Digital Data

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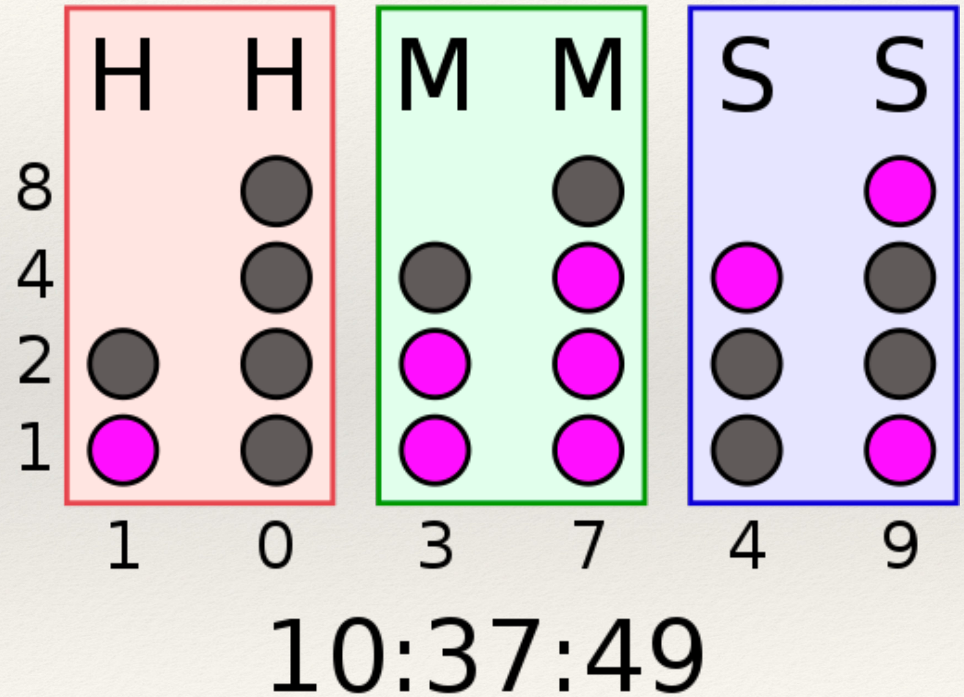


Digital Data

Binary and hexadecimal representations

Different types of numbers: natural numbers, integers, real numbers

ASCII code and UNICODE

Sound: Sampling, and Quantitizing

Images

Digital Data

Binary and hexadecimal representations

Number representation

We are used to counting in base 10:

1000	100	10	1
10^3	10^2	10^1	10^0

….. thousands hundreds tens units

Example:

1 7 3 2 ← *digits*

1000	100	10	1

$1 \times 1000 + 7 \times 100 + 3 \times 10 + 2 \times 1 = 1732$

Number representation

Computers use a different system: base 2:

1024	512	256	128	64	32	16	8	4	2	1
2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Example:

← *bits*

1	1	0	1	1	0	0	0	1	0	0
1024	512	256	128	64	32	16	8	4	2	1

1x1024+1x512+0x256+1x128+1x64+0x32+ 0x16+ 0x8 + 1x4 + 0x2 + 0x1 = 1732

Number representation

Base 10	Base 2
0	0
1	1
2	10
3	11
4	100
5	101
6	110
…	…
253	11111101
254	11111110
255	11111111
…	…

Conversion

From base 2 to base 10:

1	1	1	0	1	0	1	0	1	0	0
1024	512	256	128	64	32	16	8	4	2	1

1x1024+1x512+1x256+0x128+1x64+0x32+ 1x16+ 0x8 + 1x4 + 0x2 + 0x1 = 1876

From base 10 to base 2:

1877 %2 = 938 Remainder 1
938 %2 = 469 Remainder 0
469 %2 = 234 Remainder 1
234 %2 = 117 Remainder 0
117 %2 = 58 Remainder 1
58 %2 = 29 Remainder 0
29 %2 = 14 Remainder 1
14 %2 = 7 Remainder 0
7 %2 = 3 Remainder 1
3 %2 = 1 Remainder 1
1 %2 = 0 Remainder 1

1877 (base10) = 11101010101 (base 2)

Facts about Binary Numbers

-Each “digit” of a binary number (each 0 or 1) is called a bit

-1 byte = 8 bits

-1 KB = 1 kilobyte = 2^{10} bytes = 1024 bytes (≈1 thousand bytes)

-1 MB = 1 Megabyte = 2^{20} bytes = 1,048,580 bytes (≈ 1 million bytes)

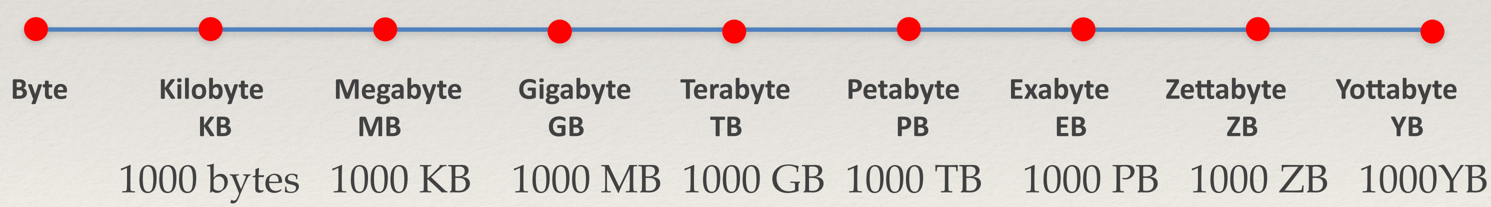
-1 GB = 1 Gigabyte = 2^{30} bytes = 1,073,741,824 bytes (≈1 billion bytes)

-1 TB = 1 Tetabyte = 2^{40} bytes = 1,099,511,627,776 bytes (≈ 1 trillion bytes)

