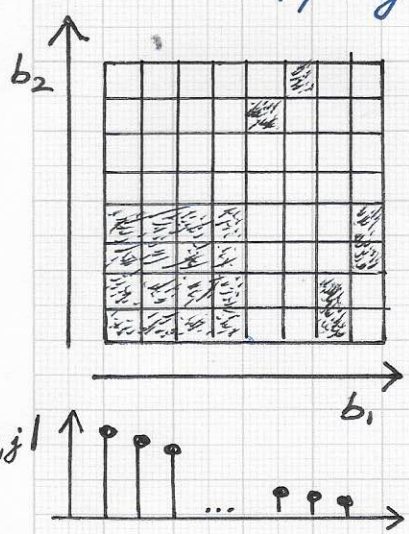
■ OBJECT AND MATERIAL EIGENFUNCTIONS – Cont'd.

• Laplacian eigenfunctions and neural networks: ...

Optimal use of dimensions and resolutions. Since a Haar wavelet approximation uses a "fully orthonormal" set of basis functions, the (absolute) magnitudes of the coefficients in the Haar wavelet expansion directly define the "importance" / "degree of information" captured by the coefficients' associated basis functions. For example, if one is given s data samples, i.e., tuples $(b_1^1, b_2^1, \ldots, b_n^1, B^1), \ldots, (b_1^s, b_2^s, \ldots, b_n^s, B^s)$, one can reasonably assume that the s expansion coefficients with largest (absolute) values define a "high-quality" representation of a class-identifying function $B(b_1, \ldots, b_n)$. The principle of using just the largest coefficients of a computed best-approximating Haar wavelet expansion is sketched in the left figure: The shown bivariate expansion based on 64 coefficients can be "used" by only using a small number of largest coefficients $c_{i,j}$. The paper by **Pulido** et al. "Data reduction using Lossy compression for cosmology and astrophysics workflows," ...



$$H(b_1, b_2) = \sum_{j=0}^{7} \sum_{i=0}^{7} c_{i,j} \cdot H_i(b_1) \cdot H_j(b_2)$$

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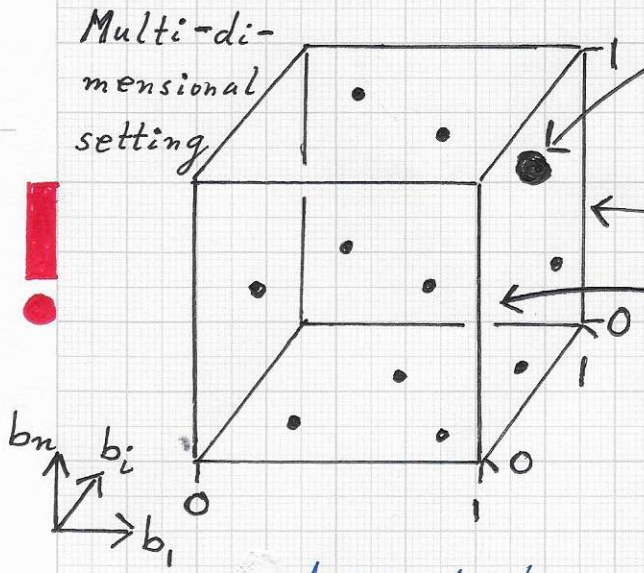
OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• Laplacian eigenfunctions and neural networks: ... Proc. XXX IUPAP Conf. on Computational Physics 2018 (CCP 2018), J. of Physics, Conf. Series, Vol. 1290, 2019 discusses how the use of lossy, large-coefficients-only wavelet approximations can be employed for efficient and error-controlled massive data processing and analysis. (A paper that considers near-optimal approximation using a piecewise linear representation of the approximating function for triangulated data is the paper "On simulated annealing and the construction of linear spline approximations for scattered data" by Kreylos and Hamann, Proc. VisSym '99, Springer, pp. 189-198.) Thus, a class-identifying function B_j, represented in a multiresolution Haar wavelet expansion, only needs to be "good enough" to satisfy the specified classification performance requirements — and this might often be achievable by using only largest-absolute-value coefficients of B_j's wavelet expansion. Further, when considering only a very small percentage of the $2^{d(r-1)}$ coefficients of B_j's expansion as a d-variate function with r resolution levels, d and r can be large(r). ...

OBJECT AND MATERIAL EIGENFUNCTIONS-Cont'd.

- Laplacian eigenfunctions and neural networks:...

It is not easy to illustrate or visualize the multivariate and multi-valued best approximations B_j — each one only using an "optimal subset" of the totally n--dimensional domain and each one using an "optimal resolution." In the following, an attempt is made to combine simple, abstract sketches and "condensed" equation to explain our setting and approach. In an n-dimensional unit hyper-cube C, one is given original sample data, with n independent components b_i^k and N dependent components B_j^k for s data totally ($k = 1 \dots s$). One can use "compact vector notation" and write this given data set as $\{(b_k; B_k)\}_{k=1}^{s}$.

