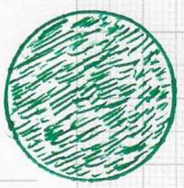
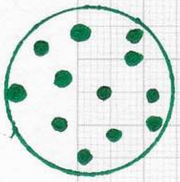
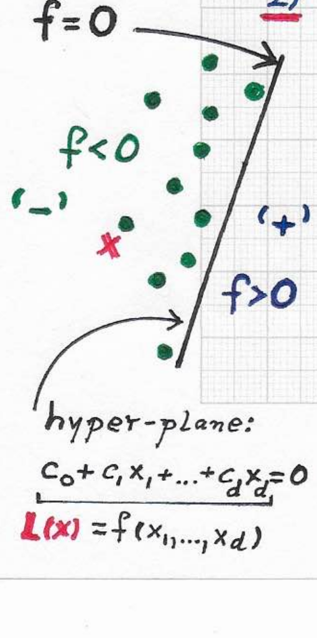
OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... It is important to review and summarize some of the major goals and issues that motivate our keen interest in approximating regions in a high-dimensional space via half-space representations. We provide an abstract, high-level partial list at this point:





1) In our application — material classification — a class cannot be defined a prior via some kind of analytical function representing the precise region in a high-dimensional "feature domain space." Instead, we are only provided with a discrete, finite sampling of this region-defining function. In other words, we do not **know** the exact definition of a continuum region that precisely establishes the complete and infinite set of all "feature points" that represent instances of material samples belonging to a specific class.

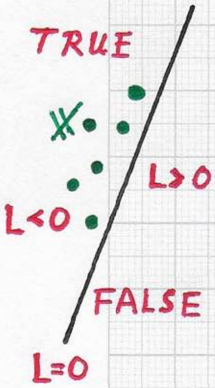


2) A line, plane or hyper-plane (in the general case) make it possible to partition space into two half-spaces, e.g., the negative (–) and positive (+) half-spaces as defined by a linear polynomial's sign when evaluating the polynomial in its negative-values and positive-values domain regions, respectively. This linear polynomial has the value zero on the (decision) boundary, i.e., a hyper-plane that separates the – and + half-spaces.

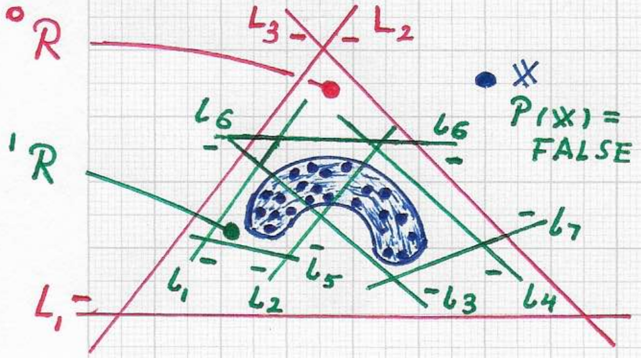
...

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... 3) The fact that a linear hyper-plane definition using the polynomial $L(x) = c_0 + c_1x_1 + \ldots + c_dx_d$ makes possible the binary splitting of the d-dimensional domain space into the region where $L(x) \leq 0$ and $L(x) > 0$ allows us to base binary Boolean decision-making (TRUE, FALSE) on the efficient evaluation of the linear polynomial $L(x)$. For example, if one based the predicate of a point x merely on one line / hyper-plane, one would define $P(x) := \text{TRUE}$, if $L(x) \leq 0$, and $P(x) := \text{FALSE}$, otherwise.



4) It is and can be assumed that the vast majority of points x are outside the region that defines all valid points representing a material class. Thus, in the spirit of efficient classification decision-making, one wants to determine that $P(x) = \text{FALSE}$ as quickly as possible, see the example shown in the figure. Here, $P(x) := L_1 L_2 L_3 = \text{FALSE}$. **NOTE: A BOOLEAN AND EXPRESSION $\bigwedge_{i=1}^{N} expr_i$ IS FALSE WHEN ONE OR MORE $expr_i$ VALUES ARE FALSE.** This fact is very important when designs "most efficent methods for Boolean expression evaluation for optimal network/gate architecture."



OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

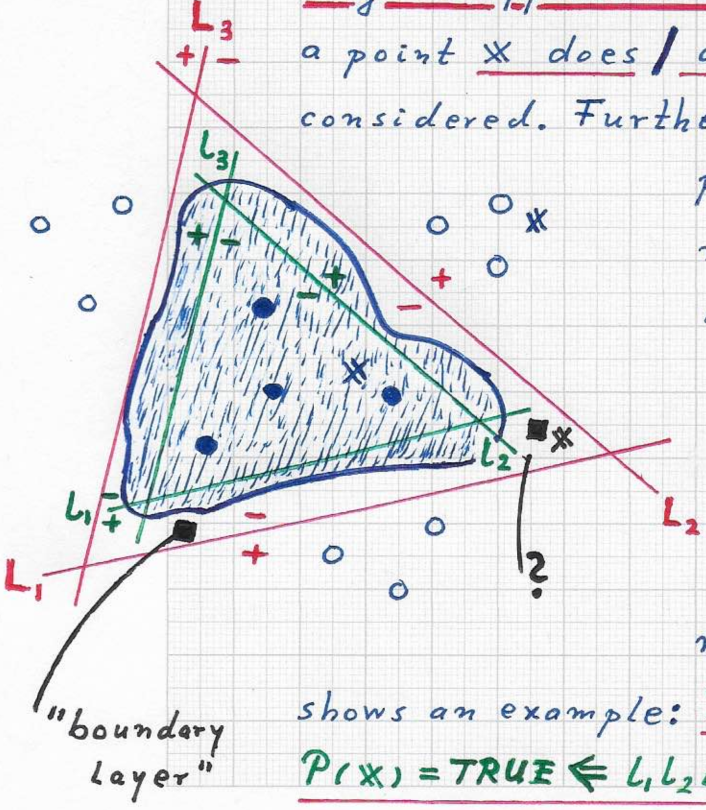
• Laplacian eigenfunctions and neural networks:...

5) The example sketched in the bottom figure on the previous page emphasizes the concept of a hierarchical, refinement-based multi-resolution approach that uses increasingly complex line/hyper-plane configurations — that make it possible to approximate the (unknown, shaded) region containing all TRUE sample points * in its interior, with a steadily decreasing approximation error for the region. In the shown example, the first, initial predicate of a point is given by the Boolean expression ${}^0P(*) := L_1 L_2 L_3$. The second, next-level predicate of * is given as ${}^1P(*) := l_1 l_2 l_5 l_6 \vee l_3 l_4 l_6 l_7 l_1$. Here, ${}^0P$ is TRUE for all points * inside region ${}^0R$ (bounded by the large outer triangle), and ${}^1P$ is TRUE for all points * inside region ${}^1R$ (bounded by the quadrilateral defined by l_1, l_2, l_5, l_6 $\cup$ the pentagon defined by l_3, l_4, l_6, l_7, l_1). For a variety of computational aspects, it is therefore highly desirable that the sequence of regions resulting from such a refinement approach represent a nested sequence, i.e., ${}^0R \supset {}^1R \supset {}^2R \supset \ldots$ One must keep in mind the significant computational complexity for high-dimensional domain spaces (*) — and optimize regions, Boolean expressions, gates. ...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

