Stratovan

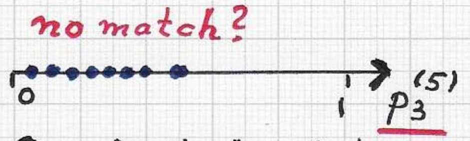
OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

- Laplacian eigenfunctions and neural networks:... The fact that (p_1, p_2) tuples are close to the corner tuple $(1,1)$ indicates a relatively high degree of similarity between the unclassified material and the stored class-6 samples — at least as far as scales 1 and 2 are concerned. The decider function ${}^{1,1,0}F_6 = {}^{1,1,0}F_6(p_1, p_2)$ is a bivariate function that, given the two scale similarities p_1 and p_2, determines an overall, "final" probability value defining the likelihood that the unclassified material segment is a class-6 segment. It is not obvious or simple to define the decider function ${}^{1,1,0}F_6$. Possibly it suffices to require that one of the values (of p_1 and p_2) is "large" to trigger a class-6 detection; possibly both values must be "large" to trigger a class-6 detection; possibly a completely different condition must be satisfied to trigger a class-6 detection. The figure on the previous page includes contours/isolines of ${}^{1,1,0}F_6$, where these contours are simple circular arcs, defined by ${}^{1,1,0}F_6(p_1, p_2) = (p_1^2 + p_2^2)^{1/2} = \text{constant}$. Using a decider function as simple as this one, it is still challenging to establish a proper threshold value for ${}^{1,1,0}F_6$, to trigger a class-6 detection, when ${}^{1,1,0}F_6 \geq t_6$. ONE MUST USE "TRAINING AND LEARNING" TO DEFINE ${}^{1,1,0}F_6$ AND t_6.

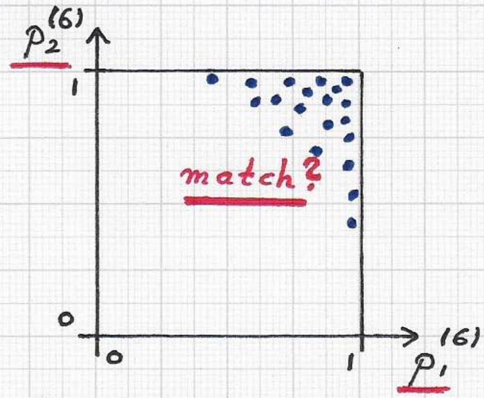
...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

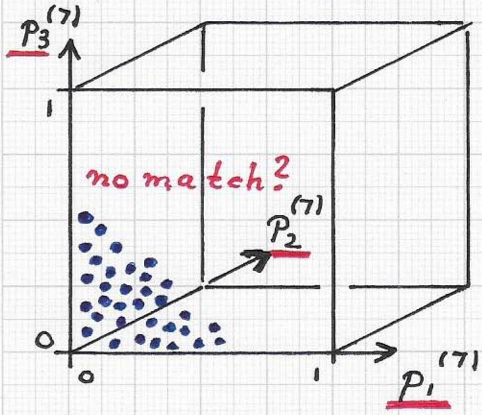
• Laplacian eigenfunctions and neural networks: ...



Similarity/match probability values of $p_3^{(5)}$ – comparing the unclassified material with samples of class **5** at scale **3**.



Similarity/match probability values (tuples) of $p_1^{(6)}$ and $p_2^{(6)}$ – comparing the unclassified material with samples of class **6** at scales **1** and **2**.



Values (triples) resulting from comparison with class **7** at scales **1**, **2** and **3**.

An even more detailed exemplary classification scenario is visualized by the figures shown on this page: The goal is to determine whether the unclassified material is potentially a class-**5** or a class-**6** or a class-**7** material. As far as "necessary" similarity comparisons are concerned, only scale-**3** behavior is considered for class-**5** comparisons (top figure); scales **1** and **2** are used for class-**6** comparisons (middle figure); and scales **1**, **2** and **3** are considered for class-**7** comparisons (bottom figure). The unclassified material segment is characterized at all scales, scales **1**, **2** and **3**. Three decider functions must be defined for this classification task: ${}^{0,0,1}F^{(5)}(p_3^{(5)})$, ${}^{1,1,0}F^{(6)}(p_1^{(6)}, p_2^{(6)})$ and ${}^{1,1,1}F^{(7)}(p_1^{(7)}, p_2^{(7)}, p_3^{(7)})$. An appropriate representation of the decider functions is stored. ...

