■ OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

- Laplacian eigenfunctions and neural networks:... ● It is not obvious how one should assign proper values to the $w^{MC}_{cl1,cl2}$ weights, associated with mis-classifications that correctly determine that a material is a threat material but incorrectly identify a material class. Nevertheless, since increasing threat class indices imply increasingly dangerous threat materials, it seems reasonable to apply the following rationale:
  - → Weights $w^{MC}_{cl1,cl2}$ with $cl2 < cl1$ are associated with mis-classifications where the system-generated material class (cl2) is a class that is less dangerous than the true class of the material (cl1). One can view these mis-classifications as cases of "under-estimation of danger." The value of $cl1-cl2$ indicates the degree of under-estimation; the larger this difference value is, the larger is the danger caused by the under-estimation. Weight values should consider the value of $cl1-cl2$.
  - → Weights $w^{MC}_{cl1,cl2}$ with $cl2 > cl1$ are associated with mis-classifications where the system-generated material class (cl2) is a class that is more dangerous than the true class of the material (cl1).

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■ OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

- Laplacian eigenfunctions and neural networks:... One can view these mis-classifications as cases of "over-estimation of danger." The value of cl2-cl1 Indicates the degree of over-estimation; the larger this difference value is, the larger is the perceived, incorrect high level of danger implied by the over-estimation. Weight values should consider the value of cl2-cl1.

In summary, it is plausible to assign values to $w^{MC}_{cl1,cl2}$ weights that are proportional to the absolute index tuple difference, i.e., $|cl2-cl1|$.

Sign : $w^{MC}_{cl1,cl2} < 0$.

Based on these guiding principles for the assignment of weight values for classification outcomes, we can derive and discuss examples. First, we consider the case of three-class classification, see the table (left). Here, the weight values are:

$w^{TN} = 1$, $w^{TP}_1 = w^{TP}_2 = 1$;

$w^{FN}_1 = -2$, $w^{FN}_2 = -4$, $w^{FP}_1 = -1$, $w^{FP}_2 = -2$;

$w^{MC}_{1,2} = -1$, $w^{MC}_{2,1} = -2$.

This set of values satisfies all principles discussed for the various classification result outcomes.

GT \ RES	0	1	2
0	1	-1	-2
1	-2	1	-1
2	-4	-2	1

↓ cl1

→ cl2

GT is the ground truth class, cl1, and **RES** is the generated result/outcome class, cl2. The shown weight values are viable value options.

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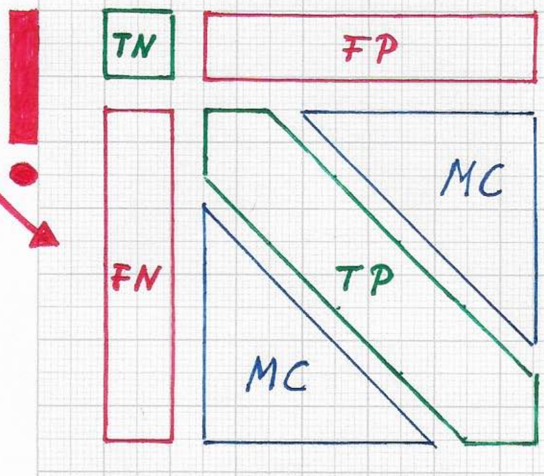
OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

- Laplacian eigenfunctions and neural networks: ...

GT \ RES	0	1	2	3	4
0	1	-1	-3	-5	-7
1	-2	2	-1	-3	-5
2	-5	-3	3	-1	-3
3	-8	-6	-4	4	-1
4	-11	-9	-7	-5	5

Five-class weight value assignment possibility

- Abstract visualization of qualitative weight variation in GT-RES space:



The six distinct regions in GT-RES space where region-specific weight value variations must be defined.

Value variation can be adhoc, linear or non-linear in the regions.

Second, we provide an example of weight value choices for a five-class classification scenario (left table). Here, one can describe the weights algebraically as follows:

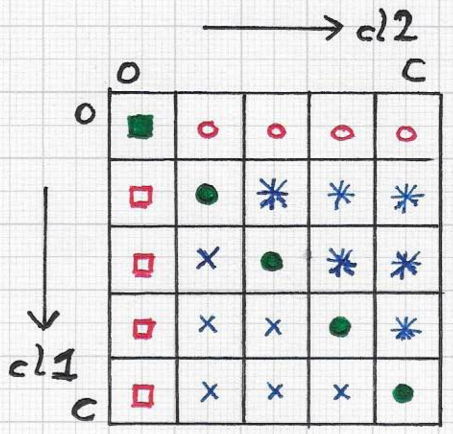
$$w^{TN} = 1 , \quad w^{TP}_{cl} = 1 + 1\cdot cl \;;$$
$$w^{FN}_{cl} = -2 - 3\cdot(cl-1) , \quad cl>0;$$
$$w^{FP}_{cl} = -1 - 2\cdot(cl-1) , \quad cl>0;$$
$$w^{MC}_{cl1,cl2} = -1 - 2\cdot(cl2-cl1-1), \quad cl1>0, cl2>cl1$$
$$w^{MC}_{cl1,cl2} = -3\cdot(cl1-1) + 2\cdot(cl2-1), \quad cl1>0, 0<cl2<cl1$$