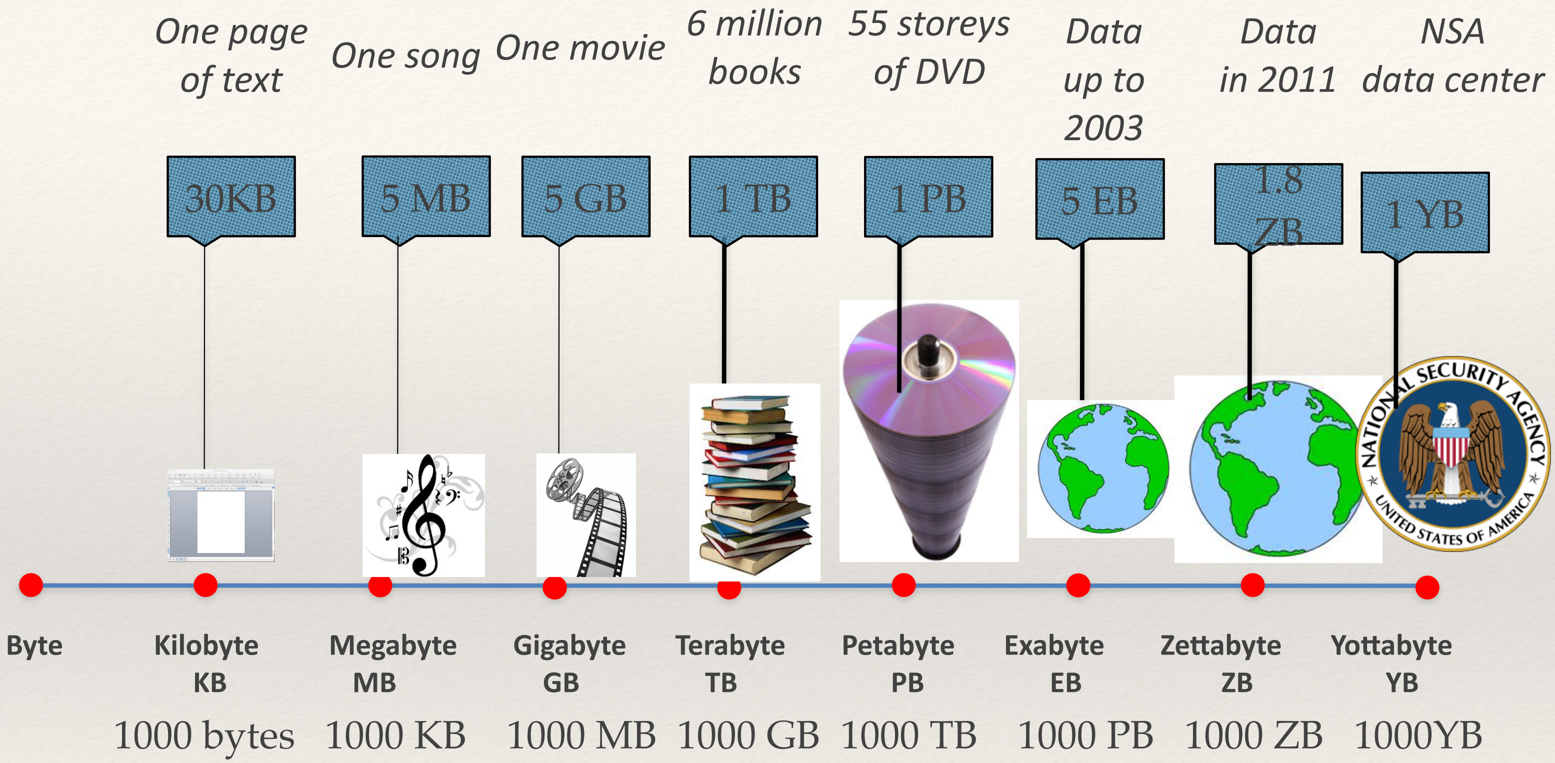
-A byte can represent numbers up to 255: 11111111 (base 2) = 255 (base 10)

