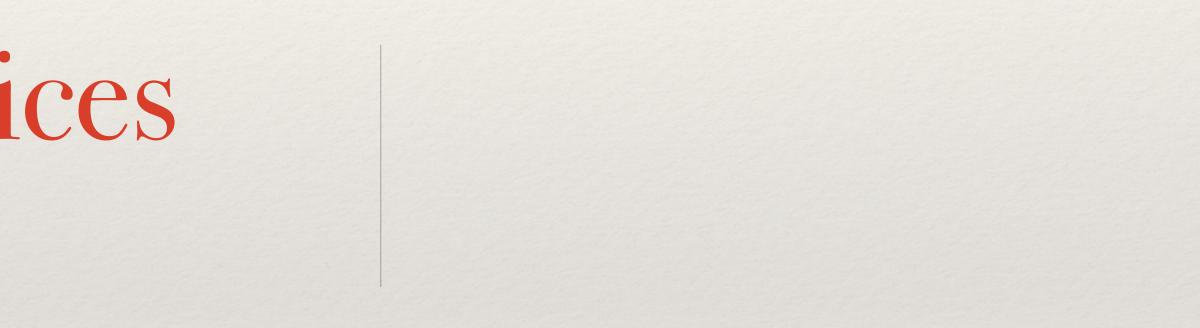
Activity 1: Good practices and puzzles



Mathematical symbols

Set of Numbers

- \mathbb{N} : set of natural numbers : 1,2,3,...
- \mathbb{Z} : set of Integers : . . . , -3, -2, -1, 0, 1, 2, 3, ...
- Q: set of rational numbers
- \mathbb{R} : set of real numbers

Set of Operations

- +: addition
- -: substration
- *x*: multiplication
- */or-:* division
- $\sqrt{:}$ square root exp: exponential log: logarithm

Solve

If <i>x</i> is a natural number	: no so
If <i>x</i> is an integer	: no so
If <i>x</i> is a rational number	: no so
If <i>x</i> is a positive real number	: <i>S</i> =
If <i>x</i> is a real number	$: S = \{$

Solve

olutions

olutions

olutions

 $\left\{\sqrt{2}\right\}$

 $: S = \left\{ -\sqrt{2}, \sqrt{2} \right\}$

Do not prove the obvious!!

Example:

Let x be a real number. Show that: $(x - 1)(x + 2) - x^2 + 2 = 2(x + 1) - x - 2$

Do not prove the obvious!!

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Let x be a real number. Show that: $(x - 1)(x + 2) - x^2 + 2 = 2(x + 1) - x - 2$

Method 1: (unsatisfactory): Develop on both side

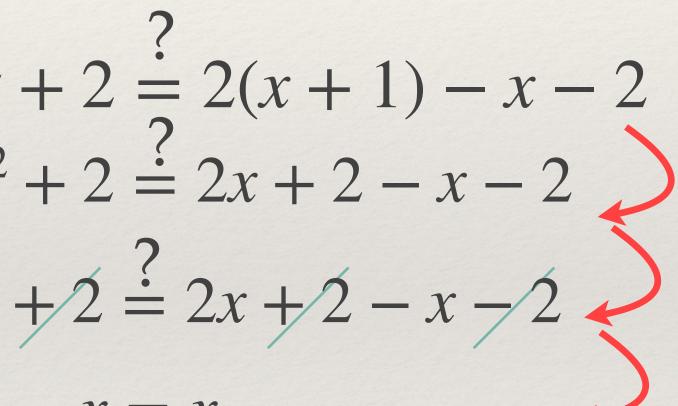
$$(x - 1)(x + 2) - x^{2}$$

$$x^{2} + 2x - x - 2 - x^{2}$$

$$x^{2} + 2x - x - 2 - x^{2}$$

But this is obvious !!!

x = x



Do not prove the obvious!!

Example:

Let x be a real number. Show that: $(x - 1)(x + 2) - x^2 + 2 = 2(x + 1) - x - 2$

Method 2: (preferred): 1) *Define*:

$$LHS = (x - 1)(x + 2) - x^{2} + 2$$
$$RHS = 2(x + 1) - x - 2$$

2) Compute LHS and RHS:

$$LHS = (x - 1)(x + 2) - x^{2} + 2$$

= $x^{2} + 2x - x - 2 - x^{2} + 2$
= x
RH

3) Compare LHS and RHS:

$$LHS = RHS$$

4) Conclusion

The property is true for all real numbers

IS = 2(x + 1) - x - 2= 2x + 2 - x - 2= x

Reason!

Example: a bottle of wine costs 11 dollars. The wine is worth 10 dollars more than the bottle. How much is the bottle worth?



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Reason!

Example: a bottle of wine costs 11 dollars. The wine is worth 10 dollars more than the bottle. How much is the bottle worth? Naively, we would say that the wine is worth 10 dollars and the bottle 1 dollar....

But then the wine would be worth 9 dollars more than the bottle!

Let W be the worth of the wine, and B the worth of the bottle. What we know:

$$\begin{cases} W+B = 11 \\ W = B + 10 \end{cases}$$

This system is easy to solve!

$$\begin{cases} B = 0.5\\ W = 10.5 \end{cases}$$

B+10+B=11 B+B=1 2B=1 B=1/2



Logic Puzzles

or knaves, who always *lie*.

inhabitants' type from their statements (*the visitor cannot ask questions*)

Example:

Can you find out what types John and Bill are?

