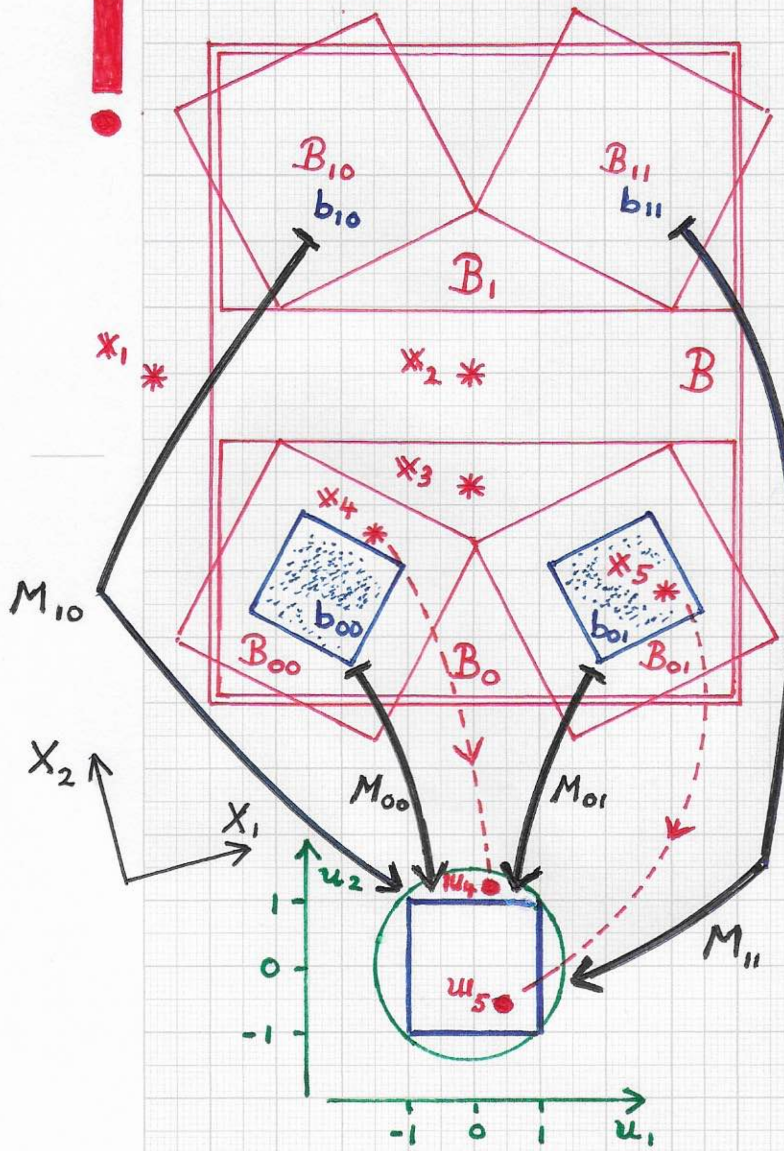
Bernd Hamann
Stratovan

7/9/2023

OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

- Laplacian eigenfunctions and neural networks: ...

Bounding box hierarchy, $D = 2$



$u_4 \Rightarrow$ probability of $*_4 > 0$
$u_5 \Rightarrow$ " of $*_5 > 0$

Decision-making process for classification of points $*_1, \ldots *_5$

In the following, we describe the use of a constructed bounding hyper-box hierarchy for decision-making and implied probability value calculations for given points to be classified. The left figure is abstract and rather complex. We discuss the sketched example in detail. Five points must be classified: $*_1, \ldots, *_5$. A hierarchy of bounding boxes for the classified sample points $\{\bullet\}$ belonging to the class being considered has been established – based on T-value ("tightness"-value) conditions: B; B_0, B_1; $B_{00}, B_{01}, B_{10}, B_{11}$. At the most detailed level of the hierarchy, **minimal** point (sub) set bounding boxes are known as well: $b_{00}, b_{01}, b_{10}, b_{11}$.

...