-The largest number represented by a binary number of size N is $2^N - 1$

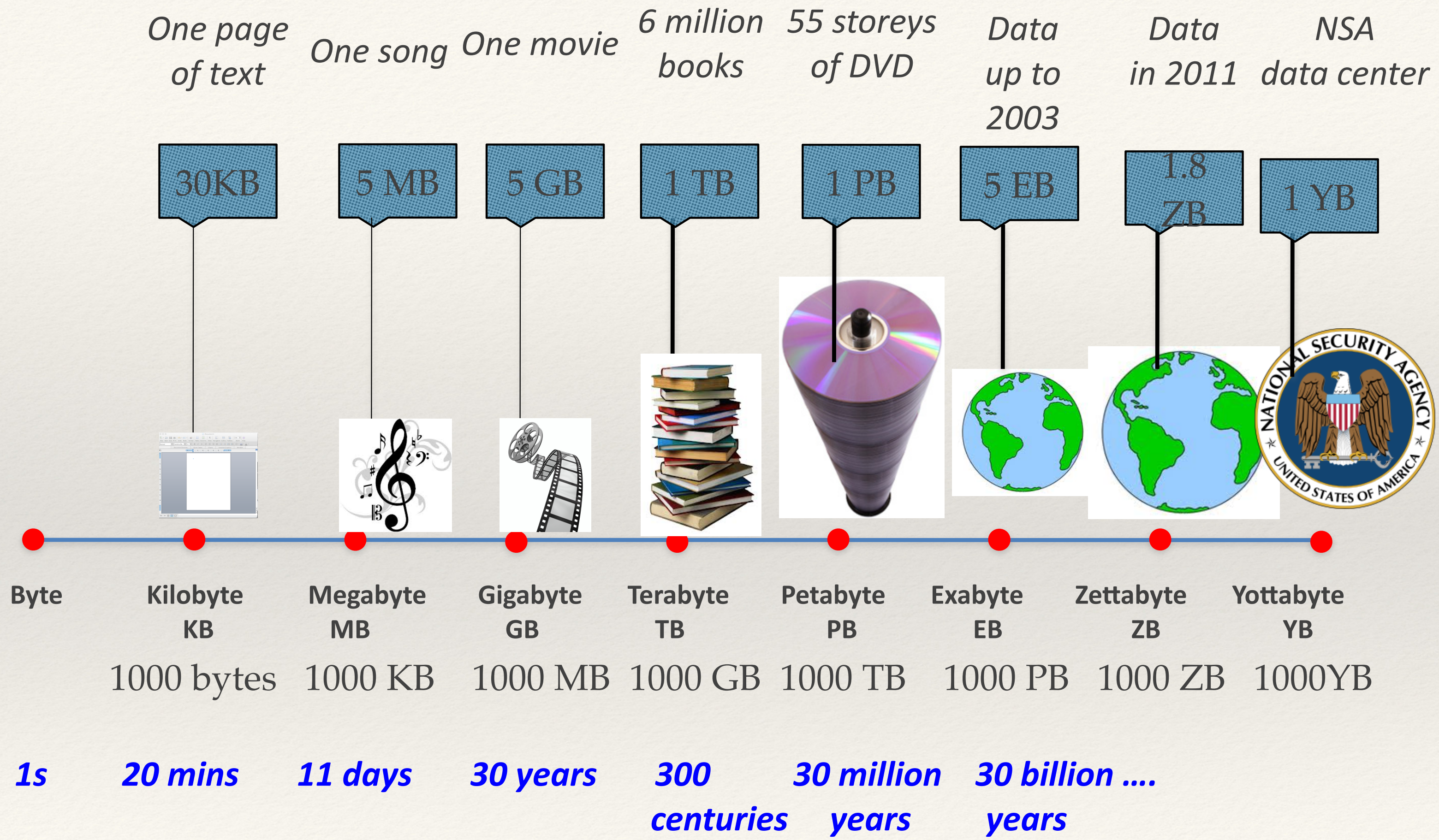
Big Data: Volume



Big Data: Volume



Big Data: Volume



Hexadecimal numbers

While base 10 and base 2 are the most common bases used to represent numbers, others are also possible: base 16 is another popular one, corresponding to hexadecimal numbers

256	16	1
16^2	16^1	16^0

The "digits" are: 0 1 2 3 4 5 6 7 8 9 A B C D E F

Example:

2	A	F
256	16	1

2x256 + 10*16 + 15x1 = 687

Hexadecimal numbers

Everything we have learned in base 10 should be studied again in other bases !!

Example: multiplication table in base 16:

x	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
1	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
2	2	4	6	8	A	C	E	10	12	14	16	18	1A	1C	1E
3	3	6	9	C	F	12	15	18	1B	1E	21	24	27	2A	2D
4	4	8	C	10	14	18	1C	20	24	28	2C	30	34	38	3C
5	5	A	F	14	19	1E	23	28	2D	32	37	3C	41	46	4B
6	6	C	12	18	1E	24	2A	30	36	3C	42	48	4E	54	5A
7	7	E	15	1C	23	2A	31	38	3F	46	4D	54	5B	62	69
8	8	10	18	20	28	30	38	40	48	50	58	60	68	70	78
9	9	12	1B	24	2D	36	3F	48	51	5A	63	6C	75	7E	87
A	A	14	1E	28	32	3C	46	50	5A	64	6E	78	82	8C	96
B	B	16	21	2C	37	42	4D	58	63	6E	79	84	8F	9A	A5
C	C	18	24	30	3C	48	54	60	6C	78	84	90	9C	A8	B4
D	D	1A	27	34	41	4E	5B	68	75	82	8F	9C	A9	B6	C3
E	E	1C	2A	38	46	54	62	70	7E	8C	9A	A8	B6	C4	D2
F	F	1E	2D	3C	4B	5A	69	78	87	96	A5	B4	C3	D2	E1

Hexadecimal numbers

Base 10	Base 2	Base 16
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Conversion: From base 2 to base 16, and back

This is in fact easy!!

-From base 2 to base 16:

Example: 11011000100

Step 1: break into groups of 4 (starting from the right):

110 1100 0100

Step 2: pad with 0, if needed:

0110 1100 0100

Step 3: convert each group of 4, using table:

6 C 4

Step 4: regroup:

6C4

11011000100 (base 2) = 6C4 (base 16)

Conversion: From base 2 to base 16, and back

From base 16 to base 2:

Example: 4FD

Step 1: split:

4 F D

Step 2: convert each "digit", using table:

0100 1111 1101

Step 3: Remove leading 0, if needed

100 1111 1101

Step 4: regroup:

10011111101

4FD (base 16) = 10011111101 (base 2)

Digital Data

Different types of numbers: natural numbers, integers, real numbers

The different set of numbers

$\mathbb{N}$	Natural numbers	$1, 2, 3, 4 \ldots,$
$\mathbb{Z}$	Integers	$\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$
$\mathbb{Q}$	Rational numbers	$\frac{a}{b}$ where a and b are integers and b is not zero
$\mathbb{R}$	Real numbers	The limit of a convergent sequence of rational numbers
$\mathbb{C}$	Complex numbers	$a + ib$ where a and b are real numbers and i is the square root of -1

Representing Integers

Unsigned integers (natural numbers):

Num

Sizes

- ➢Char: 1 bit
- ➢Unsigned short: 16 bits (2 bytes)
- ➢Unsigned int: 32 bits (4 bytes)

Representing Integers

Signed integers

s	Num

S:

- sign bit: 0 means positive, 1 means negative

Num:

- If s = 0, direct representation of the number in binary form
- If s = 1, two's complement of the number

Sizes

- ➢Char: 1 bit
- ➢Short: 16 bits (2 bytes)
- ➢int: 32 bits (4 bytes)

Representing Integers: two’s complement

The two's complement of an N-bit number is defined as its complement with respect to 2^N

The sum of a number and its two's complement is 2^N.

For instance, for the three-bit number 010, the two's complement is 110, because
$010 + 110 = 1000$ $(= 2^3 = 8)$.

The two's complement is calculated by inverting the bits and adding one.

Eight-bit signed integers

Bits	Unsigned value	Two's complement value
0000 0000	0	0
0000 0001	1	1
0000 0010	2	2
0111 1110	126	126
0111 1111	127	127
1000 0000	128	−128
1000 0001	129	−127
1000 0010	130	−126
1111 1110	254	−2
1111 1111	255	−1

IEEE Floating Point Representation

IEEE Standard 754

- ➢Established in 1985 as uniform standard for floating point arithmetic
  Before that, many idiosyncratic formats

- ➢Supported by all major CPUs

Driven by Numerical Concerns

- ➢Nice standards for rounding, overflow, underflow
- ➢Hard to make go fast
  - Numerical analysts predominated over hardware types in defining standard

IEEE Floating Point Representation

Numerical Form

$(-1)^s\ M\ 2^E$

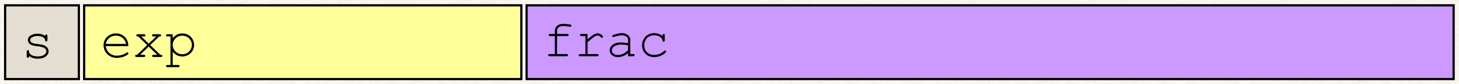
- ➢Sign bit ***s*** determines whether number is negative or positive
- ➢Significand ***M*** normally a fractional value in range [1.0,2.0).
- ➢Exponent ***E*** weights value by power of two

