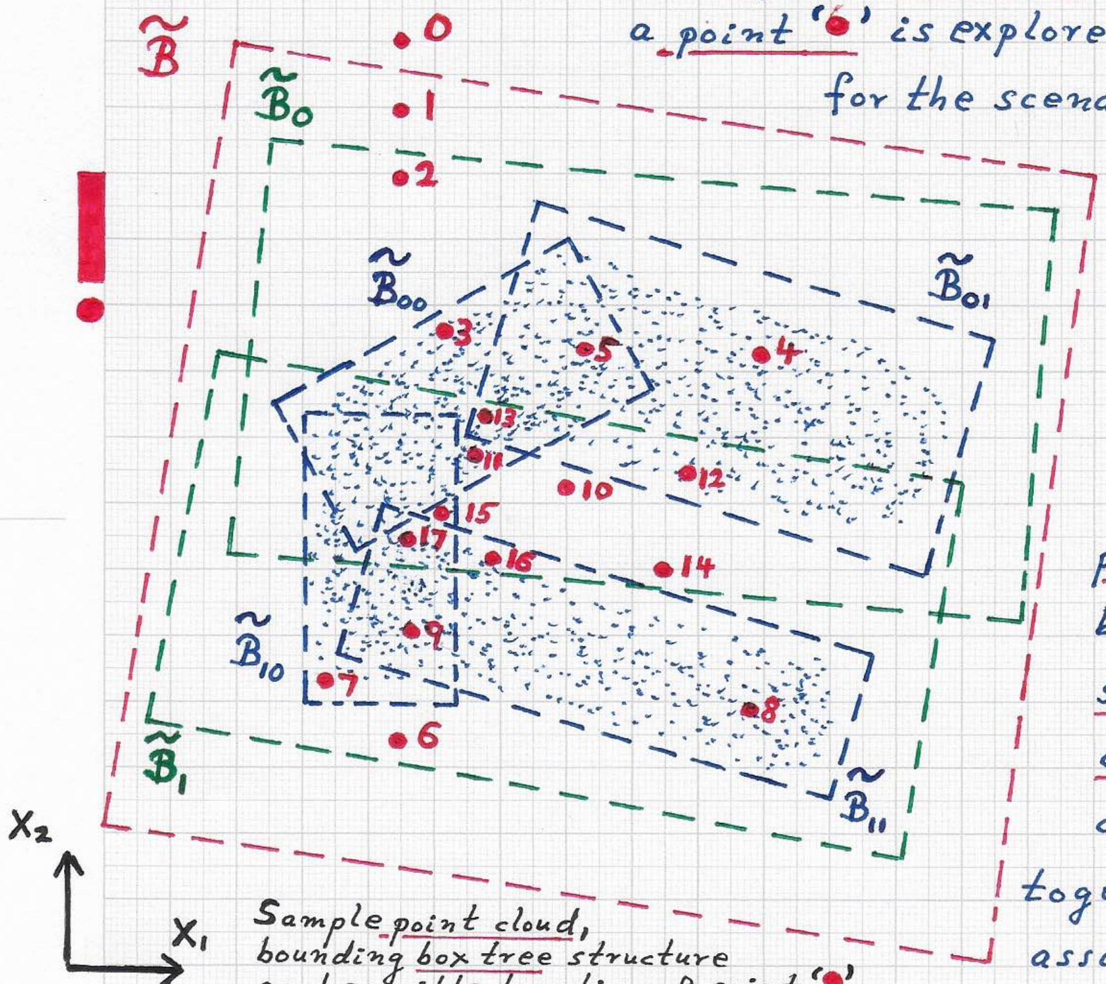
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks: ... The described method for determining a probability value P for a point '•' is explored in detail for the scenario shown in the left figure. The point cloud '⁘' represents a set of sample points that all belong to the same (material) class. The point cloud is shown together with its associated bounding box tree, consisting of partially intersecting/overlapping expanded local bounding boxes $\tilde{B}$, $\tilde{B}_0$, $\tilde{B}_1$, $\tilde{B}_{00}$, $\tilde{B}_{01}$, $\tilde{B}_{10}$ and $\tilde{B}_{11}$. The figure includes specific locations of '•' used to exemplify the 18 distinct cases one must consider when calculating the class-membership probability (P) value for point '•'. First, some of these 18 cases are discussed in detail, before summarizing all in a table.



Sample point cloud, bounding box tree structure and possible locations of point '•'.

...

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

- Laplacian eigenfunctions and neural networks: ... In the following, we discuss the decision-making and probability calculation for some of the prototypical cases:

!

