Counting

1) The pigeonhole principle:

a) If \((k+1)\) objects are placed into \(k\) boxes, then there is (at least) one box that contains at least 2 objects.

b) If \(N\) objects are placed into \(k\) boxes, then there is (at least) one box that contains at least \(\left\lceil \frac{N}{k} \right\rceil\) objects.

Example #1

If you pick 3 integers randomly,
there is at least one pair of those integers whose difference is even.

Objects: 3 integers.
Boxes: \(\square\) \(\square\) odd, even.

One of the boxes will contain at least 2 elements.
The difference between those 2 elements will be even.
Example 2

If you pick 11 integers randomly, there is at least one pair of those integers whose difference is a multiple of 7.

Objects: 11 numbers

Boxes:

1 2 3 4 5 6 7 8 9

First digit

0

The pigeon hole principle tells us that at least, one of these boxes contain at least 2 elements.

Call \( n \) and \( m \) those 2 numbers.

\[ n = 10q + d \]
\[ m = 10p + d \]

\[ n - m = 10(q - p) \]
Example 3:

Consider 6 vertices. You connect all pairs of vertices with edges.

Each edge is colored, either in blue or in red.

There is at least one triangle whose edges are of the same color.

Objects

The five edges leaving $V_1$: 
5 objects: 5 edges leaving \( V_1 \)

2 boxes: red and blue.

There is (at least) one box that contains \( \sqrt{5/2} \) objects.

There is one box that contains 3 objects (at least).

There are 3 edges leaving \( V_1 \), that share the same color.

\[ \text{Option 1: } \quad A \; B \; \text{is of color } D \]

\[ \text{Option 2: } \quad A \; B \; \text{is of color } E. \]