Encoding

`s`	`exp`	`frac`

- ➢MSB is sign bit
- ➢`exp` field encodes *E*
- ➢`frac` field encodes *M*

IEEE Floating Point Representation



Encoding

- ➢MSB is sign bit
- ➢`exp` field encodes *E*
- ➢`frac` field encodes *M*

Sizes

- ➢Single precision: 8 `exp` bits, 23 `frac` bits
  (32 bits total)
- ➢Double precision: 11 `exp` bits, 52 `frac` bits
  (64 bits total)
- ➢Extended precision: 15 `exp` bits, 63 `frac` bits
  - Only found in Intel-compatible machines
  - Stored in 80 bits (1 bit wasted)

IEEE Floating Point Representation

Special value:

`exp = 111…1`

- ➢ **`exp = 111…1, frac = 000…0`**
  - Represents value ∞ (infinity)
  - Operation that overflows
  - Both positive and negative
  - E.g., 1.0/0.0 = −1.0/−0.0 = +∞, 1.0/−0.0 = −∞

- ➢ **`exp = 111…1, frac ≠ 000…0`**
  - Not-a-Number (NaN)
  - Represents case when no numeric value can be determined
  - E.g., sqrt(−1), ∞ − ∞

Floating Point Operations

Conceptual View

- First compute exact result
- Make it fit into desired precision
  - Possibly overflow if exponent too large
  - Possibly round to fit into `frac`

Rounding Modes (illustrate with $ rounding)

	\$1.40	\$1.60	\$1.50	\$2.50	–\$1.50
Round down (-∞)	\$1	\$1	\$1	\$2	–\$2
Round up (+∞)	\$2	\$2	\$2	\$3	–\$1
Nearest Even	\$1	\$2	\$2	\$2	–\$2

Note:

1. *Round down: rounded result is close to but no greater than true result.*
2. *Round up: rounded result is close to but no less than true result.*

Unwanted noise

Computers encounter noise!

The Ariane 5 tragedy: On June 1996, the first Ariane 5 was launched… and exploded after 37 seconds

The failure of the Ariane 501 was caused by the complete loss of guidance and altitude information 37 seconds after start….due to a numerical error.

https://www.wired.com/2005/11/historys-worst-software-bugs/

Digital Data

ASCII code and UNICODE

ASCII

American Standard Code for Information Interchange

So far, we have seen how computers can handle numbers.

What about letters / characters?

The ASCII code was designed for that: it assigns a number to each character:

A-Z: 65- 90
a-z: 97-122
0-9: 48- 57

ASCII

American Standard Code for Information Interchange

Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char
0	00	Null	32	20	Space	64	40	@	96	60	`
1	01	Start of heading	33	21	!	65	41	A	97	61	a
2	02	Start of text	34	22	"	66	42	B	98	62	b
3	03	End of text	35	23	#	67	43	C	99	63	c
4	04	End of transmit	36	24	$	68	44	D	100	64	d
5	05	Enquiry	37	25	%	69	45	E	101	65	e
6	06	Acknowledge	38	26	&	70	46	F	102	66	f
7	07	Audible bell	39	27	'	71	47	G	103	67	g
8	08	Backspace	40	28	(	72	48	H	104	68	h
9	09	Horizontal tab	41	29	)	73	49	I	105	69	i
10	0A	Line feed	42	2A	*	74	4A	J	106	6A	j
11	0B	Vertical tab	43	2B	+	75	4B	K	107	6B	k
12	0C	Form feed	44	2C	,	76	4C	L	108	6C	l
13	0D	Carriage return	45	2D	-	77	4D	M	109	6D	m
14	0E	Shift out	46	2E	.	78	4E	N	110	6E	n
15	0F	Shift in	47	2F	/	79	4F	O	111	6F	o
16	10	Data link escape	48	30	0	80	50	P	112	70	p
17	11	Device control 1	49	31	1	81	51	Q	113	71	q
18	12	Device control 2	50	32	2	82	52	R	114	72	r
19	13	Device control 3	51	33	3	83	53	S	115	73	s
20	14	Device control 4	52	34	4	84	54	T	116	74	t
21	15	Neg. acknowledge	53	35	5	85	55	U	117	75	u
22	16	Synchronous idle	54	36	6	86	56	V	118	76	v
23	17	End trans. block	55	37	7	87	57	W	119	77	w
24	18	Cancel	56	38	8	88	58	X	120	78	x
25	19	End of medium	57	39	9	89	59	Y	121	79	y
26	1A	Substitution	58	3A	:	90	5A	Z	122	7A	z
27	1B	Escape	59	3B	;	91	5B	[	123	7B	{
28	1C	File separator	60	3C	<	92	5C	\	124	7C	|
29	1D	Group separator	61	3D	=	93	5D	]	125	7D	}
30	1E	Record separator	62	3E	>	94	5E	^	126	7E	~
31	1F	Unit separator	63	3F	?	95	5F	_	127	7F	□

UNICODE

ASCII only contains 127 characters (though an extended version exists with 257 characters).

This is by far not enough as it is too restrictive to the English language.

UNICODE was developed to alleviate this problem: the latest version, UNICODE 14.0 (September 2021) contains more than 140,000 characters, covering most existing languages.

For more information, see:

http://www.unicode.org/versions/Unicode14.0.0/

Digital Data

Sound: Sampling, and Quantitizing

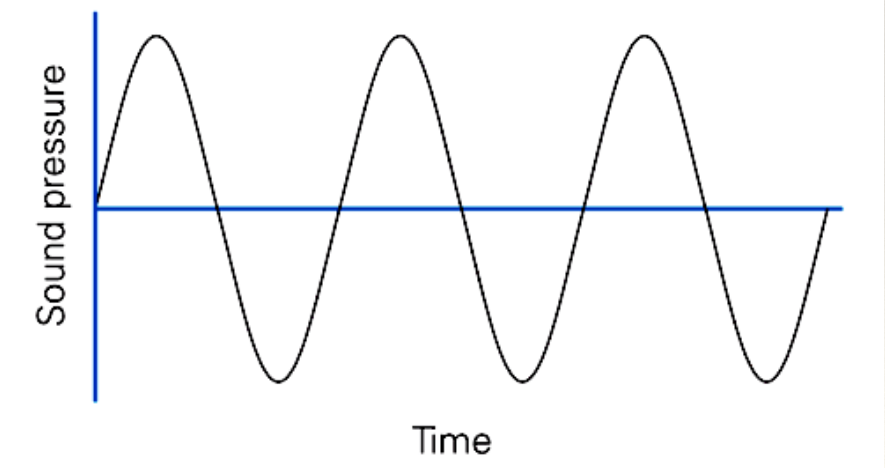
Digital Sound

Sound is produced by the vibration of a media like air or water. Audio refers to the sound within the range of human hearing.
Naturally, a sound signal is analog, i.e. continuous in both time and amplitude.

