Chapter 1

Introduction

Computerized axial tomography (CAT) and magnetic resonance imaging (MRI) have been major breakthroughs in medical imaging. Both rendering techniques supply people in the field of medical diagnosis with two-dimensional images revealing internal structures of some part of the human body. They are based on slicing a three-dimensional object and producing a sequence of two-dimensional pictures. Scanning a human's head, CAT produces slices taken perpendicular to the spine. Each slice delivers an X-ray image representing density values in a particular plane. The pictures obtained by this method show great detail in bone structures (see Figure 1.1., created by the slicing method described in chapter 2.3.). MRI, on the other hand, allows slicing a head in axial (top-to-bottom), saggital (ear-to-ear), and coronal (nose-to-back) direction directly. It also gives better insight into soft tissue. Looking at these slices requires a person's ability to intuitively interpolate between them in order to get a perception of the real three-dimensional structure. People in the medical field are quite capable in performing this task, but creating three-dimensional images on a computer screen instantaneously might be an even more powerful tool in diagnosis.

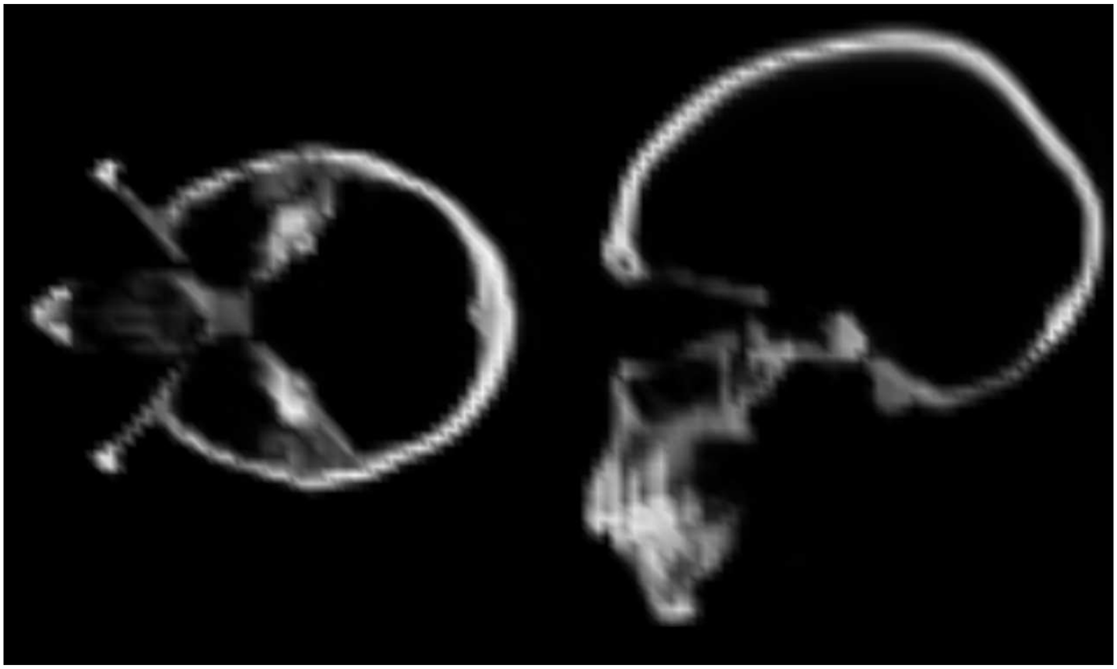

Fig. 1.1. Axial and saggital slice of human head (CAT).

CAT and MRI are examples serving well as a prelude for the topic of this dissertation. Visualizing and approximating three-dimensional data belonging to a scalar field (points in space with associated function values) is a difficult problem, too, in meteorology (temperature, pressure, velocity measurements), in geography (physical maps), physics, and mathematics itself. Mathematically, the data given can be viewed as the discretization of an unknown scalar field with three spatial variables sampled at certain points. It is quite common to approximate the data with a trivariate function over some domain (either inside the bounding box or the convex hull of the data points), evaluate the trivariate function on a regular grid and then plot the result. New methods are developed for both visualizing and modeling these data.

Several approaches exist for visualizing bivariate functions. Among the most common ones are contour lines (often called isolines) in the plane and surfaces in three-dimensional space. Using the second possibility, a point on the surface is simply given by the two spatial variables in the plane and the function value there (which yields the perpendicular distance to the plane). These techniques are extended to the trivariate case. Subdividing the three-dimensional domain of a trivariate function, simulating transparency for volumetric data, and slicing the domain with hyperplanes are methods being investigated.

A new path is followed modeling trivariate data. Traditionally, a continuous-approximating function is constructed considering each of the given data points and then evaluated. From a mathematical point of view, this is an obvious path to choose. However, some physical data sets might belong to a discontinuous scalar field. Discontinuities might have their explanation in the fact that physically different objects are present within the data. Different objects with their different properties naturally lead to rather heterogeneous function values (measurements) associated with them. Referring to the previous example of CAT and MRI, thousands of density measurements are obtained throughout a volume. In those applications, the internal structures in the volume and their actual three-dimensional geometry are of much greater interest than a trivariate function approximating all the thousands of density values.

Algorithms producing precisely these internal structures (contours of the density function) are reviewed and extended such that a topologically complete representation of the boundaries of different objects is obtained. Being surfaces in three-dimensional space, these boundaries are described in terms of triangulations. Each triangulation belonging to a particular object (a single component of a contour) can now be treated separately.

The triangulations in space might still be redundant in the sense that too many triangles might be used to approximate the shape of contours. A data reduction algorithm is developed examining the curvature of a surface. The number of triangles in nearly planar regions is reduced significantly. The method used for locally approximating the curvature of a polygonized surface in three-dimensional space generalizes nicely to the case of a trivariate function whose domain is subdivided into a set of tetrahedra. Curvature, in this generalized case, becomes more complicated, but it still helps to reveal qualitative properties of trivariate functions.

Finally, the set of reduced triangulations approximating different components of a contour is used to construct tangent-plane-continuous surfaces. A simple rational curve scheme based on degree elevated conics is used for this purpose. Briefly, methods for estimating normal vectors, needed for this construction, are discussed.

Overall, the sequence of modeling steps proposed in this dissertation makes use of intrinsic properties of the data. If the existence of different objects within a three-dimensional volume is known, and the corresponding contour level is apparent, this new strategy is more satisfactory with respect to both storage and computing efficiency.

Chapter 2 shows a variety of rendering methods for trivariate data. In chapter

3, an algorithm is described for generating a piecewise linear approximation of a contour. The result of this algorithm is a set of contour points, representing the geometry of the contour in three-dimensional space, and topological information. The topological information relates contour points to each other, determining which points form the vertices in a triangulation; it also includes the neighborhood information in the triangulation and associates each triangle with the component of a contour it belongs to. The chapter is concluded with some remarks concerning the estimation of gradients and normal vectors for three-dimensional points.

In chapter 4, principal curvatures are approximated for both surfaces and trivariate functions (precisely, for two-dimensional and three-dimensional manifolds). The concept of an osculating paraboloid (as used in differential geometry) derived from a local least squares fit serves as mathematical tool for this purpose. Chapter 5 presents a data reduction technique for a triangulated surface. Triangles are iteratively removed in areas with low curvature. In chapter 6, the reduced triangulation is used to construct an overall tangent-plane-continuous surface. Chapter 7 concludes the dissertation, summarizing the main results and mentioning possibilities for further research.

All algorithms described in this dissertation have been implemented in the programming language C on a Silicon Graphics workstation, model 4D/220 GTX.