I) Sets

a) Representation

\[ S = \{ 2, 4, 6, \ldots, 1000 \} \]

\[ S = \{ n \text{ integer} \mid 1 \leq n \leq 1000 \text{ and } n \text{ is even} \} \]

b) Domain

c) Vocabulary

- To indicate that an element \( x \) belongs to a set \( S \), we write \( x \in S \)
- \( n \in \mathbb{N} \)
- To indicate that a subset \( A \) belongs to a set \( B \), I write \( A \subset B \)

\[ S = \{ a, b, c \} \]

\[ a \in S \]

\[ \{ a \} \subset S \]
Some special subsets:

Intersection: Given two sets $A$ and $B$ in the same domain $D$, the elements of $D$ that belong to $A$ and $B$ form the intersection of $A$ and $B$: $A \cap B$.

Union: Given two sets $A$ and $B$ in the same domain $D$, the elements of $D$ that belong to $A$ or $B$ (or inclusion) form the union of $A$ and $B$: $A \cup B$.

Complement: Given a set $A$ in a domain $D$, the elements of $D$ that do not belong to $A$ form the complement of $A$ (in $D$): $\overline{A}$.
Symmetric difference: Given two sets $A$ and $B$ in the same domain $D$, the element of $D$ that are in $A$, $a$ in $B$ but not in both form the symmetric difference between $A$ and $B$. $A \Delta B$

Cartesian product: When the elements of a set are formed as a list of elements in other sets $A$ and $B$, we write that such an element belongs to the Cartesian product of $A$ and $B$: $A \times B$

Example: a pair of integers:

$$(a, b) \in \mathbb{Z} \times \mathbb{Z}$$
$$(a, b) \in \mathbb{N} \times \mathbb{R}$$
$$(a, b) \in \mathbb{Z}^2$$
\[ x \in \mathbb{R} \]

\[ P : (x, y, z) \in \mathbb{R}^3 \]

**Cardinality**

**Definition:** The number of elements in a finite set \( S \) is called the cardinality of the set and is written \( |S| \).

\[ S = \{ a, b, c \} \quad |S| = 3 \]

**Important properties:**

a) The cardinality of a Cartesian product \( A \times B \) is the product of the cardinality of \( A \) and \( B \):

\[ |A \times B| = |A| \cdot |B| \]

b) The inclusion-exclusion principle:

\[ |A \cup B| = |A| + |B| - |A \cap B| \]