How to solve such puzzles?

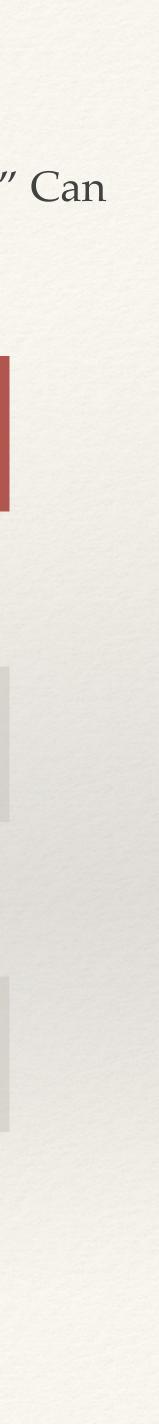
- The puzzles are set on a fictional island, Smullyan's island, where all inhabitants are either knights, who always tell the truth,
- The puzzles involve a visitor to the island who meets small groups of inhabitants. The aim is for the visitor to deduce the

Let John and Bill be two inhabitants of the island. John says, "We are the same kind," but Bill says, "We are of different kinds."



John	Bill
Knight	Knight
Knight	Knave
Knave	Knight
Knave	Knave

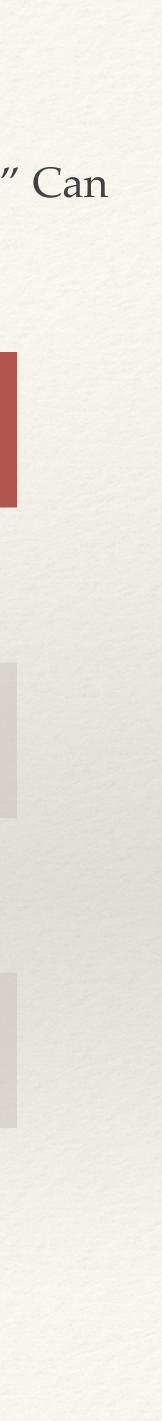




John	Bill	John's statement	Bill's statement
Knight	Knight	TRUE	FALSE
Knight	Knave	FALSE	TRUE
Knave	Knight	FALSE	TRUE
Knave	Knave	TRUE	FALSE

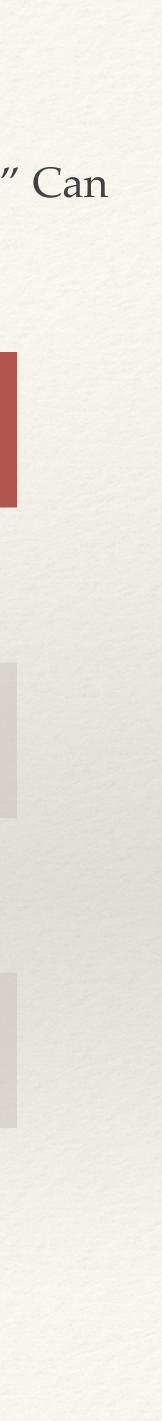


John	Bill	John's statement	Bill's statement	Validity
Knight	Knight	TRUE	FALSE	No: Bill is a knight that would lie
Knight	Knave	FALSE	TRUE	No: John is a knight that would lie
Knave	Knight	FALSE	TRUE	Yes
Knave	Knave	TRUE	FALSE	No: John is a knave that would tell the truth



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Knight	Knave	FALSE	TRUE	No: John is a knight that would lie
Knave	Knight	FALSE	TRUE	Yes
Knave	Knave	TRUE	FALSE	No: John is a knave that would tell the truth

John is a knave and Bill is a knight



Let John and Bill be two inhabitant of the island. John says, "I and Bill are not of the same kind," but Bill says, "of John and I, exactly one is a knight." Can you find out what types John and Bill are?

John	Bill	John's statement	Bill's statement
Knight	Knight	FALSE	FALSE
Knight	Knave	TRUE	TRUE
Knave	Knight	TRUE	TRUE
Knave	Knave	FALSE	FALSE



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Knight	Knight	FALSE	FALSE	No: John is a knight that would lie
Knight	Knave	TRUE	TRUE	No: Bill is a knave that would tell the truth
Knave	Knight	TRUE	TRUE	No: John is a knave that would tell the trut
Knave	Knave	FALSE	FALSE	Yes

John and Bill are knaves



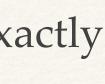
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Knight	Knave	
Knave	Knight	
Knave	Knave	

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Knave	Knight	FALSE	TRUE	Yes
Knave	Knave	TRUE	FALSE	No: John is a knave that would tell the truth
				John is a knave and Bill is a knight



Let John and Bill be two inhabitants of the island. John says something, but I can't hear what he says. Bill says, "We are both knaves" Can you find out what types John and Bill are?

John	Bill	John's statement	Bill's statement	Validity
Knight	Knight		FALSE	No: Bill is a knight that would lie
Knight	Knave		FALSE	Yes
Knave	Knight		FALSE	No: Bill is a knight that would lie
Knave	Knave		TRUE	No: Bill is a knave that would lie