To store and process sound information in a computer or to transmit it through a computer network, we must first convert the analog signal to digital form using an analog-to-digital converter (ADC); the conversion involves two steps:

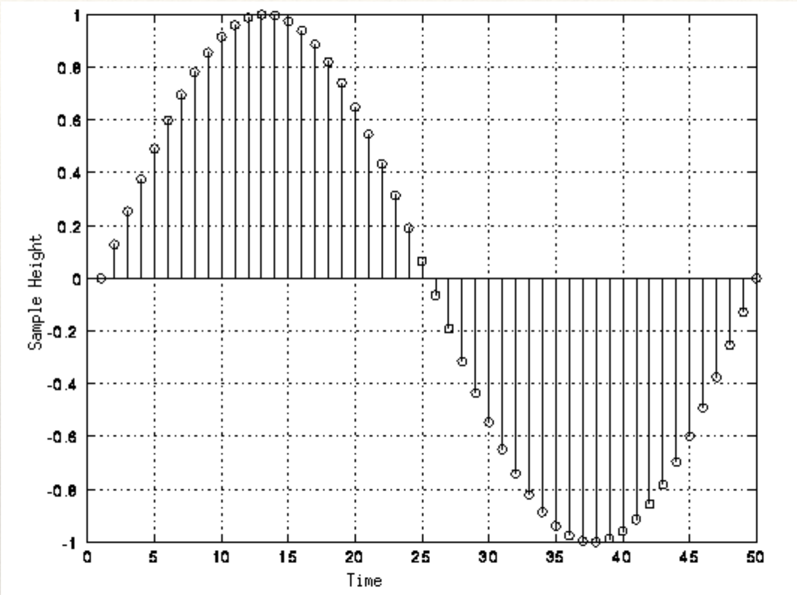
(1) sampling, and

(2) quantization.



Sampling

Sampling is the process of examining the value of a continuous function at regular intervals.

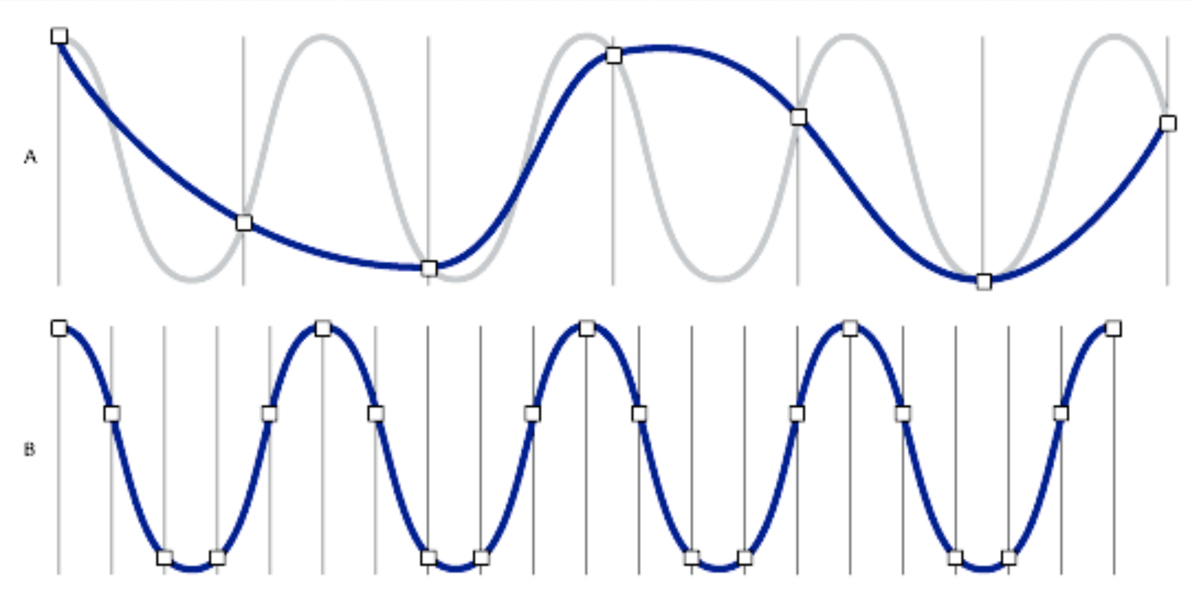


Sampling usually occurs at uniform intervals, which are referred to as sampling intervals. The reciprocal of sampling interval is referred to as the sampling frequency or sampling rate.

If the sampling is done in time domain, the unit of sampling interval is second and the unit of sampling rate is Hz, which means cycles per second.