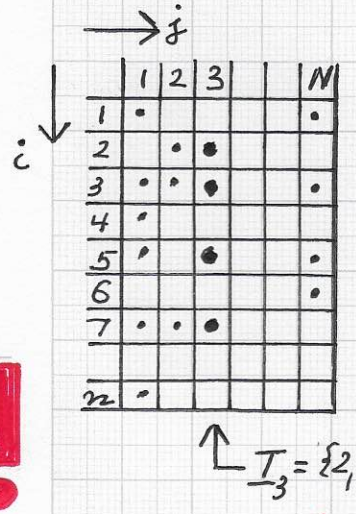


The goal is an "optimal" construction of class-identifying functions B_j (recognizing class j) using an optimal n_j-dimensional subspace of the n-dimensional domain and an optimal number of resolutions, r_j. Thus, $B_j = B_j^{n_j, r_j}$.

...

OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• Laplacian eigenfunctions and neural networks:... In other words, each class-identifying real-valued function $B_j^{n_j, r_j}$ is optimized for the necessary class-recognition performance while guaranteeing optimal computational and memory efficiency. As a consequence, the functions $B_j^{n_j, r_j}$ have different "degrees of complexity," and their formal definition is complicated. The left table shows that B_3 only uses $n_j = 4$ of the n available dimensions. The associated index set for B_3 is $\{2,3,5,7\} = I_3 \subseteq \{1,2,\ldots,n\}$.



It should be noted that ALL s given data samples are used for the optimal, best-approximation Haar wavelet expansion B_j. The fact that the value of s is "relatively small" (limited by the number of available objects/materials used for training) makes it possible to compute the coefficients of B_j in an acceptable amount of PRE-processing time. Thus, initially all functions $B_j, j = 1 \ldots N$, are n-variate functions with r levels of dyadic resolutions. IN A SUBSEQUENT OPTIMIZATION STEP IT MUST BE DETERMINED WHAT THE OPTIMALLY SELECTED SUB-SPACE DIMENSIONS (n_j) AND NEEDED RESOLUTIONS (r_j) ARE. ...

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■ OBJECT AND MATERIAL EIGENFUNCTIONS - Cont'd.

• Laplacian eigenfunctions and neural networks:...

The following illustation shows the construction of $\mathcal{B}_j^{n_j, r_j}$, $j = 1 \ldots N$:

$$\begin{array}{l} \{ |b_k; \mathcal{B}_1^k \}_{k=1}^s \longrightarrow \mathcal{B}_1(b_1, \ldots, b_n) \longrightarrow \mathcal{B}_1^{n_1, r_1}(b_\bullet, \ldots, b_\bullet) \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \underbrace{\qquad\qquad}_{I_1} \\ \vdots \qquad\qquad\quad \vdots \qquad\qquad\quad \vdots \qquad\qquad\qquad\qquad , n_j \leq n; \\ \{ |b_k; \mathcal{B}_N^k \}_{k=1}^s \longrightarrow \mathcal{B}_N(b_1, \ldots, b_n) \longrightarrow \mathcal{B}_N^{n_N, r_N}(b_\bullet, \ldots, b_\bullet) \quad I_j \subseteq \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \underbrace{\qquad\qquad}_{I_N} \quad \{1, 2, \ldots, n\}. \end{array}$$

Here, $\{ |b_k; \mathcal{B}_j^k \}_{k=1}^s = \{ (b_1^k, \ldots, b_n^k; \mathcal{B}_j^k) \}_{k=1}^s$.

The illustration summarizes the two steps used to define the desired "optimal" class-identifying functions $\mathcal{B}_j^{n_j, r_j}$. First, one constructs the functions $\mathcal{B}_j(b_1, \ldots, b_n)$ via a least-squares best approximation step, considering all given sample data in $\{ (|b_k; \mathcal{B}_k) \}_{k=1}^s$. Second, one performs a "complexity-reduction" step that determines optimal index sets I_j, $j = 1 \ldots N$, for each function $\mathcal{B}_j$, defining the functions $\mathcal{B}_j^{n_j, r_j}$. Thus, after the execution of the second step, the optimal sub-space dimensions and optimal resolution levels have been established.

At this point, one can understand the functions $\mathcal{B}_j^{n_j, r_j}$ as approximations of the original best approximations $\mathcal{B}_j$. For example, the functions $\mathcal{B}_j^{n_j, r_j}$ could be using only the largest-absolute-value coefficients of the (wavelet) expansions $\mathcal{B}_j$ – to support computation efficiency.

...