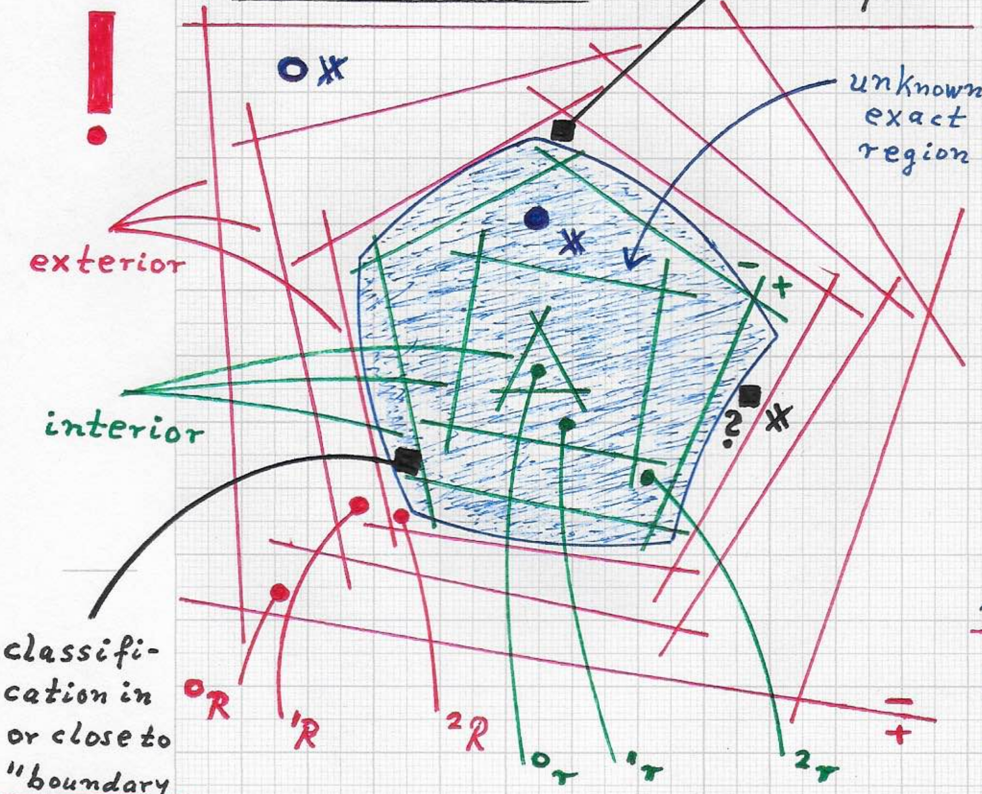
• Laplacian eigenfunctions and neural networks:...

6) The design of highly efficient – ideally optimal-efficient – algorithms and data structures for performing classification in a high-dimensional space (✱) is supremely important. A region in ✱-space that defines a point's membership in a specific class should be represented via a data structure that supports a hierarchical, multi-resolution region-approximation at different levels of precision; such a representation should be designed in a way such that a low-resolution, rather imprecise region-approximation suffices to conclude that a point ✱ does / does not belong to the class being considered. Further, the numbers of lines / hyper-planes defining such imprecise region-approximations should be **LOW**, **SMALL** numbers – thereby leading to simple Boolean expressions, permitting a "rapid rejection/acceptance of a point ✱" concerning class membership. The left figure shows an example: $P(✱) = FALSE \Leftarrow L_1 L_2 L_3 = FALSE$ (○). $P(✱) = TRUE \Leftarrow l_1 l_2 l_3 = TRUE$ (●). Boundary layer case: $L_1 L_2 L_3 = TRUE \wedge l_1 l_2 l_3 = FALSE \Rightarrow P(✱ = ■) = ?$ Refinement necessary! ...



■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:...



classification in or close to "boundary layer": deterioration of performance!

7) The left figure provides a sketch of a concept that "combines" the objectives to calculate and represent a multi-resolution structure for an increasingly precise approximation of an unknown exact region R associated with a specific class, and to make possible rapid decision-making for the rejection or acceptance (○ or ●) of a point ✳ as a point not belonging or belonging to the class. The sketch essentually shows how one can integrate the ideas of refinement and approximation to generate "two sequences of increasingly better region R approximations": An exterior sequence ^0R, ^1R, ^2R, ... and an interior sequence ^0r, ^1r, ^2r, ... are constructed. Ultimately, these sequences would "converge to the unknown exact region R." To enable a quick, efficiently computed decision of a point's class membership, one would use the index (0,1,2,...) for the order of half-space tests....