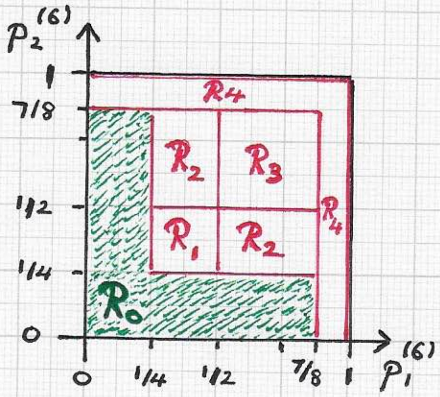
OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... Therefore, the values that must efficiently be computed as part of the "real-time classification" are the values of $p_3^{(5)}$, $p_1^{(6)}$, $p_2^{(6)}$, $p_1^{(7)}$, $p_2^{(7)}$ and $p_3^{(7)}$, and ${}^{0,0,1}F^{(5)}$, ${}^{1,1,0}F^{(6)}$ and ${}^{1,1,1}F^{(7)}$. In addition to "pre-computing" the three needed decider functions and storing their representation, the three associated threshold values for t_5, t_6 and t_7 must also be known and stored. For example, very simple analytical — but practically unrealistic — decider functions are ${}^{0,0,1}F^{(5)} = \left((p_3^{(5)})^2 \right)^{1/2}$, ${}^{1,1,0}F^{(6)} = \left((p_1^{(6)})^2 + (p_2^{(6)})^2 \right)^{1/2}$ and ${}^{1,1,1}F^{(7)} = \left((p_1^{(7)})^2 + (p_2^{(7)})^2 + (p_3^{(7)})^2 \right)^{1/2}$. Associated threshold values could be $t_5 = 1/2$, $t_6 = 1$ and $t_7 = 1$. All sample tuples $(p_1^{(6)}, p_2^{(6)})$ shown in the middle figure from the previous page satisfy ${}^{1,1,0}F^{(6)} \geq 1$; thus, it is highly likely that the unclassified material segment belongs to class 6. In this case, the $(p_1^{(6)}, p_2^{(6)})$ tuples are clustered closely to the $(1,1)$ corner, also indicating that the material is most likely a class-6 material.

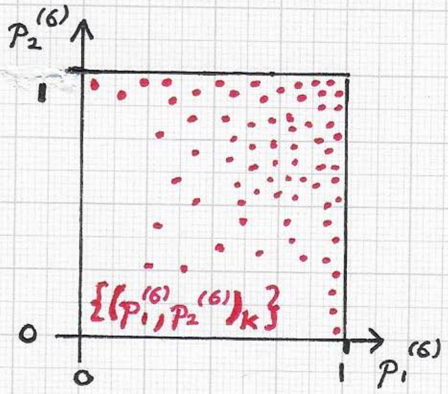
! IT SHOULD BE POINTED OUT AGAIN THAT THE MAPPING OF A TUPLE $(p_1^{(6)}, p_2^{(6)})$ TO AN OVERALL DECIDER FUNCTION VALUE ${}^{1,1,0}F^{(6)}$ CAN BE "ARBITRARILY COMPLEX: ONE MUST "TRAIN" THE CLASSIFICATION SYSTEM, TEACHING IT HOW TO ASSOCIATE, CALCULATE DECIDER FUNCTION VALUES, GIVEN $(p_1^{(6)}, p_2^{(6)})$ TUPLES. ...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:...



A priori decomposition of $(p_1^{(6)}, p_2^{(6)})$ domain square into regions with explicitly defined decider function values.



Discrete specification of finite number of decider function values at finite number of domain locations $(p_1^{(6)}, p_2^{(6)})$.

⇒ When comparing the unclassified material segment at these two scales WITH ONE class-6 sample from the sample database, ONE similarity tuple $(p_1^{(6)}, p_2^{(6)})$ results; for this tuple we must compute the decider function value, using one of these decider function definitions.

The figures (left) illustrate two approaches for potentially defining the specific decider function ${}^{1,1,0}F^{(6)}$. The first figure shows that the $(p_1^{(6)}, p_2^{(6)})$ domain, $[0,1] \times [0,1]$, is subdivided into sub-regions $R_0, \ldots, R_4$. In certain very rare cases, it might be possible to define such a domain subdivision — with associated region-specific decider function values — theoretically and analytically. For example, a very simple function is ${}^{1,1,0}F^{(6)}(p_1^{(6)}, p_2^{(6)}) = {}^{4}/_{r}$, where $(p_1^{(6)}, p_2^{(6)}) \in R_r$, $r = 0, \ldots, 4$. The second figure shows a tuple set $\{(p_1^{(6)}, p_2^{(6)})_k\}$ with a defined value set $\{({}^{1,1,0}F^{(6)})_k\}$. Thus, this method of establishing the definition of the decider function requires the system to provide a means to estimate/approximate a decider function value for an arbitrary location in the domain, using the provided discrete data.

• • •

Stratovan

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

- Laplacian eigenfunctions and neural networks:... Next, one must devise a plausible approach for comparing the unclassified material segment at scales 1 and 2 with the finite, discrete sample set of class-6 representatives in the sample database. Each comparison with a class-6 sample produces a similarity/match probability tuple $(p_1^{(6)}, p_2^{(6)})$. Further, this tuple lies somewhere in the $[0,1] \times [0,1]$ domain of the decider function; thus, we compute the value of ${}^{1,1,0}F^{(6)}$ for this tuple $(p_1^{(6)}, p_2^{(6)})$. This process is iteratively applied to all class-6 database samples. Considering the notation and indexing used earlier (see p. 12, 4/8/2022), there are k_6 material segments representing class 6 in the database. Consequently, one obtains k_6 values of ${}^{1,1,0}F^{(6)}$. The FINAL class-6 detection/classification step must therefore determine a specific value of these k_6 values — or an average of these k_6 values — to yield the FINAL value for the unclassified material segment; this FINAL value is subsequently subjected to a threshold procedure:

  Is the FINAL (real) value $\geq t_6$ (real threshold)?

  → YES : The unclassified material is classified as a class-6 material.

  → NO : The unclassified material is not a class-6 material.

...