Sampling

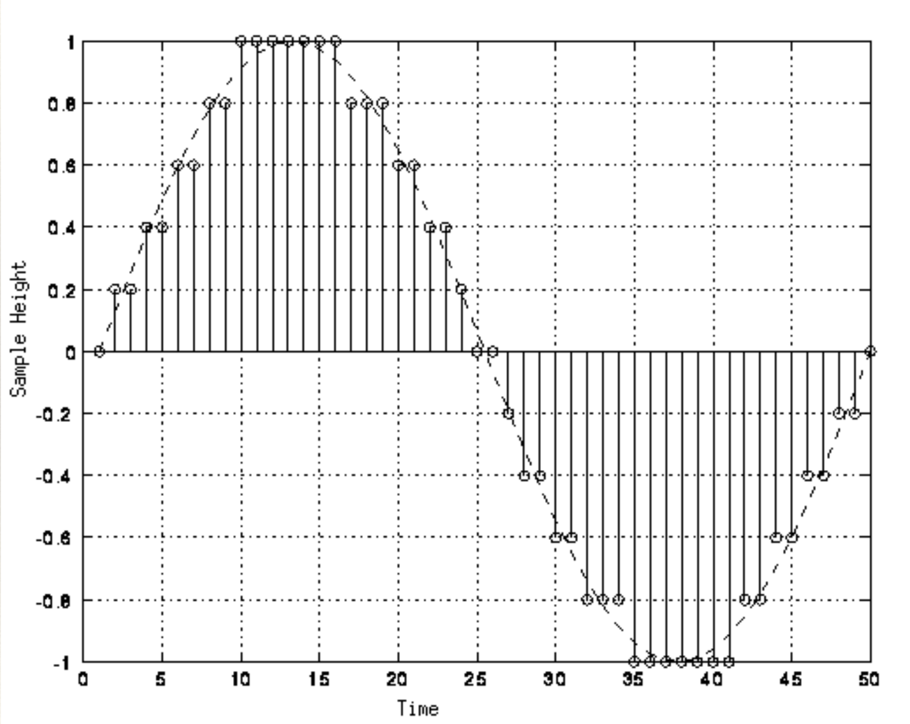
Note that choosing the sampling rate is not innocent:



A higher sampling rate usually allows for a better representation of the original sound wave. However, when the sampling rate is set to be **strictly greater than twice the highest frequency** in the signal, the original sound wave can be reconstructed without loss from the samples.
This is known as the Nyquist theorem.

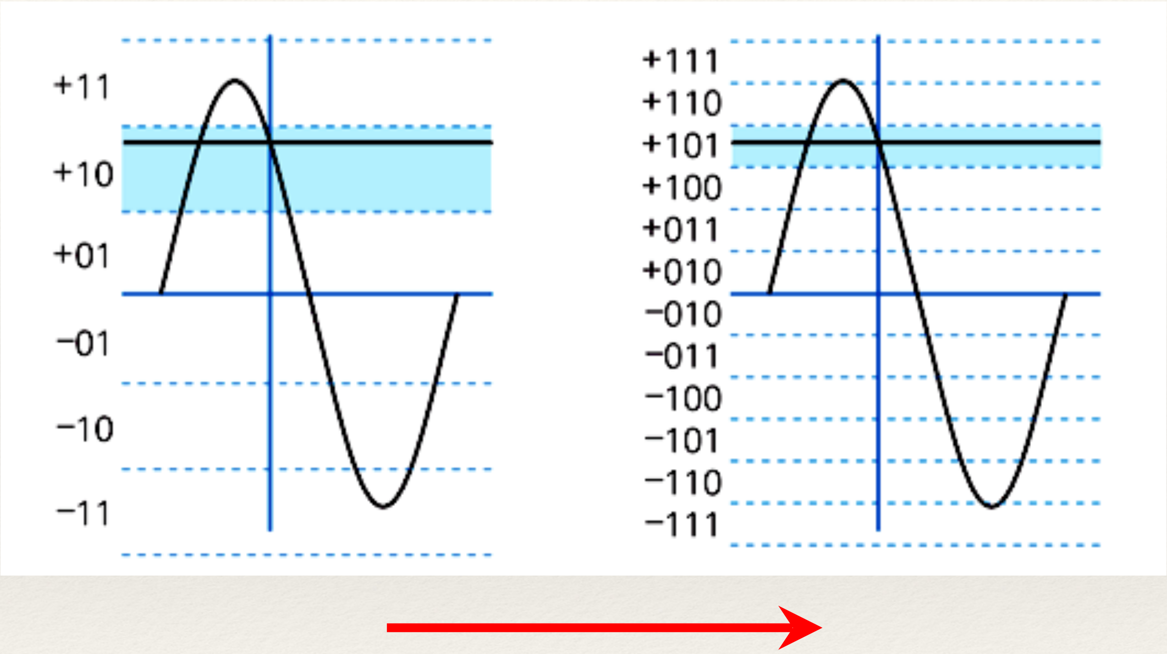
Quantization

Quantization is the process of limiting the value of a sample of a continuous function to one of a predetermined number of allowed values, which can then be represented by a finite number of bits.



Quantization

The number of bits used to store each intensity defines the accuracy of the digital sound:



Adding one bit makes the sample twice as accurate

Audio Sound

Sampling:

The human ear can hear sound up to 20,000 Hz: a sampling rate of 40,000 Hz is therefore sufficient. The standard for digital audio is 44,100 Hz.

Quantization:

The current standard for the digital representation of audio sound is to use 16 bits (i.e 65536 levels, half positive and half negative)

Audio Sound

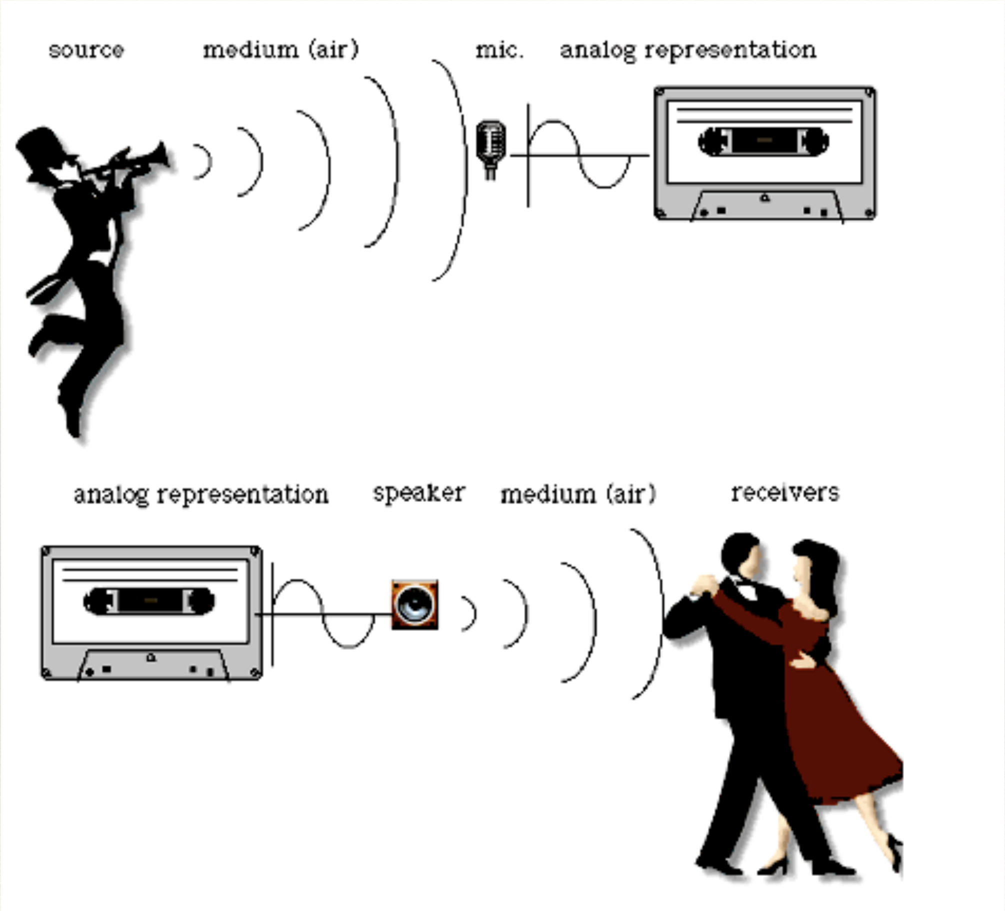
How much space do we need to store one minute of music?

