Activity 1: Good practices and puzzles

Mathematical symbols

Set of Numbers

$\mathbb{N}$: set of natural numbers : 1,2,3,...
$\mathbb{Z}$: set of Integers : . . . , − 3, − 2, − 1,0,1,2,3,...
$\mathbb{Q}$: set of rational numbers
$\mathbb{R}$: set of real numbers

Set of Operations

+: addition
−: substration
x: multiplication
/ or −: division
$\sqrt{}$: square root
exp: exponential
log: logarithm

Good practices in mathematics

Solve $x^2 = 2$

Good practices in mathematics

Solve $x^2 = 2$

If x is a **natural number** : no solutions

If x is an **integer** : no solutions

If x is a **rational number** : no solutions

If x is a **positive real number** : $S = \left\{\sqrt{2}\right\}$

If x is a **real number** : $S = \left\{-\sqrt{2}, \sqrt{2}\right\}$

Good practices in mathematics

Do not prove the obvious!!

Example:
Let x be a real number. Show that: $(x-1)(x+2) - x^2 + 2 = 2(x+1) - x - 2$

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Method 1: (unsatisfactory):
Develop on both side

$$(x-1)(x+2) - x^2 + 2 \stackrel{?}{=} 2(x+1) - x - 2$$
$$x^2 + 2x - x - 2 - x^2 + 2 \stackrel{?}{=} 2x + 2 - x - 2$$
$$x^2 + 2x - x - 2 - x^2 + 2 \stackrel{?}{=} 2x + 2 - x - 2$$
$$x = x$$

But this is obvious !!!

Good practices in mathematics

Do not prove the obvious!!

Example:
Let x be a real number. Show that: $(x-1)(x+2)-x^2+2=2(x+1)-x-2$

Method 2: (preferred):

1) Define:

$LHS=(x-1)(x+2)-x^2+2$
$RHS=2(x+1)-x-2$

2) Compute LHS and RHS:

$LHS=(x-1)(x+2)-x^2+2$
$=x^2+2x-x-2-x^2+2$
$=x$

$RHS=2(x+1)-x-2$
$=2x+2-x-2$
$=x$

3) Compare LHS and RHS:

$LHS=RHS$

4) Conclusion

The property is true for all real numbers

Good practices in mathematics

Reason!

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But then the wine would be worth 9 dollars more than the bottle!

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Example: a bottle of wine costs 11 dollars. The wine is worth 10 dollars more than the bottle. How much is the bottle worth?

Naively, we would say that the wine is worth 10 dollars and the bottle 1 dollar….

But then the wine would be worth 9 dollars more than the bottle!

Let W be the worth of the wine, and B the worth of the bottle.
What we know:

$$\begin{cases} W + B = 11 \\ W = B + 10 \end{cases}$$

B+10+B=11
B+B=1
2B=1
B=1/2

This system is easy to solve!

$$\begin{cases} B = 0.5 \\ W = 10.5 \end{cases}$$

Logic Puzzles

The puzzles are set on a fictional island, Smullyan's island, where all inhabitants are either **knights**, who always ***tell the truth***, or **knaves**, who always ***lie***.

The puzzles involve a visitor to the island who meets small groups of inhabitants. The aim is for the visitor to deduce the inhabitants' type from their statements (*the visitor cannot ask questions*)

Example:
Let John and Bill be two inhabitants of the island. John says, "We are the same kind," but Bill says, "We are of different kinds."
Can you find out what types John and Bill are?

How to solve such puzzles?

Let John and Bill be two inhabitant of the island. John says, "We are the same kind," but Bill says, "We are of different kinds." Can you find out what types John and Bill are?

John	Bill	John's statement	Bill's statement
Knight	Knight		
Knight	Knave		
Knave	Knight		
Knave	Knave		

Let John and Bill be two inhabitant of the island. John says, "We are the same kind," but Bill says, "We are of different kinds." Can you find out what types John and Bill are?

John	Bill	John's statement	Bill's statement
Knight	Knight	TRUE	FALSE
Knight	Knave	FALSE	TRUE
Knave	Knight	FALSE	TRUE
Knave	Knave	TRUE	FALSE

Let John and Bill be two inhabitant of the island. John says, "We are the same kind," but Bill says, "We are of different kinds." Can you find out what types John and Bill are?

John	Bill	John's statement	Bill's statement	Validity
Knight	Knight	TRUE	FALSE	No: Bill is a knight that would lie
Knight	Knave	FALSE	TRUE	No: John is a knight that would lie
Knave	Knight	FALSE	TRUE	Yes
Knave	Knave	TRUE	FALSE	No: John is a knave that would tell the truth

Let John and Bill be two inhabitant of the island. John says, "We are the same kind," but Bill says, "We are of different kinds." Can you find out what types John and Bill are?

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Knight	Knight	TRUE	FALSE	No: Bill is a knight that would lie
Knight	Knave	FALSE	TRUE	No: John is a knight that would lie
Knave	Knight	FALSE	TRUE	Yes
Knave	Knave	TRUE	FALSE	No: John is a knave that would tell the truth

John is a knave and Bill is a knight

Let John and Bill be two inhabitant of the island. John says, "I and Bill are not of the same kind," but Bill says, "of John and I, exactly one is a knight." Can you find out what types John and Bill are?

John	Bill	John's statement	Bill's statement
Knight	Knight	FALSE	FALSE
Knight	Knave	TRUE	TRUE
Knave	Knight	TRUE	TRUE
Knave	Knave	FALSE	FALSE

Let John and Bill be two inhabitant of the island. John says, "I and Bill are not of the same kind," but Bill says, "of John and I, exactly one is a knight." Can you find out what types John and Bill are?

John	Bill	John's statement	Bill's statement	
Knight	Knight	FALSE	FALSE	No: John is a knight that would lie
Knight	Knave	TRUE	TRUE	No: Bill is a knave that would tell the truth
Knave	Knight	TRUE	TRUE	No: John is a knave that would tell the truth
Knave	Knave	FALSE	FALSE	Yes

John and Bill are knaves

Let John and Bill be two inhabitants of the island. John says, "I am a knight or Bill is a knave," but Bill says, "of John and I, exactly one is a knight." Can you find out what types John and Bill are?

John	Bill	John's statement	Bill's statement
Knight	Knight		
Knight	Knave		
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Knight	Knave	TRUE	TRUE	No: Bill is a knave that would tell the truth
Knave	Knight	FALSE	TRUE	Yes
Knave	Knave	TRUE	FALSE	No: John is a knave that would tell the truth

John is a knave and Bill is a knight

Let John and Bill be two inhabitants of the island. John says something, but I can't hear what he says. Bill says, "We are both knaves" Can you find out what types John and Bill are?

John	Bill	John's statement	Bill's statement	Validity
Knight	Knight		FALSE	No: Bill is a knight that would lie
Knight	Knave		FALSE	Yes
Knave	Knight		FALSE	No: Bill is a knight that would lie
Knave	Knave		TRUE	No: Bill is a knave that would lie