*Name:*___________________________________
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ECS 17: Data, Logic, and Computing
Final
March 20, 2023

Notes:

1) The final is open book, open notes.
2) You have 2 hours, no more: I will strictly enforce this.
3) The final is graded over 100 points
4) Please, check your work! **Also, do show your work**

Part I Data (10 questions, each 3 points; total 30 points)
(These questions are multiple choices; in each case, find the most **plausible** answer)

1) ***What is the largest unsigned integer that can be stored on 2 bytes?***
   a. 256
   b. 255
   c. 65535
   d. 65536

2) ***Convert the binary number $(1101101111110101)_2$ to hexadecimal***
   a. #DBF5
   b. #DCF5
   c. #5FBD
   d. #5FCD

3) ***$(1110)_2 - (101)_2 =$***
   a. #B
   b. #8
   c. #A
   d. #9

4) ***Which of these sampling rates would be appropriate for a sound sample of maximum frequency 16 kHz (circle all that apply)?***
   a. 16000 Hz,
   b. 8000 Hz,
   c. 35000 Hz,
   d. 35 Hz.

5) ***Assume that you have taken a square picture with a 4 megapixel digital camera. Assume that you are printing this picture out on a printer that has approximately 4000 dots per inch. How big would the picture be? (note: 1 dot = 1 pixel)***
   a. 1 inch x 1inch
   b. 2 inches x 2 inches
   c. 0.5 inch x 0.5 inch
   d. 4 inches x 4 inches

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6) Which binary number comes right after the binary number 101?

a. 1100
b. 111
c. 102
d. 110

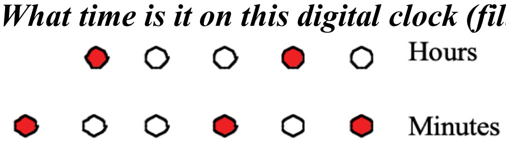
7) Decode the name whose ASCII representation is #53 #61 #6C #6C #79

a. Sally
b. Sylla
c. SALLY
d. SYLLA

8) The highest frequency note for a piano is fc=4200 Hz. Assuming that you record 1 hour of piano music with a sampling rate 3 times fc, in mono, with 16 bits resolution, what is the size of the resulting file (assuming 1MB = 1,000,000 bytes):

a. 0.9072 MB
b. 90.72 MB
c. 9.072 MB
d. 181.44 MB

9) What time is it on this digital clock (filled circle mean "on")?



a. 10:37
b. 10:41
c. 18:37
d. 18:41

10) You want to store an electronic copy of a book on your computer. This book contains 500 pages; each page contains (on average) 60 lines, and each line contains 60 characters (again, on average), including space. Each character needs 2 bytes of storage. How much space do you need to store this book (assuming 1MB = 1,000,000 bytes)?

a. 3.6 MB
b. 36 MB
c. 0.36MB
d. 360 MB

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Part II Logic (three problems; total 30 points)

1) For each of the five propositions in the table below, indicates on the right if they are always **tautologies or not** (p and q are propositions) ***(10 points)***.

Proposition	**Tautology (Yes or No)**
If 2+6 = 5 then 10 = -9	
$(p \vee \neg p) \rightarrow q$	
$(p \wedge \neg q) \rightarrow p$	
if 3+3 = 6 then 25=16+9	
$(p \wedge \neg p) \vee (\neg p \vee p)$	

2) A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. You go to this island, as you have been told that a treasure may be buried on it. You meet two inhabitants, John, and Sally. John tells you that, \`I am a knight if and only if the treasure is on the island.' Sally tells you that \`If John is a knight, then the treasure is not on the island.' Can it be determined if the treasure is on the island? Can it be determined also whether John is a knight or knave? What about Sally ? Justify your answers. ***(10 points)***

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3) Let p and q be two propositions. Use a truth table or logical equivalence to show that the proposition $[\neg(p \rightarrow \neg q)] \rightarrow [\neg(p \leftrightarrow \neg q)]$ is a tautology. ***(10 points)***

Part III. Proofs (4 questions; each 10 points; total 40 points)

1) Give a ***direct proof*** and a ***proof by contradiction*** of the proposition: if $2n^3 + 3n^2+4n+3$ is odd, then n is even, where n is a natural number.

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2) Use induction to prove that any postage value of n cents can be made with only 5-cent stamps and 6-cent stamps, whenever $n \geq 20$, n natural number.