0 The point is outside $\tilde{B}$. $P := 0$.

1 The point is outside $\tilde{B}_0$ AND outside $\tilde{B}_1$ AND inside $\tilde{B}$. $P := 0$.

2 The point is outside $\tilde{B}_{00}$ AND outside $\tilde{B}_{01}$ AND outside $\tilde{B}_1$ AND inside $\tilde{B}_0$ AND inside $\tilde{B}$. $P := 0$.

3 The point is inside $\tilde{B}_{00}$ AND outside $\tilde{B}_{01}$ AND outside $\tilde{B}_1$ AND inside $\tilde{B}_0$ AND inside $\tilde{B}$. $P := P_{00}$.

5 The point is inside $\tilde{B}_{00}$ AND inside $\tilde{B}_{01}$ AND outside $\tilde{B}_1$ AND inside $\tilde{B}_0$ AND inside $\tilde{B}$. Calculate P_{00} and P_{01}. $P := \max\{P_{00}, P_{01}\}$.

10 The point is inside $\tilde{B}_0$ AND inside $\tilde{B}_1$ AND inside $\tilde{B}$. Calculate P_0 and P_1. $P_0 \geq P_1$. Use sub-tree of $\tilde{B}_0$. The point is outside $\tilde{B}_{00}$ AND outside $\tilde{B}_{01}$. $P := 0$.

11 The point is inside $\tilde{B}_0$ AND inside $\tilde{B}_1$ AND inside $\tilde{B}$. Calculate P_0 and P_1. $P_0 \geq P_1$. Use sub-tree of $\tilde{B}_0$. The point is inside $\tilde{B}_{00}$ AND outside $\tilde{B}_{01}$. $P := P_{00}$.

13 The point is inside $\tilde{B}_0$ AND inside $\tilde{B}_1$ AND inside $\tilde{B}$. Calculate P_0 and P_1. $P_0 \geq P_1$. Use sub-tree of $\tilde{B}_0$. The point is inside $\tilde{B}_{00}$ AND inside $\tilde{B}_{01}$. Calculate P_{00} and P_{01}. $P := \max\{P_{00}, P_{01}\}$.

!

- **Note**. The probability values P_0 and P_1 (P_{00} and P_{01}) are the result of mapping $\tilde{B}_0$ and $\tilde{B}_1$ ($\tilde{B}_{00}$ and $\tilde{B}_{01}$) to the normalized domain $[-1,1]^D$ and computing "•"'s probability.

...

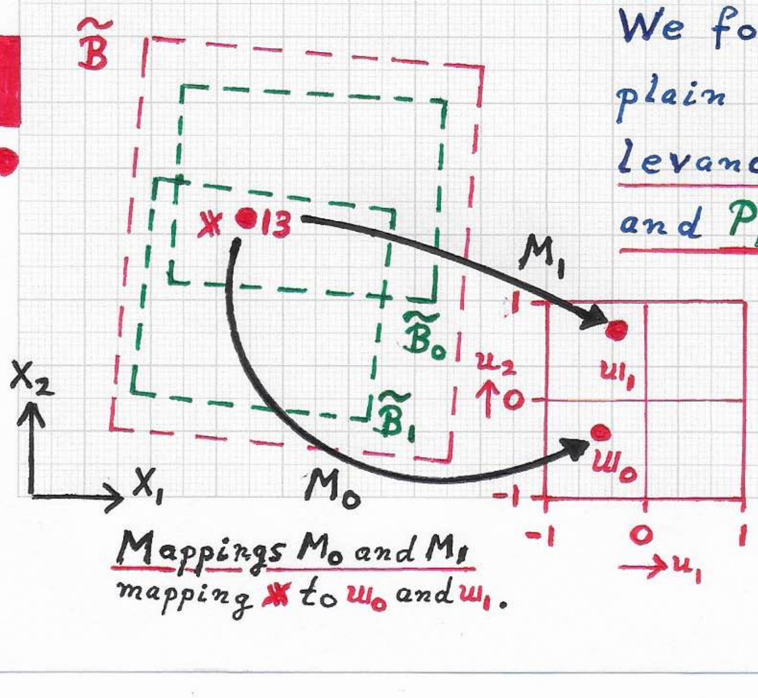
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks: ... We can now summarize the probability value computations for all 18 cases. The left table defines how to calculate the class-membership probability P for a to-be-classified point in 2D (X_1, X_2)-space. The point cloud representing sample points all belonging to the same class C has an associated binary (expanded) bounding box tree obtained after applying repeated box splitting two times to the root box $\widetilde{B}$. The resulting boxes are $\widetilde{B}$, $\widetilde{B}_0$, $\widetilde{B}_1$, $\widetilde{B}_{00}$, $\widetilde{B}_{01}$, $\widetilde{B}_{10}$ and $\widetilde{B}_{11}$ — and they are allowed to intersect. The table uses 'y' and 'n' to indicate that the point belonging to the specific case lies inside or does not lie in a particular box. ...

