**ECS 17: Data, Logic, and Computing**

**Final**

**March 20, 2023**

***Notes:***

1. The final is open book, open notes.
2. You have 2 hours, no more: I will strictly enforce this.
3. The final is graded over 100 points
4. Please, check your work! **Also, do show your work**

**Part I Data (10 questions, each 3 points; total 30 points)**

(These questions are multiple choices; in each case, find the most **plausible** answer)

1. ***What is the largest unsigned integer that can be stored on 2 bytes?***
	1. 256
	2. 255
	3. 65535
	4. 65536
2. ***Convert the binary number (1101101111110101)2 to hexadecimal***
	1. #DBF5
	2. #DCF5
	3. #5FBD
	4. #5FCD
3. ***(1110)2 –(101)2 =***
	1. #B
	2. #8
	3. #A
	4. #9
4. ***Which of these sampling rates would be appropriate for a sound sample of maximum frequency 16 kHz (circle all that apply)?***
	* + 1. 16000 Hz,
			2. 8000 Hz,
			3. 35000 Hz,
			4. 35 Hz.
5. ***Assume that you have taken a square picture with a 4 megapixel digital camera. Assume that you are printing this picture out on a printer that has approximately 4000 dots per inch. How big would the picture be? (note: 1 dot = 1 pixel)***
	* + 1. 1 inch x 1inch
			2. 2 inches x 2 inches
			3. 0.5 inch x 0.5 inch
			4. 4 inches x 4 inches
6. ***Which binary number comes right after the binary number 101?***
	1. 1100
	2. 111
	3. 102
	4. 110
7. ***Decode the name whose ASCII representation is #53 #61 #6C #6C #79***
8. Sally
9. Sylla
10. SALLY
11. SYLLA
12. ***The highest frequency note for a piano is fc=4200 Hz. Assuming that you record 1 hour of piano music with a sampling rate 3 times fc, in mono, with 16 bits resolution, what is the size of the resulting file (assuming 1MB = 1,000,000 bytes):***
	1. 0.9072 MB
	2. 90.72 MB
	3. 9.072 MB
	4. 181.44 MB
13. ***What time is it on this digital clock (filled circle mean “on”)?***

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1. 10:37
2. 10:41
3. 18:37
4. 18:41
5. ***You want to store an electronic copy of a book on your computer. This book contains 500 pages; each page contains (on average) 60 lines, and each line contains 60 characters (again, on average), including space. Each character needs 2 bytes of storage. How much space do you need to store this book (assuming 1MB = 1,000,000 bytes)?***
6. 3.6 MB
7. 36 MB
8. 0.36MB
9. 360 MB

**Part II Logic (three problems; total 30 points)**

1) For each of the five propositions in the table below, indicates on the right if they are always **tautologies or not** (*p* and *q* are propositions) ***(10 points)***.

|  |  |
| --- | --- |
| **Proposition** | **Tautology (Yes or No)** |
| If 2+6 = 5 then 10 = -9 |  |
| $$\left(p∨¬p\right)\rightarrow q$$ |  |
| $$\left(p∧¬q\right)\rightarrow p$$ |  |
| if 3+3 = 6 then 25=16+9 |  |
| $$\left(p∧¬p\right)∨\left(¬p∨p\right)$$ |  |

2) A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. You go to this island, as you have been told that a treasure may be buried on it. You meet two inhabitants, John, and Sally. John tells you that, `I am a knight if and only if the treasure is on the island.' Sally tells you that `If John is a knight, then the treasure is not on the island.’ Can it be determined if the treasure is on the island? Can it be determined also whether John is a knight or knave? What about Sally ? Justify your answers. ***(10 points)***

3) Let *p* and *q* be two propositions. Use a truth table or logical equivalence to show that the proposition $\left[¬\left(p\rightarrow ¬q\right)\right]\rightarrow \left[¬\left(p\leftrightarrow ¬q\right)\right] $is a tautology. ***(10 points)***

**Part III. Proofs (4 questions; each 10 points; total 40 points)**

1) Give a ***direct proof*** and a ***proof by contradiction*** of the proposition: if *2n3 + 3n2+4n+3* is odd, then *n* is even, where *n* is a natural number.

2) Use induction to prove that any postage value of *n* cents can be made with only 5-cent stamps and 6-cent stamps, whenever $n\geq 20$, *n* natural number.

3) Show by induction that for allnatural numbers n≥1
$$\sum\_{i=1}^{n}\left(-1\right)^{i}i^{2}=\frac{\left(-1\right)^{n}n\left(n+1\right)}{2} $$

4) Let *fn* be the *n*-th Fibonacci number (note: Fibonacci numbers satisfy *f0=0, f1=1* and ). Prove by induction that for all natural numbers $n\geq 3$,

$$\frac{f\_{1}}{f\_{2}f\_{3}}+\frac{f\_{2}}{f\_{3}f\_{4}}+\cdots +\frac{f\_{n-2}}{f\_{n-1}f\_{n}}=1-\frac{1}{f\_{n}}$$

Appendix A: ASCII table



Appendix B: Binary to Hexadecimal

|  |  |  |
| --- | --- | --- |
| Base 10 | Base 2 | Base 16 |
| 0 | 0000 | 0 |
| 1 | 0001 | 1 |
| 2 | 0010 | 2 |
| 3 | 0011 | 3 |
| 4 | 0100 | 4 |
| 5 | 0101 | 5 |
| 6 | 0110 | 6 |
| 7 | 0111 | 7 |
| 8 | 1000 | 8 |
| 9 | 1001 | 9 |
| 10 | 1010 | A |
| 11 | 1011 | B |
| 12 | 1100 | C |
| 13 | 1101 | D |
| 14 | 1110 | E |
| 15 | 1111 | F |

**Appendix C**