- 60 seconds
- 44,100 samples
-16 bits (2 bytes) per sample
- 2 channels (stereo)

S = 60x44100x2x2 = 10,534,000 bytes ≈ 10 MB !!

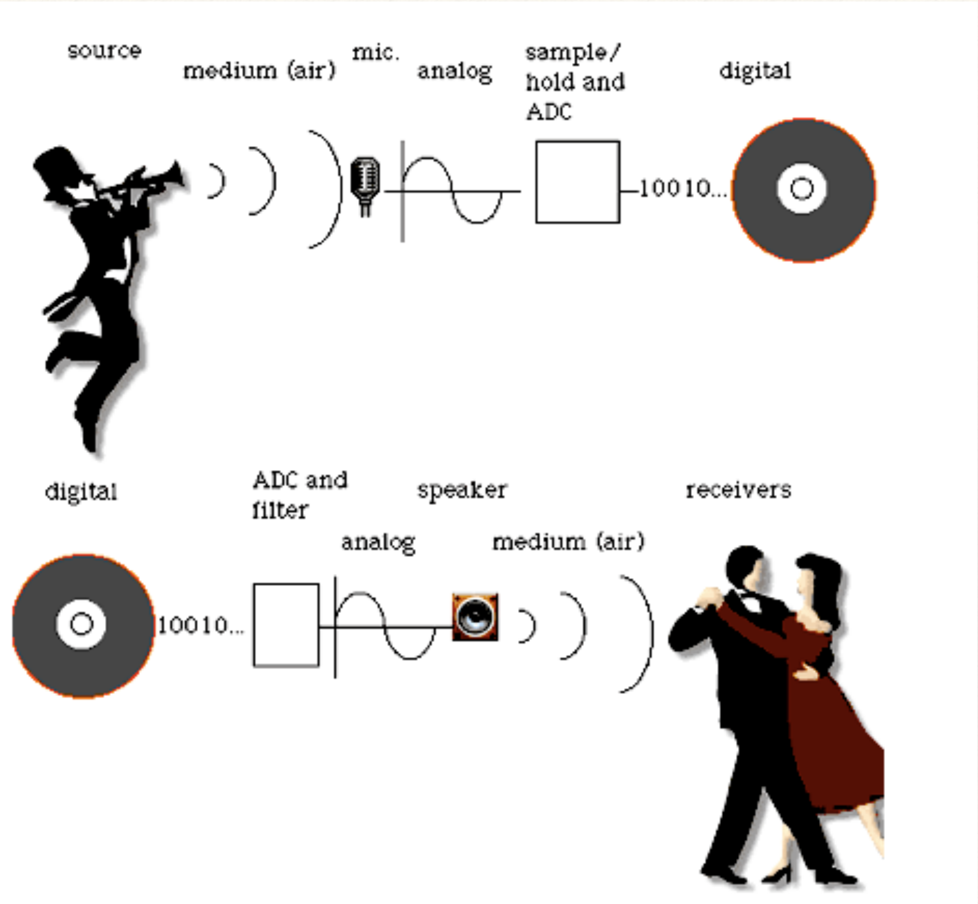
1 hour of music would be more than 600 MB !

Analog Recording



www.atpm.com/6.02/digitalaudio.shtml

DIGITAL RECORDING



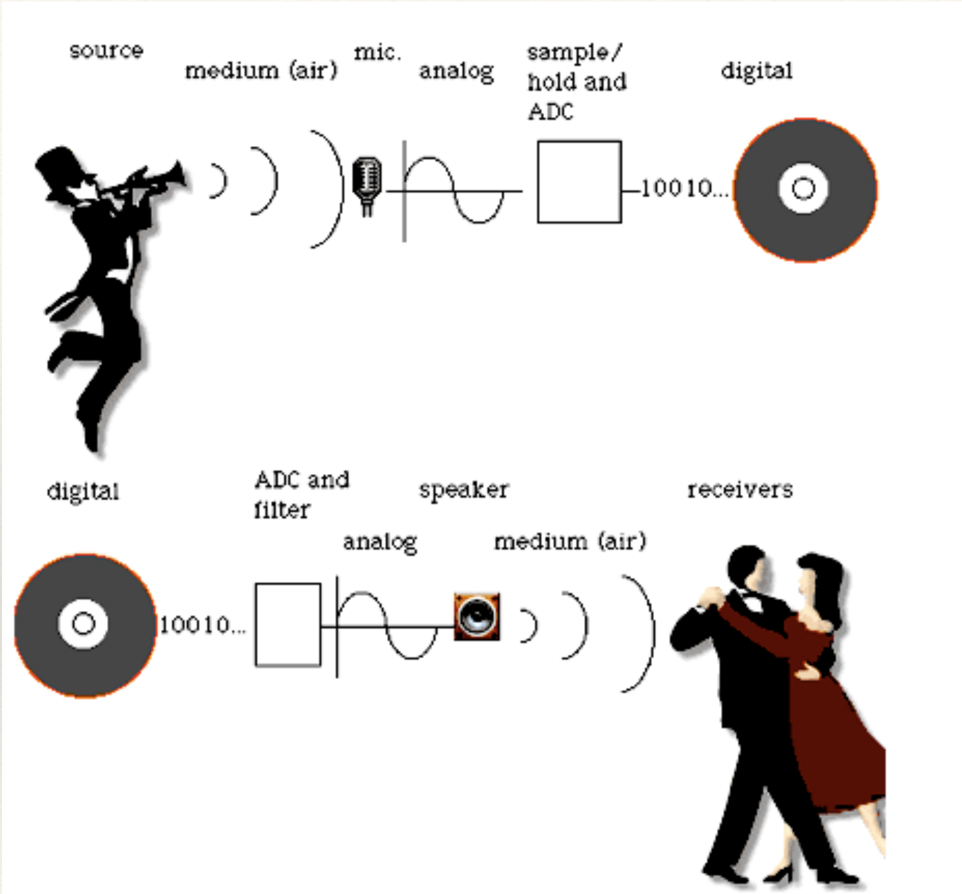
www.atpm.com/6.02/digitalaudio.shtml

DIGITAL RECORDING

Advantages of digital recording:

-Faithful
- can make multiple identical copies

-Can be processed
- compression (MP3)



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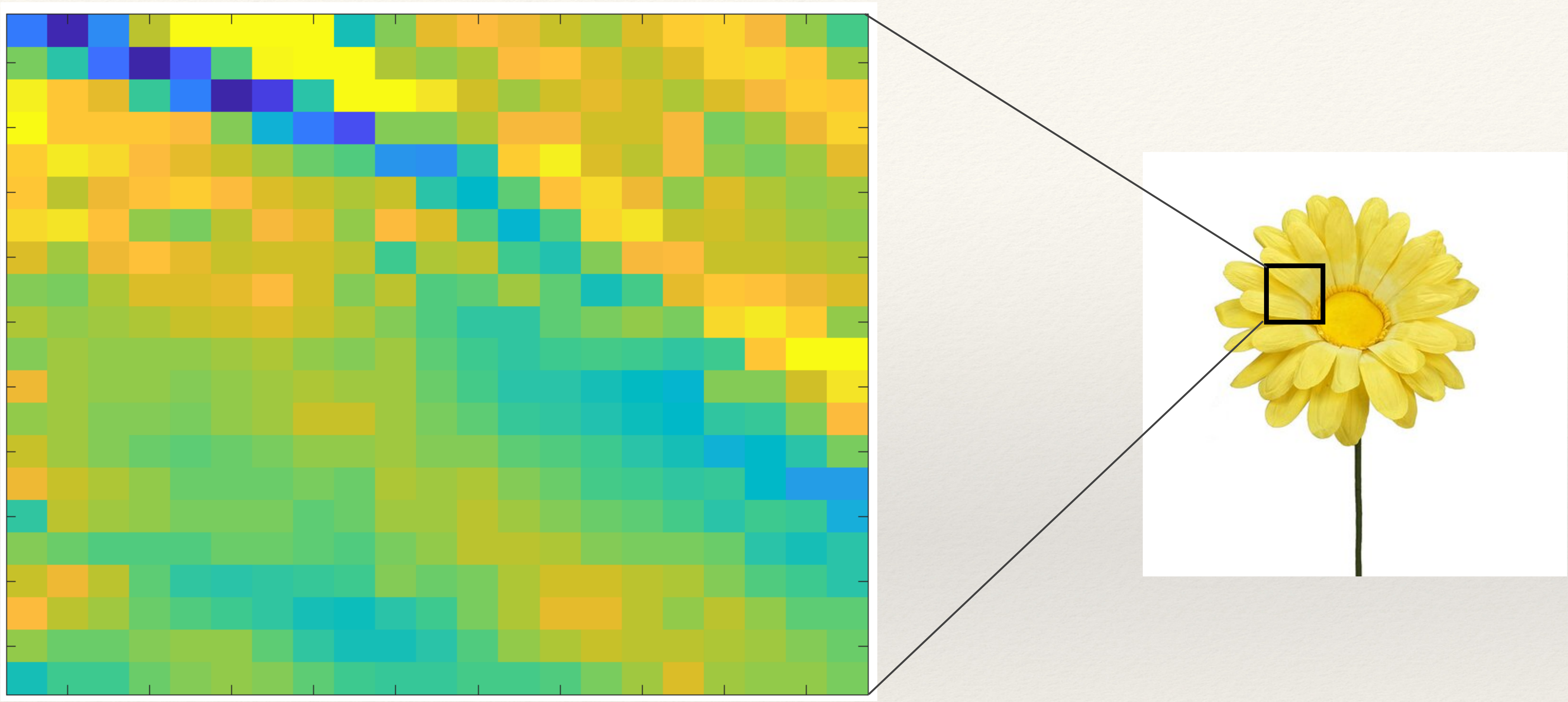
Digital Data

Images

Digital Images

Digital Images



Digital Images

Sampling:

Images are broken down into little squares: pixels
Resolution: Number of squares along each direction

Quantization:

Each pixel is characterized either as

- A binary number (0 or 1) to indicate black or white
- A natural number between 0 and 255, to indicate a gray scale
- - A set of three numbers, each between 0 and 255, to indicate the amount of Red (R), Green (G), and Blue (B)

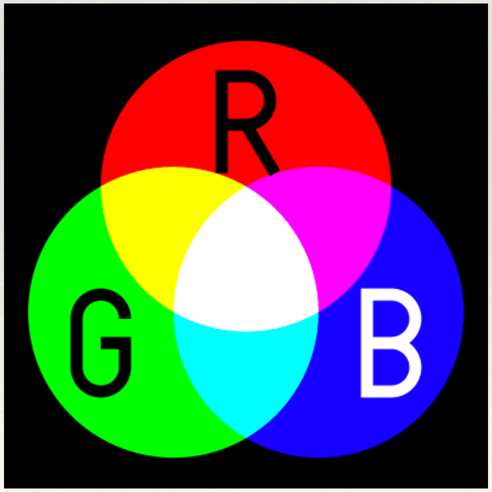
“True Color”: a pixel is represented by 24 bits, corresponding to 16,777,216 possible colors

Digital Images

The RGB color model (used for most digital representations of images)

Notation	RGB triplet
Arithmetic	(1.0, 0.0, 0.0)
Percentage	(100%, 0%, 0%)
Digital 8-bit per channel	(255, 0, 0) or sometimes #FF0000 (hexadecimal)

Mixing colors:



Digital Images

The CMYK color model (used by color printers)