▪ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks:... The oriented minimal point (sub)set bounding boxes b_{00}, b_{01}, b_{10} and b_{11} can be mapped to the normalized u-space hyper-cube $[-1,1]^D$ via box-specific linear transformations; for the example shown in the figure, these transformations applied to b_{00}, b_{01}, b_{10} and b_{11} are called M_{00}, M_{01}, M_{10} and M_{11}, respectively, in the figure. We can now describe the classification decision-making process for the points to be classified:

x_1 – This point is outside box B. The resulting probability value of this point for belonging to the class being considered, P, is zero.

x_2 – This point is inside box B AND outside box B_0 AND outside box B_1. The P-value is zero.

x_3 – This point is inside box B AND inside box B_0 AND outside box B_{00} AND outside box B_{01}. The P-value is zero.

x_4 – This point is inside box B AND inside box B_0 AND inside box B_{00} (AND ouside box b_{00}). This point is mapped via M_{00} to the point u_4. The P-value is the value of the class-specific class-membership probability function, i.e., $P = prob(x_4, u_4)$. (It is also possible to define the prob-function more simple, as $prob(u_4)$.)

...

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

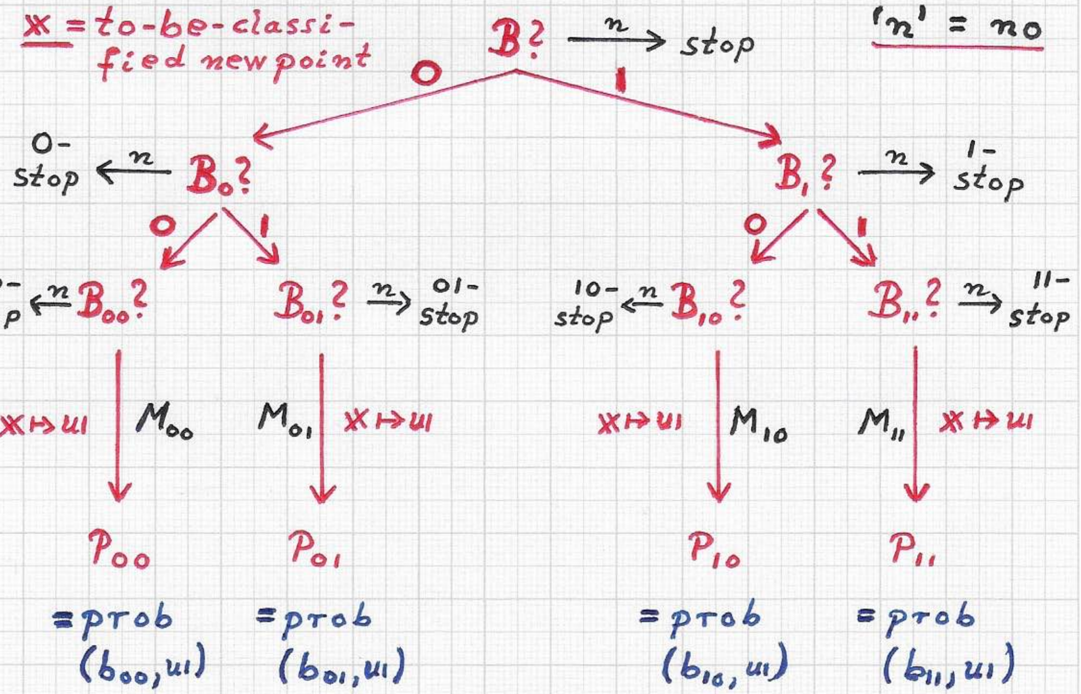
- Laplacian eigenfunctions and neural networks:... x_5 — This point is inside box B AND inside box B_0 AND inside box B_{01} (AND inside box b_{01}). This point is mapped via M_{01} to the point u_5. The P-value is given by the used probability function, i.e., $P = prob(x_5, u_5)$ or $P = prob(u_5)$.
- Note. It is assumed that the probability function prob — a function defined over u-space (and potentially "considering" the x-space representation of a point) — has been optimized to return near-optimal P-values for the specific class. Generally, $prob(u) > 0$ for u-tuples satisfying $\|u\|^2 < D$, i.e., $u_1^2 + \ldots + u_D^2 < D$, and $prob(u) = 0$ for $\|u\| \geq D$.

