	0	50	50	
2	2	2	2	T_2 T_2	0	0	2	0	0	0	100	0	C

The nine correct classifications ("C") represent 9 of 81 combinatorially possible results.

The third table we include on the next page considers two classes, 0 and 1, using three objects to be classified.

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... The provided three combinatorial tables simply serve as detailed examples to derive the general case.

truth	classification	type	number of T_0	T_1	F_0	F_1	ratio [%] of TRUE	FALSE	
0 0 0	0 0 0	T_0 T_0 T_0	3	0	0	0	100	0	C
0 0 0	0 0 1	T_0 T_0 F_1	2	0	0	1	67	33	
0 0 0	0 1 0	T_0 F_1 T_0	2	0	0	1	67	33	
0 0 0	0 1 1	T_0 F_1 F_1	1	0	0	2	33	67	
0 0 0	1 0 0	F_1 T_0 T_0	2	0	0	1	67	33	
0 0 0	1 0 1	F_1 T_0 F_1	1	0	0	2	33	67	
0 0 0	1 1 0	F_1 F_1 T_0	1	0	0	2	33	67	
0 0 0	1 1 1	F_1 F_1 F_1	0	0	0	3	0	100	
0 0 1	0 0 0	T_0 T_0 F_0	2	0	1	0	67	33	
0 0 1	0 0 1	T_0 T_0 T_1	2	1	0	0	100	0	C
0 0 1	0 1 0	T_0 F_1 F_0	1	0	1	1	33	67	
0 0 1	0 1 1	T_0 F_1 T_1	1	1	0	1	67	33	
0 0 1	1 0 0	F_1 T_0 F_0	1	0	1	1	33	67	
0 0 1	1 0 1	F_1 T_0 T_1	1	1	0	1	67	33	
0 0 1	1 1 0	F_1 F_1 F_0	0	0	1	2	0	100	
0 0 1	1 1 1	F_1 F_1 T_1	0	1	0	2	33	67	

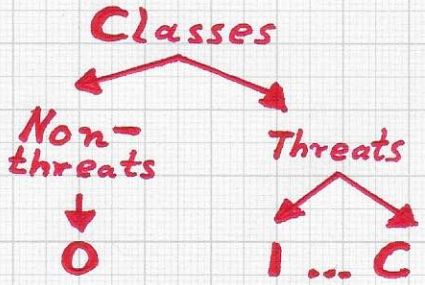
... ...

The eight correct classifications ("C") represent 8 of 64 combinatorially possible results.

vi) Second, one must establish a meaningful definition of TP, TN, FP and FN for the multi-class classification setting. On the high level, there exist two "categories" of material classes: a) threat materials, i.e., classes 1, ..., C, and b) non-threat materials, i.e., all materials not belonging to any of the classes 1,..., C, together constituting class 0. Thus, "category" a) splits into C classes, while "category" b) is not subdivided into classes. ...

OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• Laplacian eigenfunctions and neural networks:...



Non-threat materials collectively define class 0. Threat materials belong to classes 1, ..., C.

truth	classification	type
0	0	TN
	1	FP$_1$
	2	FP$_2$
	3	FP$_3$
1	0	FN
	1	TP$_1$
	2	FP$_2$
	3	FP$_3$
2	0	FN
	1	FP$_1$
	2	TP$_2$
	3	FP$_3$
3	0	FN
	1	FP$_1$
	2	FP$_2$
	3	TP$_3$

Combinatorially possible classification result types for classes 0, 1, 2, 3.

In other words, the non-threat materials define the classification result **NEGATIVE**, and the C threat materials define the classification result **POSITIVE**. Therefore, one can define the following classification result types:

- **TN**: material does not belong to any of the classes 1, ..., C and has been correctly classified as a non-threat material
- **FN**: material belongs to one of the threat classes 1, ..., C but has been incorrectly classified as a non-threat material
- **TP$_{cl}$**: material belongs to threat class cl and has been correctly classified as class-cl material
- **FP$_{cl}$**: material does not belong to threat class cl but has been incorrectly classified as a class-cl threat material

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• Laplacian eigenfunctions and neural networks:... Optimization of the values of TPs, FPs, TNs and FNs is the goal of optimizing classification performance. Thus, it is crucial to define a meaningful methodology to properly count the numbers of TPs, FPs, TNs and FNs in a multi-class classification setting. For example, one could count as follows:

- The numbers of TPs, FPs, TNs and FNs are called #TP, #FP, #TN and #FN, respectively.
- The "more specialized number of true positives of a class cl" is called $\#TP_{cl}$.
- Whenever the following classification result types are obtained, counters are incremented as follows:

type	counter changes
TN	#TN ++
FN	#FN ++
TP_{cl}	$\#TP_{cl}$++ , #TP++
FP_{cl}	$\#FP_{cl}$++ , #TP++

- A TP_{cl} result type increases the overall number #TP by one and also the number $\#TP_{cl}$ by one.
- AN FP_{cl} result type INCREASES the overall number #TP BY ONE and also the number $\#FP_{cl}$ BY ONE. (While result type FP_{cl} implies that a threat class has been detected, the threat class index has not been recognized correctly.)...

OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• Laplacian eigenfunctions and neural networks:...

type	counter changes
FP_{cl}	$\#FP_{cl}$++, #FP++

• AN FP_{cl} result type INCREASES the overall number #FP (when the material to be classified is NOT a threat-class material) and also the number $\#FP_{cl}$ by one. (This case is shown in the table above.)

truth	classification	counter changes
0	0	#TN++
	1	#FP++, $\#FP_1$++
	2	#FP++, $\#FP_2$++
1	0	#FN++
	1	#TP++, $\#TP_1$++
	2	#TP++, $\#FP_2$++ *
2	0	#FN++
	1	#TP++, $\#FP_1$++ *
	2	#TP++, $\#TP_2$++

The left table provides the possible counter increases that can combinatorially arise when materials belonging to non-threat class(es) 0 and threat classes 1, ..., C (=2) are classified. (*) In these cases, a threat-class material is recognized as a threat-class material — but the specific threat-class index is not correctly identified. We provide examples of this counting methodology in the following table:

truth / classifications	#TN	#FN	#TP	#FP	$\#TP_1$	$\#TP_2$	$\#FP_1$	$\#FP_2$	
012012012									
012012012	3	0	6	0	3	3	0	0	C
120120120	0	3	3	3	0	0	3	3	
201201201	0	3	3	3	0	0	3	3	
000000000	3	6	0	0	0	0	0	0	
111111111	0	0	6	3	3	0	6	0	
222222222	0	0	6	3	0	3	0	6	
100100100	0	6	0	3	0	0	3	0	

$\Sigma = 9$

The "C" row shows the values for the correct classification of all nine materials. • • •