Chapter 7

Conclusions

The dissertation has presented several ideas for the visualization of trivariate data. More research in this area is necessary, since computing power becomes more and more accessible, while visualization techniques, particularly for dynamical systems, are in a rather primitive state.

A new approach has been introduced for visualizing and modeling trivariate data (scalar fields). Following this approach, a contour of some trivariate function or a finite, discrete trivariate data set is approximated first and then used for modeling. This process itself can be seen as data reduction, since the contour approximation leads to a two-dimensional triangulation. This strategy might not be applicable for all real-world problems, but in the case of discontinuous scalar fields, e.g., CAT scan data, it is definitely an alternative to the standard way of constructing a trivariate interpolant for all data.

An existing technique for computing a triangular approximation to a contour of a trivariate function, the so-called *marching-cubes method*, has been corrected and improved. The contour approximation produces a continuous triangulation for which additional topological information (neighbors of triangles) is generated. Continuity of a triangulation is necessary for further modeling.

A way for approximating the two principal curvatures at the vertices in a two-dimensional triangulation in three-dimensional space has been developed and extended to the approximation of the three principal curvatures at the vertices on the three-dimensional graph of a trivariate function in four-dimensional space. This leads to a method for analyzing the smoothness of two-dimensional surfaces and of hypergraphs of trivariate functions, e.g., trivariate interpolants and approximants. Approximation schemes requiring curvature input can make use of the principal curvature approximation as well.

Most data reduction techniques can only be applied to function data, e.g., to sets of points in the plane (or space) with function values. The new triangle removal algorithm can be used for general two-dimensional triangulations in three-dimensional space. A triangulation is adaptively reduced such that at each reduction step the implied piecewise triangular approximant of the surface changes as little as possible. The same strategy could also be applied to the removal of tetrahedra in a tetrahedrization of points in three-dimensional space (with function values), taking the absolute curvature at points on the implied piecewise linear hypergraph in four-dimensional space into account.

An elegant planar curve scheme based on degree elevated conics has been developed. It is utilized as a blending technique, needed for a particular triangular surface scheme, the side-vertex method. Combining the side-vertex method with the new curve scheme results in smooth surfaces. The triangular, tangent-plane continuous surface can be viewed as an alternative to existing triangular interpolants for general two-dimensional triangulations in three-dimensional space.