On the next page, the classification decision-making process is illustrated and explained for an arbitrary x-space point x to be classified. The illustration is still based on the simple bounding box hierarchy used in the above example for decision-making for the points $x_1, \ldots, x_5$ — but the decision tree illustration applies to a general unclassified point x. Further, the decision tree illustration captures the general D-dimensional setting. The decision tree is traversed by using bounding box tests applied to x. These tests are simply written as "Box?" – standing for "Is x inside Box?"

...

■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

- Laplacian eigenfunctions and neural networks:… The decision-making process as described for the simple 2D example can be summarized and illustrated as follows:
  - Initially, set all point (sub)set-related class-membership probability values to zero, i.e., $P_{00} = P_{01} = P_{10} = P_{11} = 0$.
  - Execute the hierarchical decision-making process based on the following binary tree:

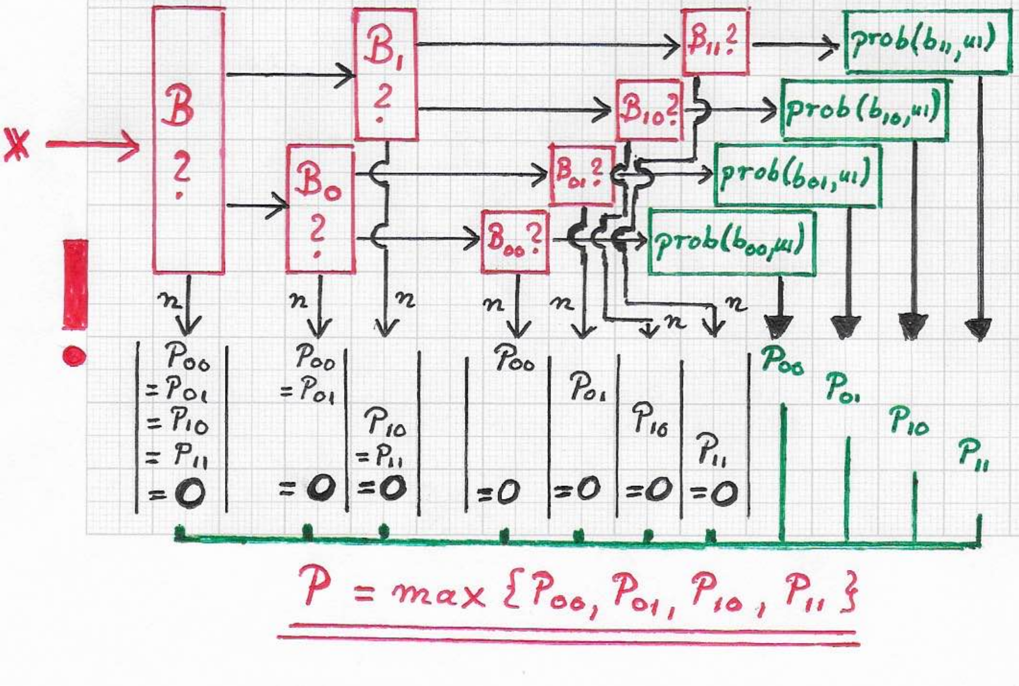


  - Define the value of the probability of ✕ to belong to the class being considered as $P = \max\{P_{00}, P_{01}, P_{10}, P_{11}\}$.

A "**stop**" decision for a node in the tree implies that the sub-tree having this node as its root is not further traversed. For example, "0-stop" means: "No processing of B_0's sub-tree".…

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

- Laplacian eigenfunctions and neural networks:... The decision tree illustration on the previous page also indicate – at the leaf level – that the point (sub)set-specific transformations M_{00}, M_{01}, M_{10} and M_{11} are used to map x to its u-space representation u. It is also emphazid that the function called "prob" – computing a point (sub)set-specific probability value for x (P_{00}, P_{01}, P_{10} or P_{11}) – receives as input the "name" of the minimal oriented point (sub)set bounding box (b_{00}, b_{01}, b_{10} or b_{11}) in addition to the u-tuple. This makes it possible – when necessary or desirable – that different "prob" functions can be employed for different $b_{i,j}$ boxes. Nevertheless, the "prob" functions all have u-space as their domain, specifically $u \in [-1,1]^D$.



$$P = \max\{P_{00}, P_{01}, P_{10}, P_{11}\}$$

The left figure illustrates the same decision-making process and P-value calculation in a way that is more computing-centric.

...