	case	$\widetilde{B}$	$\widetilde{B}_0$	$\widetilde{B}_1$	$\widetilde{B}_{00}$	$\widetilde{B}_{01}$	$\widetilde{B}_{10}$	$\widetilde{B}_{11}$	P
	0	n							0
	1	y	n	n					0
	2	y	y	n	n	n			0
	3	y	y	n	y	n			P_{00}
	4	y	y	n	n	y			P_{01}
	5	y	y	n	y	y			$\max\{P_{00}, P_{01}\}$
	6	y	n	y			n	n	0
	7	y	n	y			y	n	P_{10}
	8	y	n	y			n	y	P_{11}
	9	y	n	y			y	y	$\max\{P_{10}, P_{11}\}$
$P_0 \geq P_1$	10	y	y	y	n	n			0
	11	y	y	y	y	n			P_{00}
	12	y	y	y	n	y			P_{01}
	13	y	y	y	y	y			$\max\{P_{00}, P_{01}\}$
$P_1 > P_0$	14	y	y	y			n	n	0
	15	y	y	y			y	n	P_{10}
	16	y	y	y			n	y	P_{11}
	17	y	y	y			y	y	$\max\{P_{10}, P_{11}\}$

■ OBJECT AND MATERIAL EIGENFUNCTIONS — Cont'd.

• **Laplacian eigenfunctions and neural networks:...** The bounding box scenario shown in the figure on the previous page includes intersecting expanded bounding boxes $\tilde{B}_0$ and $\tilde{B}_1$. In this case, the described paradigm for box tree traversal ensures that the sub-tree of at most one of the tree nodes $\tilde{B}_0$ and $\tilde{B}_1$ is used for the potentially necessary consideration of the children of these nodes for P-value calculation. Thus, when a to-be-classified point lies inside $\tilde{B}$ AND inside $\tilde{B}_0$ AND inside $\tilde{B}_1$ (cases 10–17) one must compute the box-specific probability values P_0 for $\tilde{B}_0$ and P_1 for $\tilde{B}_1$ to uniquely determine the sub-tree to be used for tree traversal: If $P_0 \geq P_1$, then the sub-tree of $\tilde{B}_0$ will be used (cases 10–13); if $P_1 > P_0$, then the sub-tree of $\tilde{B}_1$ will be used (cases 14–17).



Mappings M_0 and M_1 mapping ∗ to w_0 and w_1.

We focus on case 13 to explain the computation and relevance of the values of P_0 and P_1 in detail. The left figure illustrates the essential aspects of a point ∗ representing case 13. One must decide whether to traverse the sub-tree of $\tilde{B}_0$ or $\tilde{B}_1$. •••

OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

- Laplacian eigenfunctions and neural networks:... The decision to be made concerning sub-tree choice must be made based on a quantitative measure. Such a measure is then used to determine whether the to-be-classified point $\ast$ should be assigned its class-membership probability based on the subset of the sample point cloud inside $\tilde{B}_0$ or $\tilde{B}_1$. It has been described how one can construct a linear mapping (transformation) M for mapping a bounding box in $\mathbb{X}$-space to the normalized $\mathbb{u}$-space domain $[-1,1]^D$. The figure on the previous page refers to two such mappings, M_0 and M_1, associated with $\tilde{B}_0$ and $\tilde{B}_1$, respectively. Point $\ast$ — lying inside both boxes — is mapped to its corresponding normalized $\mathbb{u}$-space locations/coordinates via M_0 and M_1. The two results are $\mathbb{u}_0$ and $\mathbb{u}_1$. Further, it is assumed that a class-membership probability function is defined for the $\mathbb{u}$-space domain square $[-1,1]^2$. When evaluating this function for $\mathbb{u}_0$ and $\mathbb{u}_1$, one obtains the needed values for P_0 and P_1, i.e., $P_0 = P_0(\mathbb{u}_0)$ and $P_1 = P_1(\mathbb{u}_1)$. For example, if the probability function decreased exponentially with increasing distance from the origin $\mathbb{u} = (0,0)^T$, then P_0 would be greater than P_1 in the illustrated scenario since $\|\mathbb{u}_0\|^2 < \|\mathbb{u}_1\|^2$ ($\mathbb{u}_0$ being closer to the origin). Thus, the decision is to choose the sub-tree of $\tilde{B}_0$.

...