■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... This view of pre-classification only considers a classification stream consisting of a single (integer) index per material segment. The fact that this class index is the result of a complex comparison and similarity-defining computation is "hidden" at this stage. The determination of a class index involves calculations with real numbers and thresholds that are also real numbers. Consequently, a class index – or possibly multiple class indices – could be communicated to and by the pre-classification module as an index – or indices – with its associated, underlying **CONFIDENCE or CERTAINTY** level. For example, the concept of a decider function $\mathcal{F}$ (p. 15, 4/12/2022) generates real numbers to define similarities between an unclassified and classified class samples, using multiple scales. Ultimately, a threshold must be defined and used to determine an unclassified material segment's class membership(s). The decider function $\mathcal{F}$ was originally defined as a real function depending on H scale-specific similarities / match probabilities $p_1, \ldots, p_H$ – and $\mathcal{F}$, by considering a subset of the p_{scale} or all p_{scale} values, computes a match probability for an unclassified material segment.

...

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks: … This match probability can be defined and used as a match indicator between the unclassified datum and just one classified material sample or all classified material samples belonging to a certain class. Further, as mentioned before, a decider function F might consider different and/or a different number of the p_{scale}-values available to serve as arguments for F. Therefore, one can understand the concept of multi-scale decider function classification as follows – on a high level: Decider functions F_1, $F_2, \ldots, F_C$ are used to calculate match probabilities between an unclassified datum and C material classes, where a class-specific decider function F_{cl} uses as its arguments a class-specific subset of the available H similarity / match probability values $p_1, p_2, \ldots, p_H$. Thus, to indicate a "detection alarm" one or multiple of the used decider functions F_{cl} must have a value larger than an application- and goal-specific – and data-dependent – threshold value(s). In the case of two materal classes to be detected, for example, one would use two decider functions F_1 and F_2 with associated detection thresholds t_1 and t_2. Consequently, the material classification procedure can lead to four possible outcomes.

…

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

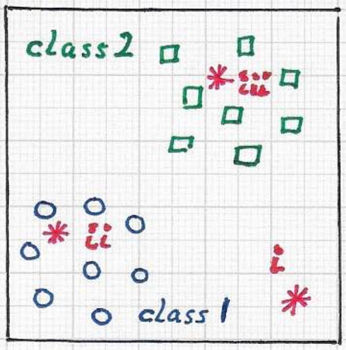
• Laplacian eigenfunctions and neural networks: … The four possible classification outcomes in the two-material case are the four possible relational cases:

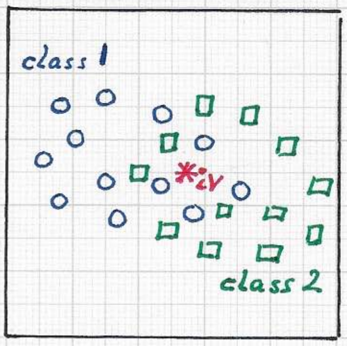
i) $F_1 < t_1 \wedge F_2 < t_2 \Rightarrow$ class index set: $\emptyset$

ii) $F_1 \geq t_1 \wedge F_2 < t_2 \Rightarrow$ " " " $\{1\}$

iii) $F_1 < t_1 \wedge F_2 \geq t_2 \Rightarrow$ " " " $\{2\}$

iv) $F_1 \geq t_1 \wedge F_2 \geq t_2 \Rightarrow$ " " " $\{1,2\}$





Abstract visualization of cases that can arise in the 2-material classification scenario. Class samples are shown as o (class 1) or □ (class 2). In some underlying feature space, the unclassified datum, *, can lead to cases i, ii, iii or iv.

The two figures (left) provide abstract illustrations of these four combinatorially possible cases. In a high-dimensional feature space, classified class-1 and class-2 samples define clusters that are "far away from each other and do not overlap" (top figure); or class-1 and class-2 samples define clusters that "overlap" (bottom figure). The top figure shows a datum that does not belong to class 1 and not to class 2 (i); a datum that belongs to class 1 (ii); and a datum that belongs to class 2 (iii). The bottom figure represents case (iv): The class-1 and class-2 clusters are "very close/overlap," and the unclassified datum is "inside the overlap region of the two clusters."

…

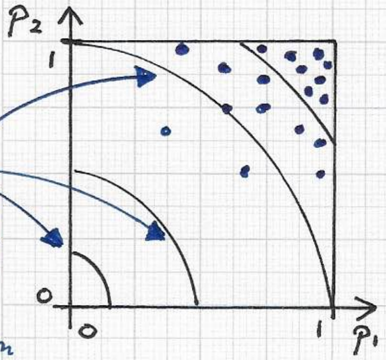
OBJECT AND MATERIAL EIGENFUNCTIONS — Cont'd.

- Laplacian eigenfunctions and neural networks:... ONCE AGAIN, ONE MUST USE AN APPRORIATE OPTIMIZATION STRATEGY, METHOD AND PRACTICAL ALGORITHM TO ESTABLISH THE VALUES OF THE THRESHOLDS $t_1, \ldots, t_C$. To summarize our setting and goals: We must detect C material threat classes. A sample database contains material samples of all classes, and each sample is represented (via eigenfunction analysis) at H scales. To determine whether a new, unclassified material can be viewed as a member of one (or multiple) of the C classes, class-specific DECIDER FUNCTIONS are defined and used. The decider functions are $F_1, \ldots, F_C$. The unclassified material is also represented via a multi-scale signature based on H scales. Nevertheless, when calculating similarity(ies) between the unclassified material and one of the sample classes (via comparison with one, several or all members of a threat class), ONE SHOULD ASSUME THAT A DECIDER FUNCTION F_{cl} OF A SPECIFIC CLASS cl CAN MAKE ITS DECISION BY CONSIDERING A SUBSET OF THE H AVAILABLE SCALE-SPECIFIC SIMILARITIES/MATCH PROBABILITIES. (The underlying hypothesis is the assumption that material classes can in many cases be characterized by certain scales.)

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... For example, if one had three scales – scale 1 (low-frequency), scale 2 (medium-frequency), scale 3 (high-frequency) – one could consider between one and three scale characterizations for material comparisons. A bit-string can be associated with a decider function to indicate the scales used, e.g., ${}^{1,1,0}F_6$ would be the decider function for class 6, and only scales 1 and 2 would be used for detecting class-6 materials. More specifically, ${}^{1,1,0}F_6 = {}^{1,1,0}F_6(p_1, p_2)$, i.e., the two arguments are p_1 and p_2, the similarities/match probabilities considering low- and medium-frequency statistical behavior.



• Example: contours/isolines of function $F = (p_1^2 + p_2^2)^{1/2}$

For a more detailed understanding, we consider the example sketch in the left figure: We are given an unclassified material segment and characterize it at scales 1, 2 and 3. We must determine whether it is potentially a class-6 material. The decider function for class 6 is ${}^{1,1,0}F_6$ and considers only scales 1 and 2. First, we compute the sample-specific similarity/match probabilities p_1 and p_2, comparing every class-6 database sample with the unclassified sample. The resulting (p_1, p_2) tuples are the points shown in the figure, clustered close to the (1,1) corner.

...