These chosen definitions of the different result/outcome types could be used to calculate weight values for any number of material classes, $cl = 0 \ldots C$. The abstract illustration (left) represents the fact that one must establish weight values for six qualitatively distinct regions. The TN and TP regions must have positive values; the FN, FP and MC regions must have negative values.

...

OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

- Laplacian eigenfunctions and neural networks:... For example, one can define linear functions - i.e., linearly varying discrete weight values - with the six distinct result/outcome regions. The illustration (left) shows the six weight groups for which one must define linear functions:



■	○	○	○	○
□	●	✳	✳	✳
□	×	●	✳	✳
□	×	×	●	✳
□	×	×	×	●

Weight groups

{■} $w^{TN} = c^{TN}$, $c^{TN} > 0$

{●} $w^{TP}_{cl} = c^{TP} + \alpha^{TP} \cdot cl > 0$,
$\alpha^{TP} > 0$, $cl = 1 \ldots C$

{□} $w^{FN}_{cl} = c^{FN} + \alpha^{FN} \cdot cl < 0$,
$\alpha^{FN} < 0$, $cl = 1 \ldots C$

{○} $w^{FP}_{cl} = c^{FP} + \alpha^{FP} \cdot cl < 0$, $\alpha^{FP} < 0$, $cl = 1 \ldots C$

{×} $w^{MC}_{cl1,cl2} = c^{MC} + \alpha^{MC} \cdot cl1 + \beta^{MC} \cdot cl2 < 0$,
$cl1 = 2 \ldots C$, $cl2 = 1 \ldots (cl1-1)$

{✳} $\overline{w}^{MC}_{cl1,cl2} = \overline{c}^{MC} + \overline{\alpha}^{MC} \cdot cl1 + \overline{\beta}^{MC} \cdot cl2 < 0$,
$cl1 = 1 \ldots (C-1)$, $cl2 = (cl1+1) \ldots C$.

Again, these weight values/weight value functions must be defined by an expert who knows how the overall classification system performance should be impacted (via these weights) - to indicate properly when the performance **P** must reflect extremely good and extremely poor material class and threat recognition.

•••

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

- Laplacian eigenfunctions and neural networks:... We consider the three-class classification scenario to demonstrate the application of the principles discussed for a more complex characterization of system performance.

→ cl2

cl1 ↓

	0	1	2
0	20	●	●
1	●	4	●
2	●	●	8

GT

/20

/4

/8

15	3	2
1	3	0
4	1	3

RESULT

/32

$3/4$	$3/20$	$1/10$
$1/4$	$3/4$	0
$1/2$	$1/8$	$3/8$

SCALED RESULT

$15/32$	$3/32$	$1/16$
$1/32$	$3/32$	0
$1/8$	$1/32$	$3/32$

SCALED RESULT (alternative)

1	-1	-2
-2	1	-1
-4	-2	1

WEIGHTS

(for the various result outcomes)

The proper generalization of the formula for performance $\mathcal{P}$ from page 5 (12/7/2022) is the following formula $(C=2)$:

$$\begin{aligned} \underline{\underline{\mathcal{P}}} = {} & W^{TN}/s_0 \cdot TN + \sum_{cl=1}^{2} W^{TP}_{cl}/s_{cl} \cdot TP_{cl} \\ & + \sum_{cl=1}^{2} W^{FN}_{cl}/s_{cl} \cdot FN_{cl} + \sum_{cl=1}^{2} W^{FP}_{cl}/s_0 \cdot FP_{cl} \\ & + \sum_{cl1=1}^{2} \sum_{\substack{cl2=1 \\ cl1 \neq cl2}}^{2} W^{MC}_{cl1,cl2}/s_{cl1} \, MC_{cl1,cl2} \, . \end{aligned}$$

!

Here, s_{cl} denotes the number of class-cl material segments present in the ground truth, GT (20, 4 and 8). The RESULT table shows how the system has classified cl1 as cl2 data. The (Left) SCALED RESULT table is obtained by dividing the RESULT data in a specific row by the row-specific number of actual materials of that cl1 class (20, 4 and 8). The WEIGHTS table summarizes a particular weight choice.

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