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3) Show by induction that for all natural numbers $n \geq 1$

$$\sum_{i=1}^{n} (-1)^i i^2 = \frac{(-1)^n n(n+1)}{2}$$

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4) Let f_n be the n-th Fibonacci number (note: Fibonacci numbers satisfy $f_0=0$, $f_1=1$ and $f_n + f_{n+1} = f_{n+2}$). Prove by induction that for all natural numbers $n \geq 3$,

$$\frac{f_1}{f_2 f_3} + \frac{f_2}{f_3 f_4} + \cdots + \frac{f_{n-2}}{f_{n-1} f_n} = 1 - \frac{1}{f_n}$$

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Appendix A: ASCII table

Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char
0	00	Null	32	20	Space	64	40	@	96	60	`
1	01	Start of heading	33	21	!	65	41	A	97	61	a
2	02	Start of text	34	22	"	66	42	B	98	62	b
3	03	End of text	35	23	#	67	43	C	99	63	c
4	04	End of transmit	36	24	$	68	44	D	100	64	d
5	05	Enquiry	37	25	%	69	45	E	101	65	e
6	06	Acknowledge	38	26	&	70	46	F	102	66	f
7	07	Audible bell	39	27	'	71	47	G	103	67	g
8	08	Backspace	40	28	(	72	48	H	104	68	h
9	09	Horizontal tab	41	29	)	73	49	I	105	69	i
10	0A	Line feed	42	2A	*	74	4A	J	106	6A	j
11	0B	Vertical tab	43	2B	+	75	4B	K	107	6B	k
12	0C	Form feed	44	2C	,	76	4C	L	108	6C	l
13	0D	Carriage return	45	2D	-	77	4D	M	109	6D	m
14	0E	Shift out	46	2E	.	78	4E	N	110	6E	n
15	0F	Shift in	47	2F	/	79	4F	O	111	6F	o
16	10	Data link escape	48	30	0	80	50	P	112	70	p
17	11	Device control 1	49	31	1	81	51	Q	113	71	q
18	12	Device control 2	50	32	2	82	52	R	114	72	r
19	13	Device control 3	51	33	3	83	53	S	115	73	s
20	14	Device control 4	52	34	4	84	54	T	116	74	t
21	15	Neg. acknowledge	53	35	5	85	55	U	117	75	u
22	16	Synchronous idle	54	36	6	86	56	V	118	76	v
23	17	End trans. block	55	37	7	87	57	W	119	77	w
24	18	Cancel	56	38	8	88	58	X	120	78	x
25	19	End of medium	57	39	9	89	59	Y	121	79	y
26	1A	Substitution	58	3A	:	90	5A	Z	122	7A	z
27	1B	Escape	59	3B	;	91	5B	[	123	7B	{
28	1C	File separator	60	3C	<	92	5C	\	124	7C	|
29	1D	Group separator	61	3D	=	93	5D	]	125	7D	}
30	1E	Record separator	62	3E	>	94	5E	^	126	7E	~
31	1F	Unit separator	63	3F	?	95	5F	_	127	7F	□

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Appendix B: Binary to Hexadecimal

Base 10	Base 2	Base 16
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

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Appendix C

The ECS17 Potato Prayers

1) Thou shalt not say "there exists k" without mentioning the domain of k.

2) Thou shalt not say "it is obvious"

3) If p and q are two propositions, then $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$. This is the basis for the proof by contrapositive.

3) If p and q are two propositions, then $p \rightarrow q \Leftrightarrow \neg p \vee q$. This is the basis for the proof by contradiction.

4) An integer n is even if and only if there exists and integer k such that $n = 2k$. We say also that n is a multiple of 2.

5) An integer n is odd if and only if there exists and integer k such that $n = 2k + 1$.

6) BEWARE of divisions and square roots when you are working with integers.

Proofs that you can use without proving them again

We can use the following results without having to validate them:

1) Let n be an integer. Then:
   a) If n is even, then $n + 1$ is odd
   b) if n is odd, then $n + 1$ is even

2) Let n be an integer. Then:
   a) n is even, if and only if n^2 is even
   b) n is odd, if and only if n^2 is odd

3) $\forall n \in \mathbb{Z}, \quad n(n + 1)$ is even.

4) $\sqrt{2}$ is irrational.