Stratovan

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... THE ULTIMATE AND OVERALL GOAL IS OPTIMIZATION OF CLASSIFICATION SYSTEM PERFORMANCE. There must be a well-defined cost function and a continually ongoing process/optimization that, at all times, monitors the evolving values of this cost function over time and regularly adjusts (hyper-) parameter values; the objective is a nearly instantaneous reaction of the system when certain material characteristics of threat materials change and/or the detection of specific threat materials becomes more or less important. For example, one could design a system that incorporates a PERMANENTLY EXECUTING SIMULATED ANNEALING MODULE that always attempts to adjust the most important parameter values in the spirit of the objective of system optimization. One can think of or define a system cost function as a weighted combination of the respective guiding performance metrics. As the system must operate in a "real-time environment," the system optimization process must also be viewed as "optimization of system performance subject to algorithmic complexity and computational time constraints. Most importantly, one must focus on the applicable metrics involving true/false positive/negative behavior. ...

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:… We assume that (C+1) classes must be considered: Class 0 is the "collective class" that is used as class of all NON-threat materials. Classes 1, …, C are C different threat-material classes. Regarding these C threat materials, it is expected that the classification system can detect the specific class a threat material belongs to. Thus, at the "lowest level of system performance," we can — during system training, testing and validation — define and determine the values of the following multi-class classification system parameters:

- TN — no. of TRUE NEGATIVE detections
- TP_{cl} — no. of TRUE POSITIVE detections of class-cl material
- FP_{cl} — no. of FALSE POSITIVE detections of non-threat materials classified as cl
- FN_{cl} — no. of FALSE NEGATIVE detections of a class-cl threat material classified as a class-0 material
- $MC_{cl1,cl2}$ — no. of MIS-CLASSIFICATIONS of a class-cl1 threat material classified as a class-cl2 threat material

Correct system results are TN and TP. The worst result is FN. An MC result detects at least the threat – but not the correct material. "Some" FP results can be tolerated. …

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... The table/matrix included here (left) allows one to precisely record the statistical performance of a classification system. The matrix diagonal ($i=j$, $i=0...C$) records the numbers of all correct classifications, where the ground truth (GT) class of a material is correctly identified by the classification system (RES).

GT \ RES	0	1	...	j	...	C-1	C
0	TN	FP_1	...	FP_j	...	FP_{C-1}	FP_C
1	FN_1	TP_1	...	$MC_{1,j}$	...	$MC_{1,C-1}$	$MC_{1,C}$
...	...	...	...	...	...	...	...
i	FN_i	$MC_{i,1}$	...	$MC_{i,j}$	...	$MC_{i,C-1}$	$MC_{i,C}$
...	...	...	...	...	...	...	...
C-1	FN_{C-1}	$MC_{C-1,1}$	...	$MC_{C-1,j}$	...	TP_{C-1}	$MC_{C-1,C}$
C	FN_C	$MC_{C,1}$	...	$MC_{C,j}$	...	$MC_{C,C-1}$	TP_C

GT = ground truth; RES = classification result.

Row 0 of the matrix records (for $j=1...C$) the numbers of mis-classifications of a class-0 material being classified as a material belonging to one of the threat classes (1...C). Column 0 records (for $i=1...C$) the numbers of mis-classifications of threat materials (classes 1...C) as class-0 materials. The remaining matrix entries ($i,j=1...C$, $i\neq j$) represent the numbers of mis-classifications of a class-i threat as a class-j threat. **The perfect, error-free classification system can have non-zero values only in the matrix diagonal ($i=j$, $i=0...C$) and thus defines the upper bound for classification performance.**

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... • **Note**. From a theoretical perspective – and practically relevant – a given unclassified material segment could be classified as a segment that i) does not belong to any of the classes (0...C) being considered; ii) does belong to one of the classes (0...C) being considered; or iii) does belong to more than one of the classes (0...C) being considered. This fact, of course, also impacts the values recorded in the GT-RES matrix (previous page) characterizing system performance.

On page 4 (11/17/2022), a "simpler version" of the GT-RES matrix is provided to summarize statistical system performance for a three-class classification example, using classes **0**, **1** and **2**. The GT-RES matrix for that example is

GT \ RES	0	1	2
0	TN	FP_1	FP_2
1	FN_1	TP_1	$MC_{1,2}$
2	FN_2	$MC_{2,1}$	TP_2

=

GT \ RES	0	1	2
0	15	3	2
1	1	3	0
2	4	1	3

.

In this example, the number of to-be-classified material segments is **32**. The diagonal values of the error-free GT-RES matrix are $TN = 20$, $TP_1 = 4$ and $TP_2 = 8$. The GT-RES matrix "completely" characterizes the statistical performance of the classification system. Should it be necessary to compute a "single-number performance metric," one must devise a method to combine the GT-RES matrix values in a meaningful, application-specific weighted fashion.

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OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... An extremely simple and straightforward approach to defining an overall summary metric, i.e., a single number, could assign values to the individual classification outcomes (TN, TP_{cl}, FN_{cl}, FP_{cl}, $MC_{cl1,cl2}$). For example, the table (left) assigns values between −1 and 1 to the five possible outcomes.

outcome	value
TN	1
TP_{cl}	1
$MC_{cl1,cl2}$	−1/4
FP_{cl}	−1/2
FN_{cl}	−1

The value 1 is assigned to the two optimal outcomes (TN, TP_{cl}), and the value −1 is assigned to the worst outcome (FN_{cl}). In the case of S material segments to be classified, the overall metric of an error-free classification system is characterized by the equation $TN + TP_1 + \ldots + TP_C = S$. A system with worst possible performance satisfies the equation $FN_1 + FN_2 + \ldots + FN_C = -S$. (The classes are $0, \ldots, C$.) The general formula for this simple scoring scheme is:

$$P = \left(TN + \sum_{cl=1}^{C} TP_{cl} \right) - \left(\sum_{cl=1}^{C} FN_{cl} + \frac{1}{2} \sum_{cl=1}^{C} FP_{cl} + \frac{1}{4} \underbrace{\sum_{cl1=1}^{C} \sum_{cl2=1}^{C}}_{cl1 \neq cl2} MC_{cl1,cl2} \right).$$

The value of this performance metric P varies between $-S$ and S. Should a normalization of this value be desirable, one can use the mapping

$$P_{norm} = (P + S)/2S. \qquad (P_{norm} \in [0,1].)$$

...