OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:… Considering our 32-material example once again (see table, left), the resulting performance values for P and P_{norm} are

GT \ RES	0	1	2
0	15	3	2
1	1	3	0
2	4	1	3

$$P = (15+6) - (5 + \tfrac{1}{2}\cdot 5 + \tfrac{1}{4}\cdot 1)$$
$$= 21 - 7.75 = 13.25 \quad \text{(range: } -32 \ldots 32\text{)}$$
$$P_{norm} = (13.75 + 32)/64 = 181/256 = .71$$

• **Note.** Regarding the recorded classification system failures, one can also compute normalized total FN rate, normalized total FP rate and normalized total MC rate — normalizing with respect to all segments, S:

$$P^{FN}_{norm} = 1/S \; \Sigma^{c}_{cl=1} \; FN_{cl} = 5/32 = .16$$
$$P^{FP}_{norm} = 1/S \; \Sigma^{c}_{cl=1} \; FP_{cl} = 5/32 = .16$$
$$P^{MC}_{norm} = 1/S \; \Sigma^{c}_{\substack{cl1=1 \\ cl1 \neq cl2}} \; \Sigma^{c}_{cl2=1} \; MC_{cl1,\,cl2} = 1/32 = .03$$

In our driving application — detection and classification of threat materials — one must or should assume that different materials have different degrees of importance or weights associated with them. The table (left) shows the general case, where weights w are defined for every possible (GT, RES) index tuple. One must define the specific weight values and their use in the P-computation. …

…	0	1	2
0	W^{TN}	W^{FP}_1	W^{FP}_2
1	W^{FN}_1	W^{TP}_1	$W^{MC}_{1,2}$
2	W^{FN}_2	$W^{MC}_{2,1}$	W^{TP}_2

Weight table.

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... The use of such weights – W^{TN}, W^{TP}_{cl}, W^{FP}_{cl}, W^{FN}_{cl} and $W^{MC}_{cl1,cl2}$ – will become important when defining a classification performance metric that considers the fact that certain threat materials are extremely "dangerous" and other threat materials are much less "dangerous." The weight values must reflect the implied importance of the respective threat classes for classification. Of course, only an expert is able to define the values of the weights, based on historical experience and understanding of the different effects of different threat materials. Another relevant aspect to be taken into account is the overall RELATIVE OCCURRENCE OF MATERIAL CLASSES. For example, this information might be known from large historical records about material class occurrences. A ground truth fractional occurrence definition of (C+1) classes could be established as follows:

- The historical record informs us that the fractional, relative occurrences of classes $0,\dots,C$ are $u_0,\dots,u_C$, where $u_{cl} \geq 0$ and $\sum_{cl=0}^{C} u_{cl} = 1$.
- Thus, a representative finite-sample data set having S samples/segments will include s_0 class-0, ..., and s_C class-C samples such that $s_{cl}/S \approx u_{cl}$.

...

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... Therefore, one should generally combine classification outcome numbers (for TN, TP, FP, MC, FN cases) with the (a priori known or postulated) relative occurrences **u** of the considered (C+1) classes and the specified weights **w** of all possible classification outcomes for each class. On a high level of description, one can articulate specific goals when incorporating class weights / classification outcome weights and relative material class occurrences into the definition of a more general classification system performance metric. Possible meaningful goals include:

1) The correct or incorrect classification of any material class should have an "associated importance" that is inversely proportional to the relative occurrence value of **u** of a class. In other words, the classification outcome for a material class that is of relatively higher (lower) occurrence should carry a relatively lower (higher) level of importance. The rationale for this goal is the following thought: The vast majority of materials is defined by harmless objects; the processing of these objects is not of (relatively) high importance. Certain material classes arise rarely; their processing is of high importance. ...

OBJECT AND MATERIAL EIGENFUNCTIONS — Cont'd.

• Laplacian eigenfunctions and neural networks:... More generally, one could argue that the processing and classification of a material class (harmless or dangerous) that occurs very often can be viewed less important than processing/classifying a rarely observed material class. "Good values" of proportionality factors must be determined experimentally. (One should multiply u_{cl} by a factor.)

2) The assignment of meaningful weight values for all $(C+1)^2$ classification outcomes (of types TN, TP, FP, FN and MC) is rather challenging, subjective and experience-based. The following principles are examples:

- One should fix the value of the weight w^{TN}. All other weight values can thus be defined relative to w^{TN}. **Sign: $w^{TN} > 0$.**
- Values chosen for w_{cl}^{FN} weights should reflect the known or suspected degree of danger caused by a threat material of class **cl**. Assuming that threat classes are sorted by degree of danger — with class 1 being the least dangerous and class **C** being the most dangerous class — weight values should monotonically increase from w_1^{FN} to w_C^{FN}.

Sign: $w_{cl}^{FN} < 0$.

...

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

- Laplacian eigenfunctions and neural networks:…
  - Values chosen for w_{cl}^{FP} weights should reflect the known or suspected degree of work and impact caused by detecting a threat material of class **cl**. Again, weight values should monotonically increase from w_1^{FP} to w_C^{FP}. Since an FP classification does **NOT** imply the real danger of an actual threat event – unlike an FN classification – the values of w_{cl}^{FP} weights can/should be (substantially) lower than the values of w_{cl}^{FN} weights.
  Sign: $w_{cl}^{FP} < 0$.
  - One could argue that the values for w_{cl}^{TP} weights should be defined as follows: $w_{cl}^{TP} := w^{TN}$, $cl = 1 \ldots C$. This definition would be justified by the fact that all TP and the TN classifications represent perfect system behavior and should therefore imply that $w_1^{TP} = w_2^{TP} = \ldots = w_C^{TP} = w^{TN}$; and the actual value should be the maximally allowable positive weight value. Considering the fact that threat class indices reflect the degree of danger, one could also define larger weights to more dangerous classes, i.e., $w^{TN} \leq w_1^{TP} \leq \ldots \leq w_C^{TP}$.
  Sign: $w_{cl}^{TP} > 0$.

…